# **Soft Banach Algebra: Theory and Applications**



## **1 . Introduction**

 In topology, compact spaces and KC-spaces are of great importance in mathematics and applied sciences. Understanding the properties of these spaces provides a strong foundation for developing theories and solving problems in a wide range of fields, from pure mathematical analysis to practical applications in engineering and sciences .The definition of  $\mathcal{K}c$  – space (which every compact subset is closed)was presented by  $[1]$  and new concepts were introduced through the definition of the following topological spaces  $\mathcal{K}(\alpha c)$  – spaces (which every compact subset is  $\alpha$  – closed),  $\alpha \mathcal{K}(\alpha c)$  – spaces (which every  $q -$  compact subset is  $q -$ closed) by S. K. Jassim and H. G. Ali<sup>[2]</sup>. In this research work, the aim was to introduce new concepts of spaces, which is named  $g(Kc)$  –spaces . New definitions were also introduced, which are On Weaker Forms of  $g(\mathcal{K}c)$  –spaces and Co–g – compact topologies.

 A soft Banach algebra is a mathematical structure that combines elements of both Banach algebras and fuzzy sets. In a traditional Banach algebra, operations like addition and multiplication are defined in a precise, deterministic manner. Soft Banach algebras, on the other hand, introduce a degree of fuzziness or uncertainty in these operations.

In a soft Banach algebra, elements are associated with fuzzy sets, which assign degrees of membership to points in a given set. The operations of addition and multiplication are then extended to operate on these fuzzy sets in a way that respects the underlying algebraic structure.

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 This concept finds applications in areas where uncertainty or imprecision play a significant role, such as in fuzzy mathematics, decision making, and optimization problems. Soft Banach algebras provide a framework to model and analyze situations where exact values are not always available or applicable.

## **2. Soft Algebras**

The concept of soft set theory has been initiated by Molodtsov in 1999 as a general mathematical tool for modeling uncertainties. He also pointed out several application of this theory solving many practical problems in economics, engineering, social sciences, medical sciences etc. the<br>maid tool many practical  $P(X)$  denote eory has been initiated by Molodtsov in 1993.<br>*A* E also pointed out several application of the ngineering, social sciences, medical sciences at *X* be an initial universe set and *E* be the set  $A \subseteq E$ .

Throughout the lecture, let  $X$  be an initial universe set and  $E$  be the set of parameters.  $P(X)$  denote the power set of  $X$  and  $A \subseteq E$ .

## **Definition (2.1)**

A pair from general finities. The diso pointed out<br>
in sin economics, engineering, social science hout the lecture, let X be an initial univer<br>
ver set of X and  $A \subseteq E$ .<br> **ion (2.1)**<br>
(*F,A*) denoted by  $F_A$  is called a soft set<br> denoted by  $F_A$  is called a soft set over X, where F is a function given by  $F : A \to P(X)$ . In Introduction the set of the set of the samplicable.<br> **F** : A refultion the set of the se other words the soft set over *X* is a parameterized family of subsets of the universal set *X* . For a particular **Definition (2.1)**<br>
A pair (*F*,*A*) denoted by  $F_A$  is called a soft<br>
other words the soft set over *X* is a paramet<br>  $e \in A$ ,  $F(e)$  may be considered the set of  $e$ <br>  $F(e) = \phi$ , i.e.  $F_A = (F, A) = {F(e) \in P(X) : e}$ may be considered the set of  $e$  - approximate elements of the soft set  $(F, A)$  and if  $e \notin A$ , then ers.  $P(X)$  denote<br>ven by  $F : A \rightarrow P(X)$ . In<br>al set X. For a particular<br> $(F, A)$  and if  $e \notin A$ , then  $\rightarrow$  *P*(*X*). In<br> *e* a particular<br> *e*  $\notin$  *A*, then Fhroughout the lecture, let *X* be an ini<br>*he* power set of *X* and  $A \subseteq E$ .<br>**Definition (2.1)**<br>A pair (*F*,*A*) denoted by *F<sub>A</sub>* is called<br>other words the soft set over *X* is a pa<br> $e \in A$ , *F*(*e*) may be considered the  $F(e) = \phi$ , i.e.  $F_A = (F, A) = \{F(e) \in P(X) : e \in A\}$ urnal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
finds applications in areas where uncertainty or imprecision play a significant role, such as<br>
ematics, decision making, and optimization proble set over *X*, where *F* is a function<br>prized family of subsets of the univapproximate elements of the soft<br> $\in A$ <br> $S(X)$ , and called soft Power Set. *F* a parameterized ramity of subsets of<br>  $f(e) = P(X)$ :  $e \in A$ }<br> *F* (*X*):  $e \in A$ }<br> *F* (*e*) =  $\phi$  for any  $e \in A$ .<br> *F* (*e*) =  $\phi$  for any  $e \in A$ .<br> *F* if *F*(*e*) = *X* for any  $e \in A$ .  $(X): e \in A$ <br> *Figure 44* (*K*), and called soft Power Set.<br>  $= \phi$  for any  $e \in A$ .<br>  $F(e) = X$  for any  $e \in A$ .<br>  $e \in A$  such that  $A(e) \neq \phi$ .

The set of all soft sets over  $X$  is denoted by  $S(X)$ , and called s , and called soft Power Set.<br> $e \in A$ . *A* called soft Power Set.<br> *A* .<br>  $\forall e \in A$ .<br>  $A(e) \neq \phi$ .

## **Definition (2.2)**

A soft set  $F_A$  over X is said to be

- 1. Null soft set, denoted by  $\phi$  if  $F(e) = \phi$  for any  $e \in A$ .
- 2. Absolute soft set, denoted by *X* if  $F(e) = X$  for any  $e \in A$ .  $e \in A$ .
- 3. Non null soft set if there is at least  $e \in A$  such that  $A(e) \neq \emptyset$ .  $= \phi$  for any  $e \in A$ .<br>  $F(e) = X$  for any  $e \in A$ .<br>  $e \in A$  such that  $A(e) \neq \phi$ .

## **Definition (2.3)**

Let  $F_A, G_B \in S(X)$ . we say that

- $=\phi$ , i.e.  $F_A = (F, A) = \{F(e) \in P(X) : e \in A\}$ <br>set of all soft sets over X is denoted by  $S(X)$ , an<br>**nition (2.2)**<br>ff set  $F_A$  over X is said to be<br>ull soft set, denoted by  $\phi$  if  $F(e) = \phi$  for any  $e \in$ <br>solute soft set, denoted 1.  $F_A$  and  $G_B$  are soft equal (or  $F_A$  soft equals  $G_B$ ), which we write as  $F_A = G_B$ , if  $A = B$  $F_A = G_B$ , if  $A = B$  $A = B$ soft set, denoted by  $\phi$  if  $F(e) = \phi$  for<br>lute soft set, denoted by *X* if  $F(e) =$ <br>null soft set if there is at least  $e \in A$  s<br>ion (2.3)<br> $G_B \in S(X)$ . we say that<br>d  $G_B$  are soft equal (or  $F_A$  soft equals<br> $F(e) = G(e)$  for all *e* and *e* a soft equals  $G_B$  *e* soft equals  $G_B$  *e* which we write as *P* and *e*  $\in$  *A* . *G<sub>B</sub>*), which we write as  $F_A = G_B$ , if  $A = B$ <br> $F_A \subseteq G_B$  if  $A \subseteq B$  and  $F(e) \subseteq G(e)$  for all  $e \in A$ <br>iff  $F_A \subseteq G_B$  and  $G_B \subseteq F_A$ *F* equals  $G_B$ , which we write as  $F_A = G_B$ , if  $A =$ <br>oted by  $F_A \subseteq G_B$  if  $A \subseteq B$  and  $F(e) \subseteq G(e)$  for<br> $F_A = G_B$  iff  $F_A \subseteq G_B$  and  $G_B \subseteq F_A$ <br>and denoted by  $F_A \subset G_B$  if  $A \subset B$  and  $F(e) \subset$
- and  $F(e) = G(e)$  for all
- 2.  $F_A$  is a soft subset of  $G_B$ , and denoted by  $F_A \subseteq G_B$  if  $A \subseteq B$  and  $F(e) \subseteq G(e)$  for all we write as  $F_A = G_B$ , if  $A = B$ <br> $A \subseteq B$  and  $F(e) \subseteq G(e)$  for all  $e \in A$ .<br>and  $G_p \subset F$ .  $F_A = G_B$ , if  $A = B$ <br> $F(e) \subseteq G(e)$  for all  $e \in A$ .  $e \in A$  .

Hence 
$$
F_A = G_B
$$
 iff  $F_A \subseteq G_B$  and  $G_B \subseteq F_A$ 

), which we write as  $F_A = G_B$ , if  $A = B$ <br>  $\subseteq G_B$  if  $A \subseteq B$  and  $F(e) \subseteq G(e)$  for all  $e \in A$ .<br>  $F_A \subseteq G_B$  and  $G_B \subseteq F_A$ <br>  $\exists$  by  $F_A \subset G_B$  if  $A \subset B$  and  $F(e) \subset G(e)$  for rite as  $F_A = G_B$ , if  $A = B$ <br> *B* and  $F(e) \subseteq G(e)$  for all  $e \in A$ .<br>  $G_B \subseteq F_A$ <br>
if  $A \subset B$  and  $F(e) \subset G(e)$  for 3.  $F_A$  is a soft proper subset of  $G_B$ , and denoted by  $F_A \subset G_B$  if  $A \subset B$  and  $F(e) \subset G(e)$  for ch we write as  $F_A = G_B$ , if  $A = B$ <br>
if  $A \subseteq B$  and  $F(e) \subseteq G(e)$  for all  $e \in A$ .<br>  $G_B$  and  $G_B \subseteq F_A$ <br>  $F_A \subset G_B$  if  $A \subset B$  and  $F(e) \subset G(e)$  for as  $P_A - O_B$ ,  $H_A - B$ <br>
and  $F(e) \subseteq G(e)$  for all  $e \in A$ .<br>  $\subseteq F_A$ <br>  $A \subset B$  and  $F(e) \subset G(e)$  for if  $A = B$ <br> *F*(*e*) for all  $e \in A$ .<br> *F*(*e*)  $\subset$  *G*(*e*) for all  $e \in A$ . is a sort subset of  $G_B$ , and denoted by  $F_A$ <br>
Hence  $F_A = G_B$  iff<br>
is a soft proper subset of  $G_B$ , and denoted<br>  $e \in A$ .

### **Definition (2.4)**

A soft set  $F_A$  over X is called

1.A soft point and its denoted by  $P_e = (e, F(e))$ , if exactly one arizmi (JIKh) Volume:8 Issue:2 Year: 202<br>  $x^x = \{(e, F(e))\}$ , if exactly one  $e \in A$ ,  $F(e)$ <br>
all  $y \in A | \{e\}$ .<br>
X such that  $F(e) = \{x\}$  for all  $e \in A$ . warizmi (JIKh) Volume:8 Issue:2 Year<br>  $p_e^x = \{(e, F(e))\}$ , if exactly one  $e \in A$ <br> *r* all  $y \in A | \{e\}$ .<br>  $\in X$  such that  $F(e) = \{x\}$  for all  $e \in A$ 2.4)<br>
<sup>4</sup> over *X* is called<br>
and its denoted by  $P_e^x = \{(e, F(e))\}$ , if ex<br>  $x \in X$  and  $F(y) = \phi$  for all  $y \in A | \{e\}$ . Iraqi Al-Khwarizmi (JIKh) Volume:8<br> *S* called<br> *F*(*y*) =  $\phi$  for all  $y \in A \setminus \{e\}$ <br> *F*(*y*) =  $\phi$  for all  $y \in A \setminus \{e\}$ <br> *F* there is  $x \in X$  such that  $F(e) = \{x\}$ *zmi* (*JIKh*) Volume:8 Issue:2 Year: 2024 pa<br>  $\{(e, F(e))\}$ , if exactly one  $e \in A$ ,  $F(e) = \{x \}$ <br>  $y \in A | \{e\}$ .<br>
Such that  $F(e) = \{x\}$  for all  $e \in A$ .  $y^{p_e} = \{(e, F(e))\}$ , if exactly one  $e \in A$ ,  $F$ <br>for all  $y \in A | \{e\}$ .<br> $x \in X$  such that  $F(e) = \{x\}$  for all  $e \in A$ . For all  $P$  is the *F* (*e*) = {*x* } if exactly one  $e \in A$ ,  $F(e) = \{x\}$ <br>  $F(e) = \{x\}$  for all  $e \in A$ .  $\in A$ ,  $F(e) = \{x\}$ <br> $e \in A$ .

for some  $x \in X$  and  $F(y) = \varphi$  for all  $y \in A \setminus \{e\}$ .

2. A singleton soft set if there is  $x \in X$  such that  $f'(e) = \{x \}$  for all

#### **Definition (2.5)**

Let  $F_A \in S(X)$ . An element  $x \in X$  is said to be belongs to the soft set  $F_A$  over X, denoted by  $x \in F_A$  if ft set  $F_A$  over  $X$  is called<br>soft point and its denoted by  $P_e^x = \{(e, F(e))$ <br>or some  $x \in X$  and  $F(y) = \phi$  for all  $y \in A \mid \{e\}$ <br>singleton soft set if there is  $x \in X$  such that<br>**nition (2.5)**<br> $F_A \in S(X)$ . An element  $x \in X$  is  $\neq \phi$  for all  $y \in A | \{e\}$ .<br>  $\phi$  is  $x \in X$  such that  $F(e) = \{x\}$  for all  $e \in A$ <br>  $x \in X$  is said to be belongs to the soft set  $F_A$ <br>  $\phi$  ar words, we say that  $x \in F_A$  read as  $X$  belon  $x \in F_A$  if 1.A soft point and its denoted by  $P_e = \{(e, e, e) \}$ <br>for some  $x \in X$  and  $F(y) = \phi$  for all  $y \in A$ <br>2. A singleton soft set if there is  $x \in X$  such<br>**Definition (2.5)**<br>Let  $F_A \in S(X)$ . An element  $x \in X$  is said to<br> $x \in F(e)$  for all for all  $e \in A$ . In other words, we say that  $x \in F_A$  read as  $x$  belongs to the soft set  $F_A$ oft set if there is  $x \in X$  such that  $F(e) = \{x \}$ <br>An element  $x \in X$  is said to be belongs to th<br> $e \in A$ . In other words, we say that  $x \in F_A$  rea<br>(e) for all  $e \in A$ .  $(e) = \{x\}$  for all  $e \in A$ .<br>
parameters of the soft set  $F_A$  over  $X$ , denoted by  $x \in F_A$  read as  $X$  belongs to the soft set  $F_A$ whenever  $x \in F(e)$  for all *x*  $\in$  *x* and  $F(y) = \phi$  for all  $y \in A | \{e\}$ .<br> *ton soft set if there is*  $x \in X$  such that  $F(e)$ <br>
(2.5)<br>
(*X*). An element  $x \in X$  is said to be belongs<br>
or all  $e \in A$ . In other words, we say that  $x \in I$ <br>  $x \in F(e)$  for al *e e x* is said to be belongs to the soft set *i* ther words, we say that  $x ∈ F_A$  read as *x* bel  $e ∈ A$ . *x* ∈ *X* is said to be belongs to the soft set  $\in$  *A*. In other words, we say that  $x \in F_A$  read as  $\infty$  b  $\in$  *N* or all  $e \in A$ .<br>*x* ∈ *X*,  $x \notin F_A$ , if  $x \notin F(e)$  for some  $e \in A$ . *z x* such that  $F(e) = \{x\}$  for all  $e \in A$ .<br>*is said to be belongs to the soft set*  $F_A$  over .<br>*ds*, we say that  $x \in F_A$  read as  $x$  belongs to to  $x \notin F(e)$  for some  $e \in A$ . *e* a to the soft set  $F_A$  over  $X$ , denoted by  $x \in F_A$ <br> *e*  $F_A$  read as  $X$  belongs to the soft set  $F_A$ <br>  $e \in A$ . An element  $x \in X$  is said to be belongs to the soft set  $F_A$  over  $\lambda e \in A$ . In other words, we say that  $x \in F_A$  read as  $x$  belongs to the soft of  $\lambda$  for all  $e \in A$ .<br>  $y \in X$ ,  $x \notin F_A$ , if  $x \notin F(e)$  for some  $e \in A$ .<br>
Exerc  $x \in F(e)$  for all  $e \in A$ . In other words, we say t<br>whenever  $x \in F(e)$  for all  $e \in A$ .<br>Note that for any  $x \in X$ ,  $x \notin F_A$ , if  $x \notin F(e)$  for<br>**Definition (2.6)**<br>Let X be a nonempty set and A be a nonempty<br>1. The function  $\varepsilon : A$ 

Note that for any , if  $x \notin F(e)$  for some

### **Definition (2.6)**

Let  $X$  be a nonempty set and  $A$  be a nonempty parameter set.

1. The function  $\varepsilon: A \to X$  is said to be a soft element of *X* .

2. A soft element  $\varepsilon$  of is said to belongs to a soft set B of X, which is denoted by  $\varepsilon \in \overline{B}$ , if for all  $e \in B$  if. pty set and *A* be a nonempty parameter set.<br>  $A \rightarrow X$  is said to be a soft element of *X*.<br>
of is said to belongs to a soft set *B* of *X*, v<br>  $e \in B$  if

#### **Definition (2.7)**

Let  $\Box$  be the set of real numbers and  $B(\Box)$  be the collection of all nonempty bounded subsets of and ngs to a soft set *B* of *X*<br> $B(\Box)$  be the collection of<br>ction  $F : A \rightarrow B(\Box)$  is ca *A* taken as a set of parameters. The function  $F : A \to B(\square)$  is called a soft real set. It is denoted by  $(F, A)$  or For some  $e \in A$ .<br> *F* alternation of *X*.<br> **F** alternation of *X* and  $f$  and and  $A$ <br> $(F, A)$  or  $F_A$ *F* all nonempty bounded<br>alled a soft real set. It is<br>*F*(*e*) is a subset of the s tion  $F: A \to B(\square)$  is called a soft real set. It<br>regative soft real set if  $F(e)$  is a subset of th<br> $e \in E$ . The set of all nonnegative  $(E)^*$  denote the set of all nonnegative

1. A soft real set  $F_A$  is said to be nonnegative soft real set if is a subset of the set of

nonnegative real numbers for each  $e \in E$ .

2. Let  $\Box$  (E) denotes the set of all soft real sets .Also  $\Box$  (E)<sup>\*</sup> denote the set of all nonnegative real set  $F_A$  is said to be nonnegative and<br>gative real numbers for each  $e \in E$ .<br>(*E*) denotes the set of all soft real sets

soft real sets.

If specifically  $F_A$  is a singleton soft set, then identifying  $F_A$  with the corresponding soft element, it will be called a soft real number and denoted  $r$ ,  $s$ ,  $t$  etc. hence  $\Box$  (*E*  $*$  denote the set of all nonnegative<br>  $*$  with the corresponding soft element,<br>  $(E)$  denote the set of all sort real num denote the set of all sort real numbers. ft real sets .Also  $\Box$  (*E*)<sup>\*</sup> denote the set of all not<br>set, then identifying  $F_A$  with the corresponding<br> $1 \tilde{r}$ ,  $\tilde{s}$ ,  $\tilde{t}$  etc. hence  $\Box$  (*E*) denote the set of all<br> $\tilde{0}(e) = 0$ ,  $\tilde{1}(e) = 1$  for all  $e$ with the corresponding soft element, it will  $E$ ) denote the set of all sort real numbers.<br> $e \in E$ , respectively.

0, 1 are the soft real numbers where  $0(e) = 0$ ,  $1(e) = 1$  for all  $e \in E$ , respectively.

### **Definition (2.8)**

Let  $F_A, G_B \in S(X)$ 

Journal of Iraqi Al-Khwarizmi (JIKh) Vo<br> **nition (2.8)**<br>  $F_A, G_B \in S(X)$ <br>
the union of  $F_A$  and  $G_B$  over X, denoted by  $F_A \cup$ <br>  $F_A \cup G_B \subseteq C$ 1. The union of  $F_A$  and  $G_B$  over X, denoted by  $F_A \cup G_B$  is the soft set  $H_C$  where  $C = A \cup B$  and *F<sub>A</sub>*  $\cup$  *G<sub>B</sub>* is the soft set  $H_c$  where  $C = A \cup B$  and pages:  $44-68$ <br>*C* =  $A \cup B$  and for all  $e \in C$ , Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year:<br> **n** (2.8)<br>  $e \in S(X)$ <br>
ion of  $\begin{aligned} F_A \text{ and } G_B \text{ over } X \text{, denoted by } F_A \cup G_B \text{ is the soft set } H_C \\ e \in C \text{,} \\ H(e) = \begin{cases} F(e), & e \in A \mid B \\ G(e), & e \in B \mid A \\ F(e) \cup G(e), & e \in A \cap B \end{cases} \\ \text{written as } F_A \cup G_B = H_C \\ \text{ersection$ varizmi (JIKh) Volume:8 Issue:2 Year:<br>  $(e) =\begin{cases} F(e), & e \in A \mid B \\ G(e), & e \in B \mid A \\ F(e) \cup G(e), & e \in A \cap B \end{cases}$ .<br>
er X, denoted by  $\begin{aligned} F_A \cap G_B &\text{is the soft s} \\ F_A \cap G_B &\text{is the soft s} \end{aligned}$ IIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br> *F*<sub>*k*</sub>  $\cup$  *G<sub><i>B*</sub> is the soft set  $H_c$  where  $C = A \cup B$  and<br> *F*(*e*),  $e \in A \mid B$ <br> *G*(*e*),  $e \in B \mid A$ <br>
(*e*)  $\cup$  *G*(*e*),  $e \in A \cap B$ <br>
Period by  $F_A \cap G_B$  is the soft set *H e G e e B A* (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
ed by  $F_A \cup G_B$  is the soft set  $H_c$  where  $C = A \cup B$  and<br>  $F(e)$ ,  $e \in A \mid B$ <br>  $G(e)$ ,  $e \in B \mid A$ <br>  $F(e) \cup G(e)$ ,  $e \in A \cap B$ <br>
denoted by  $F_A \cap G_B$  is the soft set  $H_c$  where  $C = A \cap B$ <br> cmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
moted by  $F_A \cup G_B$  is the soft set  $H_c$  where  $C = A \cup B$  and<br>  $= \begin{cases} F(e), & e \in A \mid B \\ G(e), & e \in B \mid A \\ F(e) \cup G(e), & e \in A \cap B \end{cases}$ <br>  $X$ , denoted by  $F_A \cap G_B$  is the soft set  $H_c$  where  $C$ ii (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
Med by  $F_A \cup G_B$  is the soft set  $H_C$  where  $C = A \cup B$  and<br>  $\begin{cases} F(e), & e \in A \mid B \\ G(e), & e \in B \mid A \\ F(e) \cup G(e), & e \in A \cap B \end{cases}$ <br>
, denoted by  $F_A \cap G_B$  is the soft set  $H_C$  where  $C = A \cap B$ <br> and  $G_B$  over  $X$ , denoted by  $F_A \cup G_B$  is the soft set  $H_C$  where  $C = A$ <br>  $H(e) = \begin{cases} F(e), & e \in A \mid B \\ G(e), & e \in B \mid A \\ F(e) \cup G(e), & e \in A \cap B \end{cases}$ <br>  $F_A \cup G_B = H_C$ <br>  $f \in H_A$  and  $G_B$  over  $X$ , denoted by  $F_A \cap G_B$  is the soft set  $H_C$  where  $\cup$  *B* and<br> $C = A \cap B$ *e C* , *H e F e G e* ( ) ( ) ( ) *FR* and *FR* and *FR*  $H_c$  where  $C = A \cup B$  and  $A \mid B$ <br> *FR*  $A \cap B$ <br> *FR*  $\cap G_B = H_c$ <br> *FR*  $\cap G_B = H_c$ Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 - pages: 44-68<br> **straition (2.8)**<br>
(**Finition (2.8)**<br>
(**Fig. 6.6** (*Fig.*  $G_x$  over  $X$ , denoted by  $F_X \cup G_x$  is the soft set  $H_x$  where  $C = A \cup B$  and<br>
for al **finition (2.8)**<br>
IF  $F_s$ ,  $G_s \in S(X)$ <br>
The union of  $\overline{F}_A$  and  $G_s$  over  $X$ , denoted by  $F_s \cup G_{\overline{s}}$  is the soft set  $H_c$  where  $C = A \cup B$  and<br>  $H(c) = \begin{cases} F(c), & c \in A \mid B \\ G(c), & c \in B \mid A \end{cases}$ <br>
and is written as  $F_s \cup G_s = H_c$ <br>
The Learnal of leasi Al-Klossnizmi (JIKly Volume:8 Issue.2 Year. 2024 pages: 44-68<br>
Definition (2.8)<br>
1. The union of <sup>F<sub>x</sub></sup> and <sup>G</sup><sub>F</sub> over X, denoted by  $F_x \vee G_y$  is the soft set  $H^2$  where  $C = A \vee B$  and<br>
for all  ${}^E C$ , Journal of least Al Kleoarizm (JIKb) Volume N issue 2 Year 2024 pages: 44 68<br>
Definition (2.8)<br>
Let  $F_*(G_i \leq S(X)$ <br>  $\int_{\Omega} \cos (t - t_{\text{end}}) \frac{F_*(s)}{s}$ , denoted by  $F_*(s)G_0$  is the soft of  $\frac{H_0}{s}$  where  $C = A \cup B$  and<br>  $\int$ 

hwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68

\n
$$
X \text{ , denoted by } F_A \cup G_B \text{ is the soft set } H_C \text{ where } C = A \cup B \text{ and}
$$
\n
$$
H(e) = \begin{cases} F(e), & e \in A \mid B \\ G(e), & e \in B \mid A \\ F(e) \cup G(e), & e \in A \cap B \end{cases}
$$
\n•c.

\nover  $X$  , denoted by  $F_A \cap G_B$  is the soft set  $H_C$  where  $C = A \cap B$ 

\n○  $\cap G(e)$ , and is written as  $F_A \cap G_B = H_C$ .

and is written as  $T_A \cup \mathbf{G}_B - \mathbf{H}_C$ .

2. The intersection of  $F_A$  and  $G_B$  over  $X$ , denoted by  $F_A \cap G_B$  is the soft se  $e \in A \mid B$ <br>  $e \in B \mid A$ <br>  $e \in A \cap B$ <br>  $F_A \cap G_B$  is the soft set  $H_C$  where  $C = A \cap B$ <br>  $F_A \cap G_B = H_C$ is the soft set  $H_c$  where  $C = A \cap B$ 

and for all  $e \in C$ ,  $H(e) = F(e) \cap G(e)$ , and is written as  $T_A \cap G_B = H_C$ .

#### **Definition (2.9)**

Let X be a linear space over a field F and let A be a parameter set. Let  $F_A$  and  $G_A$  be two soft set over *X* and  $\lambda \in F$ . Define be a parameter set. Let  $F_A$  and  $G_A$  be two a<br> $e \in A$  $\vec{r}_B$  is the soft set  $H_c$  where  $C = A \cap B$ <br>  $\vec{r}_A \cap G_B = H_c$ <br>
eter set. Let  $F_A$  and  $G_A$  be two soft set over<br>  $n$  for all  $e \in A$ <br>  $A = \{1, 2, 3, \dots, n\}$  be the set of<br>
is 0},  $i = 1, 2, \dots, n$ . *H*(*e*) = *F*(*e*) ∩ *G*(*e*), and is written as  $F_A \cap G_B = H_C$ .<br>
cc over a field F and let *A* be a parameter set. Let  $F_A$  and  $G_A$  be two soft set ove<br>  $x \in F(e)$ ,  $y \in G(e)$  for all  $e \in A$ <br> *F*(*e*) for all  $e \in A$ <br>
soft se 2. The intersection of  $F_A$  and  $G_B$  over  $\chi$ , denoted by  $F_A \cap G_B$  is the soft set  $H_C$  where  $C = A \cap B$ <br>and for all  $e \in C$ ,  $H(e) = F(e) \cap G(e)$ , and is written as  $F_A \cap G_B = H_C$ .<br>**Definition (2.9)**<br>Let  $\chi$  be a linear space over

1. 
$$
(F+G)(e) = {x+y : x \in F(e), y \in G(e)}
$$
 for all  $e ∈ A$ 

2.  $(\lambda F)(e) = {\lambda x : x \in F(e)}$  for all

*e* and let *A* be a parameter set. Let  $F_A$  and<br>  $G(e)$  for all  $e \in A$ <br>  $e \in A$ <br>  $(X, A)$ , then<br>  $x_n : x_i \in F(e), \quad i = 2, 3, \dots, n$  for all  $e \in A$ If  $F_1, F_2, \dots, F_n$  are *n* soft sets over  $(X, A)$ , then The intersection of  $F_A$  and  $G_B$  over  $X$ , denot<br>and for all  $e \in C$ ,  $H(e) = F(e) \cap G(e)$ , and is<br>finition (2.9)<br>t  $X$  be a linear space over a field  $F$  and let  $A$ <br>and  $\lambda \in F$ . Define<br> $(F+G)(e) = \{x + y : x \in F(e), y \in G(e)\}$  for all<br> $(\lambda$  $(F_1 + F_2 + \cdots, F_n)(e) = \{x_1 + x_2 + \cdots + x_n : x_i \in F(e), \quad i = 2, 3, \dots, n\}$  for all  $e \in A$  $e \in A$ :<br>
if space over a field F and let A be a parameter set. Let  $F_A$  and<br>
befine<br>  $x + y : x \in F(e), y \in G(e)$  for all  $e \in A$ <br>  $\exists : x \in F(e)$  for all  $e \in A$ <br>
re n soft sets over  $(X, A)$ , then<br>  $(e) = \{x_1 + x_2 + \cdots + x_n : x_i \in F(e), i = 2, 3, \cdots, n\}$ 

### **Example (2.10)**

Consider the Euclidian *n*-dimensional space  $\Box$  " over  $\Box$ . Let  $A = \{1, 2, 3, \dots, n\}$  be the set of parameters. Let  $F: A \to P(\square^n)$  be defined as follows : for all  $e \in A$ <br>= {1, 2, 3, ..., *n*} be the set of<br>0},  $i = 1, 2, \dots, n$ .

$$
F(i) = \{t \in \square^n : i \text{ -th co} - \text{ordinate of } t \text{ is } 0\}, i = 1, 2, \cdots, n
$$

Then *F* is a soft linear space or soft linear space of  $\Box$  <sup>n</sup> over  $\Box$ .

### **Theorem (2.11)**

for all soft sets  $F_A$  and  $G_A$  over X and  $\lambda \in F$ .

### Proof :

$$
(\lambda(F+G))(e) = \{\lambda z : z \in (F+G)(e)\} = \{\lambda(x+y) : x \in F(e), y \in G(e)\} = \{\lambda x + \lambda y : x \in F(e), y \in G(e)\}
$$

$$
(\lambda F + \lambda G)(e) = \{x' + y' : x' \in (\lambda F)(e), y' \in (\lambda G)(e)\} = \{x'' + \lambda y'' : x'' \in F(e), y'' \in G(e)\}
$$

Hence the result follows.

#### **Theorem (2.12)**

Let  $F_A$  be a soft set over X *X*

1. If e the result follows.<br> **rem** (2.12)<br>  $F_A$  be a soft set over *X*<br>  $x \in X$ , then  $x + F_A$  is a soft set over *X* defin<br>  $(x + F)(e) = \frac{x + F}{2}$ , then bllows.<br>  $x + F_A$  is a soft set over *X* defined as fo<br>  $(x + F)(a) = f x + y : y \in F$ is a soft set over *X* defined as follows :

$$
x \text{ set over } X \text{ defined as follows :}
$$
\n
$$
(x + F)(e) = \{x + y : y \in F(e)\} \text{ for all } e \in A
$$

2. If  $M \subseteq X$ , then  $M + F_A$  is a soft set over X defined as follows : **r**<sub>*A*</sub> be a soft set over *X*<br>  $X \in X$ , then  $x + F_A$  is a soft set over *X* define<br>  $(x + F)(e) = \{x + y\}$ <br>  $M \subseteq X$ , then  $M + F_A$  is a soft set over *X* define<br>  $M + F_A = \left| \int (x + F_A)$ , i.e.  $(x + F_A)$ 

1 of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68  
\n' + y': x' ∈ (λF)(e), y' ∈ (λG)(e)} = {λx" + λy": x" ∈ F(e), y" ∈ G(e)}  
\nllows.  
\n
$$
+ F_A
$$
 is a soft set over X defined as follows :  
\n
$$
(x + F)(e) = {x + y : y ∈ F(e)} \text{ for all } e ∈ A
$$
  
\n
$$
M + F_A
$$
 is a soft set over X defined as follows :  
\n
$$
M + F_A = \bigcup_{x \in M} (x + F_A), \text{ i.e. } (x + F)(e) = {x + y : y ∈ F(e)} \text{ for all } e ∈ A
$$
  
\n
$$
A \circ B \circ B \circ B \circ C
$$

#### **Definition (2.13)**

qi Al-Khwarizmi (JIKh) Volume:<br>  $x' \in (\lambda F)(e), y' \in (\lambda G)(e)$ } = { $\lambda x''$ <br>  $X$ <br>
s a soft set over X defined as follo<br>  $(x+F)(e) = \{x + y : y \in F(e) \}$ <br>  $\int_{A}^{x}$  is a soft set over X defined as f<br>  $\int_{A}^{x} = \bigcup_{x \in M} (x + F_A)$ , i.e.  $(x+F)(e) = \$ *M* of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year<br>  $f' + y' : x' \in (\lambda F)(e), y' \in (\lambda G)(e)$  =  $\{\lambda x'' + \lambda y'' : x'' \in F$ <br>
lows.<br> *A w K*  $F_A$  is a soft set over *X* defined as follows :<br>  $(x + F)(e) = \{x + y : y \in F(e)\}$  for all  $e \in M + F_A$  is a Fraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
y':  $x' \in (\lambda F)(e), y' \in (\lambda G)(e)$ } =  $\{\lambda x'' + \lambda y'' : x'' \in F(e), y'' \in G(e)\}$ <br>
svs.<br>
wer  $X$ <br>  $\Gamma_A$  is a soft set over  $X$  defined as follows :<br>  $(x + F)(e) = \{x + y : y \in F(e)\}$  for Kh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>  $F(r(\epsilon)) = \{ \lambda x'' + \lambda y'' : x'' \in F(\epsilon), y'' \in G(\epsilon) \}$ <br>  $F(r(\epsilon)) = \{ x + y : y \in F(\epsilon) \}$  for all  $\epsilon \in A$ <br>  $F(r(\epsilon)) = \{ x + y : y \in F(\epsilon) \}$  for all  $\epsilon \in A$ <br>  $F(r(\epsilon)) = \{ x + y : y \in F(\epsilon) \}$  for all  $\epsilon \in A$ <br>  $F(r(\epsilon))$  for Let X be a linear space over a field F and let A be the parameter set . A soft set  $F_A$  over X is said to be a soft linear space or soft vector space of X over F if  $F(e)$  is a subspace of X for all  $e \in A$ .  $(e) = {x + y : y \in F(e)}$  1<br> *F*(*e*) is a subspace of *X*  $e \in X$  is said to be a<br> $e \in A$ . *A* be the parameter set . A soft set  $F_A$  over<br>*x* F if  $F(e)$  is a subspace of *X* for all  $e \in$ <br> $x \in X$  and  $F_A$  be a *G F A A* En  $M + F_A$  is a soft set over X defined as follow<br>  $M + F_A = \bigcup_{x \in M} (x + F_A)$ , i.e.  $(x + F)(e) = \{x + y :$ <br>
(a)<br>
(b)<br>
(c)<br>
(d)<br>
(d)<br>
(d)<br>
(d)<br>
F and let A be the parameter :<br>
(or soft vector space of X over F if  $F(e)$  is a sub<br>
(c)<br> over *X* is said to be a<br>
Il  $e \in A$ .<br>
ver F if<br>  $\alpha G_A + \beta G_A \subseteq G_A$ 

#### **Definition (2.14)**

Let X be a linear space over a field F. Let  $x \in X$  and  $F_A$  be a

A soft set  $G_A$  over X is said to be a soft subspace of a soft linear space  $F_A$  of X over F if *Example 1*  $F_A$  be a<br>oft subspace of a soft linear space  $F_A$  of  $X$  or  $e \in A$ . pace of a soft linear space of  $X$ <br>*G*(*e*) is a subspace of  $X$ *X* over F if<br> $e \in A$ .

- 1.  $G_A \subseteq F_A$ , i.e.  $G_A(e) \subseteq F_A(e)$  for all
- 2.  $G_A$  is a soft linear space of X over F, i.e.  $G(e)$  is a subspace of X for all  $e \in A$ .

#### **Theorem (2.15)**

Learnal of leapi ALK bearizoni (IIKh) Volume 8 1saac2 Year 2024 pages 44.68<br>  $(\lambda F + \lambda O)(x) = (x^2 - y^2)$ ,  $x^2 \le \lambda F(x)$ ,  $y^2 \in \lambda G(\rho x) = -(\lambda x^2 - \lambda y^2)$ ,  $x^2 \in F(x)$ ,  $y^2 \in O(e)$ <br>
Hence the result fullows.<br>
Learner and actioner  $\chi$ A soft subset  $G_A$  of a soft linear space  $F_A$  of X over F is a soft subspace of  $F_A$  iff  $\alpha G_A + \beta G_A \subseteq G$ for all  $\alpha, \beta \in F$ . set  $G_A$  over X is said to be a soft subsp<br>  $\equiv F_A$ , i.e.  $G_A(e) \subseteq F_A(e)$  for all  $e \in A$ .<br>
is a soft linear space of X over F, i.e. (<br>
em (2.15)<br>
subset  $G_A$  of a soft linear space  $F_A$  of X<br>  $\alpha, \beta \in F$ .  $e \in A$  . (and of Iraqi, Al-Khwarizmi (JIKh) Voltamest Issue2 Year: 2024 pages: 44-68<br>
( $x^2 - y^2, y^2 \in (LF)(\delta, \gamma') \in (L(G)(\delta))$  = = { $Ax^2 - Ay^2, y^2 \in F(\epsilon), y^2 \in G(\epsilon)$ }<br>
follows.<br>
tertuver X<br>
tertuver X<br>
tertuver X<br>  $x + F_1$  is a suif set over subset *G<sub>A</sub>* of a soft linear space *F<sub>A</sub>* of *X* over<br> *α*, *β* ∈ F.<br>
Let *F<sub>A</sub>* be a soft linear space of *X* over F<br>
uppose that *G<sub>A</sub>* is a soft subspace of *F<sub>A</sub>*, then<br>  $\equiv A$ , then  $(\alpha G + \beta G)(e) = \{x' + y' : x' \in \alpha G(e), \}$  *x y G e* ( ) Its a subspace of  $X$  for all  $e \in A$ .<br>
(be a<br>
ft linear space  $F_A$  of  $X$  over  $F$  if<br>  $F_A$  over  $F$  if  $\alpha G_A + \beta G_A \subseteq G_A$ <br>
a soft subspace of  $F_A$  iff  $\alpha G_A + \beta G_A \subseteq G_A$ <br>
is a subspace of  $X$  for all  $e \in A$ .<br>  $G(e)$  =  $\{\alpha x + \beta y$ *G F<sub>A</sub>*, i.e.  $G_A(e) \subseteq F_A(e)$  for all  $e \in A$ .<br>
s a soft linear space of *X* over *F*, i.e.  $G(e)$  is a subspace of *X* for all  $e \in A$ .<br> **am (2.15)**<br>
subset  $G_A$  of a soft linear space  $F_A$  of *X* over *F* is a soft subspac

Proof :

Let  $F_A$  be a soft linear space of X over F F<sub>1</sub>

Suppose that  $G_A$  is a soft subspace of  $F_A$ , then  $G(e)$  is a subs  $G(e)$  is a subspace of  $X$ is a subspace of X for all  $e \in A$ .

for all 
$$
\alpha, \beta \in F
$$
.  
\nProof:  
\nLet  $F_A$  be a soft linear space of X over F  
\nSuppose that  $G_A$  is a soft subspace of  $F_A$ , then  $G(e)$  is a subspace of X for all  $e \in$   
\nLet  $e \in A$ , then  $(\alpha G + \beta G)(e) = \{x' + y' : x' \in \alpha G(e), y' \in \lambda G(e)\} = \{\alpha x + \beta y : x, y \in G(e)\}$   
\nSince  $x, y \in G(e)$  and  $\alpha, \beta \in F$ , then  $\alpha x + \beta y \in G(e)$ , so  $(\alpha G + \beta G)(e) \subseteq G(e)$   
\nHence  $\alpha G_A + \beta G_A \subseteq G_A$  for all  $\alpha, \beta \in F$ .

Since  $x, y \in G(e)$  and  $\alpha, \beta \in F$ , then  $\alpha x + \beta y \in G(e)$ , so

Hence  $\alpha G_A + \beta G_A \subseteq G_A$  for all

Conversely, let the given condition hold.

For Journal of Iraqi Al-Khwarizmi (<br>
versely, let the given condition hold.<br>  $e \in A$  let  $x, y \in G(e)$  and  $\alpha, \beta \in F$ , then and  $\alpha, \beta \in F$ , then

urnal of Iraqi Al-Khwarizmi (JIKh) Volume:8<br> *x*,  $y \in G(e)$  and  $\alpha, \beta \in F$ , then  $(\alpha G + \beta G)(e) =$ <br>  $G_A \subseteq G_A$  for all  $\alpha, \beta \in F$ , i.e.  $(\alpha G + \beta G)(e) \subseteq$ l-Khwarizmi (JIKh) Volume:8 Issue:2 Y<br>
ition hold.<br>  $\alpha, \beta \in F$ , then  $(\alpha G + \beta G)(e) = {\alpha x + \beta}$ <br>  $\alpha, \beta \in F$ , i.e.  $(\alpha G + \beta G)(e) \subseteq G(e)$ , so (Kh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>  $(\alpha G + \beta G)(e) = {\alpha x + \beta y : x, y \in G(e)}$ <br>  $\alpha G + \beta G)(e) \subseteq G(e)$ , so  ${\alpha x + \beta y : x, y \in G(e)} \subseteq G(e)$ <br>  $\alpha, \beta \in F$ , i.e.  $G(e)$  is a subspace of X for all  $e \in A$ .<br>
(e) for all  $e \in A$ . Since  $\alpha G_A + \beta G_A \subseteq G_A$  for all  $\alpha, \beta \in F$ , i.e.  $(\alpha G + \beta G)(e) \subseteq G(e)$ , so *Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 4*<br> *Findal Findal A A A A A A A A G A G A G A B G A B G A B E A Let X, y C G e D A* EXECTIVE EXECT:<br>
For September 1917<br>
September 1923<br>
For September 1923<br>
For September 1923<br>
For September 1924<br>
For Sept JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>  $(\alpha G + \beta G)(e) = {\alpha x + \beta y : x, y \in G(e)}$ <br>  $(\alpha G + \beta G)(e) \subseteq G(e)$ , so  ${\alpha x + \beta y : x, y \in G(e)} \subseteq G(e)$ <br>  $d \alpha, \beta \in F$ , i.e.  $G(e)$  is a subspace of X for all  $e \in A$ .<br>
(e) for all  $e \in A$ . fear: 2024 pages: 44-68<br>  $y: x, y \in G(e)$ <br>  $\{\alpha x + \beta y: x, y \in G(e)\} \subseteq G(e)$ <br>
behavior of X for all  $e \in A$ . *Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024*<br> *xely, let the given condition hold.*<br> *A* let  $x, y \in G(e)$  and  $\alpha, \beta \in F$ , then  $(\alpha G + \beta G)(e) = {\alpha x + \beta y : x, y \in G}$ <br>  $\alpha G_A + \beta G_A \subseteq G_A$  for all  $\alpha, \beta \in F$ , i.e.  $(\alpha$ -Khwarizmi (JIKh) Volume:8 Issue:2 Year: 20<br>
tion hold.<br>  $x, \beta \in F$ , then  $(\alpha G + \beta G)(e) = {\alpha x + \beta y : x, y \in \alpha, \beta \in F}$ , i.e.  $(\alpha G + \beta G)(e) \subseteq G(e)$ , so  ${\alpha x + \beta x, y \in G(e)}$  and  $\alpha, \beta \in F$ , i.e.  $G(e)$  is a subspace<br>
i.e.  $G_A(e) \subseteq F_A(e)$  for all Kh) Volume:8 Issue:2 Year: 2024 page<br>  $(\alpha G + \beta G)(e) = {\alpha x + \beta y : x, y \in G(e)}$ <br>  $\alpha G + \beta G)(e) \subseteq G(e)$ , so  ${\alpha x + \beta y : x, y \in \alpha, \beta \in F}$ , i.e.  $G(e)$  is a subspace of X for all  $e \in A$ .  $=\{\alpha x + \beta y : x, y \in G(e)\}$ <br>  $\subseteq G(e)$ , so  $\{\alpha x + \beta y : x, G(e)\}$  is a subspace of X  $e \subseteq G(e)$ <br> $e \in A$ . hwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pm<br>
n hold.<br>  $\beta \in F$ , then  $(\alpha G + \beta G)(e) = {\alpha x + \beta y : x, y \in G(e)}$ <br>  $\beta \in F$ , i.e.  $(\alpha G + \beta G)(e) \subseteq G(e)$ , so  ${\alpha x + \beta y : x \in G(e)}$  and  $\alpha, \beta \in F$ , i.e.  $G(e)$  is a subspace of 2<br>  $G_A(e) \subseteq F_A(e)$  for al  $(e) = \{ax + \beta y : x, y \in G(e)\}\$ <br>
(e)  $\subseteq G(e)$ , so  $\{ax + \beta y : x, y \in G(e)\} \subseteq G(e)\$ <br> *e.*  $G(e)$  is a subspace of *X* for all  $e \in A$ .<br>  $e \in A$ .

Hence  $\alpha x + \beta y \in G(e)$  for all  $x, y \in G(e)$  and  $\alpha, \beta \in F$ , i.e.  $G(e)$  is a subspace of X for all  $e \in A$ .

Since  $G_A$  is a soft subset  $F_A$ , i.e.  $G_A(e) \subseteq F_A(e)$  for all

Therefore  $G_A$  is a soft subspace of  $F_A$ .

### **Corollary (2.16)**

If  $G_A$  and  $H_A$  are soft subspaces of  $F_A$  of X over F, then  $G_A + H_A$  and  $\lambda F_A$  are soft subspaces of  $G(e)$  is a subspace of *X* for all  $e \in A$ .<br>
4.<br>  $G_A + H_A$  and  $\lambda F_A$  are soft subspaces of  $F_A$  of X over F. F.  $G_A$  and  $H_A$  are soft subspaces of  $F_A$  of  $X$  over  $F$ , then  $G_A + H_A$  and  $\lambda F_A$  are soft subsorption of  $X$  over  $F$ .<br> **rollary** (2.17)<br>  $G_i$ ) baa family of soft subspace of  $F_A$  of  $X$  over  $F$ , then  $\bigcap_{i \in J} G_i$  is a spaces of  $F_A$  of  $X$  over  $F$ , then  $G_A + H_A$  and  $\lambda F_A$  are soft subspaces of<br>ubspace of  $F_A$  of  $X$  over  $F$ , then  $\bigcap_{i \in J} G_i$  is a soft subspace of  $F_A$  of  $X$  over  $F$ .<br>field  $F$  and let  $A$  be the parameter set. A s erefore  $G_A$  is a soft subspace of  $F_A$ .<br> **rollary** (2.16)<br>  $G_A$  and  $H_A$  are soft subspaces of  $F_A$  of  $X$  over  $F$ , then  $G_A + H_A$  and  $\lambda F_A$  are soft subspaces of<br>
of  $X$  over  $F$ .<br> **rollary** (2.17)<br>  $(G_i)$  haa family of

### **Corollary (2.17)**

If  ${ \bf i}$ <br>**Example 12.17**  ${ G_i }$  baa family of soft subspace baa family of soft subspace of  $F_A$  of X over F, then  $\bigcap G_i$  is a soft subspace of  $\bigcap_{i \in J} G_i$  is a soft subset  $G_i$  is a soft subspace of  $F_A$  of X over F.  $F$ .

 $\in J$ 

### **Definition (2.18)**

pace of  $F_A$  of  $X$  over  $F$ , then  $\bigcap_{i \in J} G_i$  is a soft subspace of  $F_A$  of  $X$  over  $F$ .<br>d  $F$  and let  $A$  be the parameter set. A soft set  $F_A$  over  $X$  is said to be  $F(e)$  is a subalgebra of  $X$  for all  $e \in A$ . Let X be an algebra over a field F and let A be the parameter set. A soft set  $F_A$  over X is said to be a soft algebra of X over F if  $F(e)$  is a subalgebra of X for all  $e \in A$ . **(G<sub>i</sub>**) baa family of soft subspace of  $F_A$ <br> **finition (2.18)**<br>
t X be an algebra over a field F and level of algebra of X over F if  $F(e)$  is a su<br>
s very easy to see that in a soft algebra<br>  $(xy)\tilde{z} = x(y\tilde{z})$ <br>  $x(y + \tilde{z$ *A* be the parameter set. A soft set  $F_A$  over  $X$ <br> *x* balgebra of  $X$  for all  $e \in A$ .<br>
the soft elements satisfy the properties :<br>  $y \tilde{z}$ <br>  $x, y, \tilde{z} \in F_A$  and for any soft scalar  $\lambda$ ,<br>  $x, y, \tilde{z} \in F_A$  and for any ft subspace of  $F_A$  of  $X$  over  $F$ , then  $\bigcap_{i \in J} G_i$  is a soft subspace of  $F_A$  of  $X$  over  $F$ .<br>
or a field  $F$  and let  $A$  be the parameter set. A soft set  $F_A$  over  $X$  is said to be<br>  $F$   $F$  if  $F(e)$  is a subalgebr over F, then  $\bigcap_{i \in J} G_i$  is a soft subspace of  $F_A$  of  $X$  over F.<br>
e the parameter set. A soft set  $F_A$  over  $X$  is said to be<br>
bra of  $X$  for all  $e \in A$ .<br>
ft elements satisfy the properties :<br>  $\in F_A$  and for any soft

It is very easy to see that in a soft algebra the soft elements satisfy the properties :

- 1.  $(xy)\overline{z} = x(y\overline{z})$
- 2.  $x(y + z) = xy + xz$  and
- 3.  $\lambda(xy) = (\lambda x)y = x(\lambda y)$  where for all  $x, y, z \in F_A$  and for any soft scalar  $e \in A$

and  $(\lambda x)(e) = \lambda(e)x(e)$  for all

#### **Definition (2.19)**

Let  $F_A$  be a soft algebra of  $X$  over F F<sub>1</sub>

- 1.  $F_A$  is called a commutative soft algebra if  $xy = yx$  for all
- $\tilde{z} \in F_A$  and for any soft scalar  $\lambda$ ,<br> *d*  $(\lambda x)(e) = \lambda(e)x(e)$  for all  $e \in A$ <br> *xy* = *yx* for all *x*,  $y \in F_A$ <br>
mtity of  $F_A$  if  $x \tilde{e} = \tilde{e}x = x$  for all *x*. s and *x* (*e*) for all  $e \in A$ <br>  $x, y \in F_A$ <br>  $= e^x x = x$  for all  $x \in F_A$ 2. A soft element gebra of *X* over F<br>pommutative soft algebra if  $xy = yx$  for all  $\tilde{e} \in F_A$  is called the soft identity of  $F_A$  if  $x \tilde{e}$ is called the soft identity of  $F_A$  if  $xe = ex = x$  for all any soft scalar  $\lambda$ ,<br>  $(e)x(e)$  for all  $e \in A$ <br>  $1 \quad x, y \in F_A$ <br>  $x \tilde{e} = \tilde{e}x = x$  for all  $x \in F_A$  $x \in F_A$

3. A soft element al of Iraqi Al-Khwarizmi (JIKh) Volume:8 Is<br>  $x \in F_A$  is said to be invertible if it has inverse is said to be invertible if it has inverse in  $F_A$ , i.e. if there exists a arnal of Iraqi Al-Khwarizmi (JIKh) Volume:<br>  $x \in F_A$  is said to be invertible if it has inv<br>  $y \in F_A$  such that  $xy = yx = e^x$  and the y is cannot Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 4<br>to be invertible if it has inverse in  $F_A$ , i.e. if there exists a<br> $xy = yx = \tilde{e}$  and the y is called the inverse of  $\tilde{x}$ , denoted<br>be non-invertible soft element of

 soft element such that  $xy = yx = e$  and the y is called the inverse of  $\tilde{x}$ , denoted by

 $x^{-1}$ . Otherwise  $x^{-1}$  is said to be non-invertible soft element of  $F_A$ .

#### **3. Soft Normed Spaces**

Let X be a linear space over a field F, X is also our initial universe set and A be a nonempty set of parameters. Let X be the absolute soft linear space, i.e.,  $X(e) = X$ , for all  $e \in A$ . We use the notation x, y, z to denote soft ble if it has inverse in  $F_A$ , i.e. if there exists<br>and the y is called the inverse of  $\tilde{x}$ , denote<br>tible soft element of  $F_A$ .<br>lso our initial universe set and A be a nonem<br> $X(e) = X$ , for all  $e \in A$ . We use the notation *i* and *x* and *x* is said to be non-invertible soft element of  $F_A$ .<br> **3. Soft Normed Spaces**<br>
Let *X* be a linear space over a field F, *X* is also our initial universe set and *A* be a nonempty set of parameters.<br>
Let ty set of parameters.<br> $x, y, \tilde{z}$  to denote soft<br>tote a particular type **3. Soft Normed Spaces**<br>
Let *X* be a linear space over a field F, *X* is also our initial universe set and *A* be a nonempty set c<br>
Let *X* be the absolute soft linear space, i.e.,  $X(e) = X$ , for all  $e \in A$ . We use the not be non-invertible soft element of  $F_A$ .<br>
eld F, X is also our initial universe se<br> *r* space, i.e.,  $X(e) = X$ , for all  $e \in A$ .<br> *i*  $\tilde{r}, \tilde{s}, \tilde{t}$  to denote soft real numbers wh<br>  $\tilde{r}(e) = r$ , for all  $e \in A$  etc. For e our initial universe set and *A* be a nonempty s<br>  $e = X$ , for all  $e \in A$ . We use the notation *x*, y<br>
soft real numbers whereas  $\tilde{r}, \tilde{s}, \tilde{t}$  will denote<br>  $e \in A$  etc. For example  $\tilde{0}$  is the soft real n<br>
not rela etc. For example 0 is the soft real number such that 3. Soft Normed Spaces<br>Let X be a linear space over a field F,<br>Let X be the absolute soft linear space<br>vectors of a soft linear space and  $\tilde{r}, \tilde{s}, \tilde{s}$ ,<br>of soft real numbers such that  $\tilde{r}(e) = \tilde{0}(e) = 0$ , for all , for all  $e \in A$ . Note that, in general, r is not related to r. space over a field F, *X* is also our initial un solute soft linear space, i.e., *X*(*e*) = *X*, for al linear space and  $\tilde{r}$ ,  $\tilde{s}$ ,  $\tilde{t}$  to denote soft real numbers such that  $\tilde{r}(e) = r$ , for all  $e \in A$  etc. *r* . e:8 Issue:2 Year: 2024 pages: 44-68<br>werse in  $F_A$ , i.e. if there exists a<br>called the inverse of  $\tilde{x}$ , denoted by<br>ment of  $F_A$ .<br>universe set and A be a nonempty set of parameter<br>r all  $e \in A$ . We use the notation  $x, y, \$ me:8 Issue:2 Year: 2024 pages: 44-68<br>inverse in  $F_A$ , i.e. if there exists a<br>is called the inverse of  $\tilde{x}$ , denoted by<br>lement of  $F_A$ .<br>al universe set and A be a nonempty set of parameters.<br>for all  $e \in A$ . We use the n foof real numbers such that  $\tilde{r}(e) = r$ , for all  $e \in A$  etc. For  $e = 0$ , for all  $e \in A$ . Note that, in general,  $\tilde{r}$  is not related to <br> **inition (3.1)**<br> *X* be the absolute soft linear space. The function  $|| \cdot ||$ : tors of a soft linear space and  $\tilde{r}, \tilde{s}, \tilde{t}$  to denot<br>oft real numbers such that  $\tilde{r}(e) = r$ , for all<br> $\rho = 0$ , for all  $e \in A$ . Note that, in general,  $\tilde{r}$  is<br>**inition** (3.1)<br>*X* be the absolute soft linear spa

#### **Definition (3.1)**

Let X be the absolute soft linear space. The function  $\|\cdot\|: SE(X) \to \Box(A)^*$  is said to be a soft norm on the soft linear space X, if  $\|\cdot\|$  satisfies the following conditions : solute soft linear space . The function  $\|\cdot\|$ : *X*<br>*X*, if  $\|\cdot\|$  satisfies the following conditions<br> $x \in X$ 

1.  $||x|| \ge 0$  for all  $x \in X$ 

2. 
$$
||x|| = \tilde{0} \text{ iff } x = \tilde{0}
$$

- 3.  $||r \cdot x|| = |r| ||x||$  for all  $x \in X$  and for every soft scalar  $\tilde{r}$ , and for every soft scalar *r* , *x*,  $y \in X$ <br> *x*,  $y \in X$ <br> *x*,  $y \in X$
- 4.  $||x+y|| \le ||x|| + ||y||$  for all

The soft linear space X with a soft norm  $\|\cdot\|$  on X is said to be a soft normed linear space and is denoted by  $(X, \|\cdot\|, A)$  or call  $x \in X$ <br>  $\therefore x = 0$ <br>  $\|x\|$  for all  $x \in X$  and for every soft scalar  $\tilde{r}$ ,<br>  $\|x\| + \|y\|$  for all  $x, y \in X$ <br>
ear space X with a soft norm  $\| \cdot \|$  on X is sa<br>  $(X, \|\cdot\|, A)$  or  $(X, \|\cdot\|)$ .<br>
...2) X and for every soft scalar  $\tilde{r}$ ,<br>  $x, y \in X$ <br>
ith a soft norm  $\|\cdot\|$  on X is said to be<br>  $(X, \|\cdot\|)$ . oft linear space X with a soft norm  $\|\cdot\|$ <br>
ed by  $(X, \|\cdot\|, A)$  or  $(X, \|\cdot\|)$ .<br> **ple (3.2)**<br>
(A) be the set of all soft real numbers<br>  $x|$ , for all  $x \in \Gamma(A)$ , where  $|x|$  denotes

#### **Example (3.2)**

Let be the set of all soft real numbers. Then the function  $\rightarrow$   $\Box$  (*A*)<sup>\*</sup> is said to be a soft norm on the<br>soft normed linear space and is<br> $\Box$  (*A*)  $\rightarrow$   $\Box$  (*A*)<sup>\*</sup> which is defined by<br>al numbers, is a soft norm on  $\Box$  (*A*) and<br>space. With the same argument  $\Box \rightarrow \Box (A)^*$  is said to be a soft norm on the<br>a soft normed linear space and is<br> $\Box \Box (A) \rightarrow \Box (A)^*$  which is defined by<br>real numbers, is a soft norm on  $\Box (A)$  and<br>l space. With the same argument The soft linear space *X* with a soft inducted by  $(X, \|\cdot\|, A)$  or  $(X, \|\cdot\|)$ .<br> *xample* (3.2)<br> *x*  $\|\cdot\|$   $(x, \|\cdot\|)$  and  $\|\cdot\|$  and  $\|x\| = |x|$ , for all  $x \in \Box$  (*A*), where  $\|x\|$  ince  $SS(\Box (A)) = \Box$ , thus  $(\Box, \|\cdot\|, A)$ , for all  $\|\cdot\|$  for all *x*, *y*∈*X*<br>space *X* with a soft norm  $\|\cdot\|$  on *X* is said<br> $\|\cdot\|$ , *A*) or  $(X, \|\cdot\|)$ .<br><br>ne set of all soft real numbers. Then the func<br> $x \in \Box$  (*A*), where  $|x|$  denotes the modulus of<br> $y = \Box$ , thus  $(\$ , where |x| denotes the modulus of soft real numbers, is a soft norm on  $\Box$  (A) and  $A$  and  $A$  and  $B$ since  $SS(\Box(A)) = \Box$ , thus  $(\Box, \Vert \cdot \Vert, A)$  or  $(\Box, \Vert \cdot \Vert)$  is a soft normed space. With the same argument  $S = 0$  iff  $x = 0$ <br>  $x \leq 0$ <br>  $\left| \left| \left| x \right| \right| \right|$  for all  $x \in X$  and for every soft soft  $\left| \left| \left| x \right| \right| \right|$  for all  $x, y \in X$ <br>
oft linear space  $X$  with a soft norm  $|| \cdot ||$  on ed by  $(X, || \cdot ||, A)$  or  $(X, || \cdot ||)$ .<br> **ple (3.2)** *x*,  $y \in X$ <br>
ith a soft norm  $\| \cdot \|$  on *X* is said to be a soft r<br>  $(X, \| \cdot \|)$ .<br>
soft real numbers. Then the function  $\| \cdot \|$ :  $\Box$  (*A*<br>
here  $|x|$  denotes the modulus of soft real nu<br>  $(\Box, \| \cdot \|, A)$  or  $(\Box, \| \cdot \|)$  is a soft The same to be a set of  $\|\cdot\|$ :<br>tes the modulus of soft rea<br> $(\Box, \Vert \cdot \Vert)$  is a soft normed spe. 3.  $\|\vec{r} \cdot \vec{x}\| = |\vec{r}||\|\vec{x}\|$  for all  $x \in X$  and for every<br>4.  $\|\vec{x} + \vec{y}\| \le \|\vec{x}\| + \|\vec{y}\|$  for all  $x, y \in X$ <br>The soft linear space  $X$  with a soft norm  $\|$ <br>denoted by  $(X, \|\cdot\|, A)$  or  $(X, \|\cdot\|)$ .<br>**Example (3.2)**<br>Let  $\$  $SS(\square(A)) = \square$  is also a soft normed space. *x* is said to be a soft normed linear space and is<br> **n** the function  $\|\cdot\|: \Box(A) \to \Box(A)^*$  which is defined by<br>
nodulus of soft real numbers, is a soft norm on  $\Box(A)$  and<br>
is a soft normed space. With the same argument<br>  $x$ 

#### **Example (3.3)**

Let X be a normed space. In this case, for every  $x_e \in SV(X)$ ,  $||x_e|| = |e| + ||x||$  is a soft norm.

Proof :

Journal of Iraqi Al-Khwarizmi (JIKh)  
\nProof:  
\n1. Let 
$$
x_e \in SV(X)
$$
, then  $||x_e|| = |e| + ||x|| \ge 0$   
\n2. Let  $x_e \in SV(X)$ , then  $||x_e|| = 0$  iff  $|e| + ||x|| = 0$   
\n3. Let  $x_e \in SV(X)$  and for every soft scalar  $\tilde{r}$ , then  
\n
$$
||\tilde{r} \cdot x_e|| = ||(r \cdot x)_{re}|| = |re| + ||r||
$$

irnal of Iraqi Al-Khwarizmi (JIKh) Volume:8<br>  $x_e \in SV(X)$ , then  $||x_e|| = |e| + ||x|| \ge 0$ <br>  $SV(X)$ , then  $||x_e|| = 0$  iff  $|e| + ||x|| = 0$ , iff  $e =$ *x*<sub>e</sub> $\|\cdot\|e\| + \|x\| \ge 0$ <br>  $\bar{0} = 0$  iff  $|e| + \|x\| = 0$ , iff  $e = 0$  and  $x = 0$  iff  $x_e = 0$ <br> *x* of scalar  $\tilde{r}$  then 2. Let  $x_e \in SV(X)$ , then  $||x_e|| = 0$  iff  $|e| + ||x|| = 0$ , iff  $e = 0$  and  $x = 0$  iff  $x_e = 0$ Journal of Iraqi Al-Khwarizmi (JIKh) Volur<br>
Let  $x_e \in SV(X)$ , then  $||x_e|| = |e| + ||x|| \ge 0$ <br>  $x_e \in SV(X)$ , then  $||x_e|| = 0$  iff  $|e| + ||x|| = 0$ , iff<br>  $V(X)$  and for every soft scalar  $\tilde{r}$ , then ii (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>  $+||x|| \ge 0$ <br>  $e| + ||x|| = 0$ , iff  $e = 0$  and  $x = 0$  iff  $x_e = 0$ <br>
ar  $\tilde{r}$ , then

3. Let  $x_e \in SV(X)$  and for every soft scalar r, then

$$
\|\tilde{r} \cdot x_e\| = \|(r \cdot x)_{re}\| = |re| + \|r \cdot x\| = |r|(|r| + \|x\|) = |\tilde{r}|\|x_e\|
$$

4. Let  $x_e, y_e \in SV(X)$ , then

Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68  
\nof:  
\n1. Let 
$$
x_e \in SV(X)
$$
, then  $||x_e|| = |e| + ||x|| \ge 0$   
\n2. Let  $x_e \in SV(X)$ , then  $||x_e|| = 0$  iff  $|e| + ||x|| = 0$ , iff  $e = 0$  and  $x = 0$  iff  $x_e = 0$   
\net  $x_e \in SV(X)$  and for every soft scalar  $\tilde{r}$ , then  
\n
$$
||\tilde{r} \cdot x_e|| = ||(r \cdot x)_{re}|| = |re| + ||r \cdot x|| = |r|(|r| + ||x||) = |\tilde{r}|| ||x_e||
$$
\net  $x_e, y_{e'} \in SV(X)$ , then  
\n
$$
||x_e + y_{e'}|| = ||(x + y)_{(e + e')}|| = |e + e'| + ||x + y|| \le |e| + |e'| + ||x|| + ||y|| = (|e| + ||x||) + (|e'| + ||y||) = ||x_e|| + ||y_e||.
$$

#### **Theorem (3.4)**

1-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 4<br>
en  $||x_e|| = |e| + ||x|| \ge 0$ <br>  $||x_e|| = 0$  iff  $|e| + ||x|| = 0$ , iff  $e = 0$  and  $x = 0$  iff  $x_e = 0$ <br>  $||x_e|| = 0$  iff  $||e|| + ||x|| = 0$ , iff  $e = 0$  and  $x = 0$  iff  $x_e = 0$ <br>  $||x_e|| = ||(r \cdot x)_e||$ *r*  $\left\{\mathbf{x}_k\mathbf{x}_k\right\}$  *r*  $\left\{\mathbf{x}_k\mathbf{x}_k\mathbf{x}_k\right\}$  *x*  $\left\{\mathbf{x}_k\mathbf{x}_k\mathbf{x}_k\right\}$  *r*  $\left\{\mathbf{x}_k\mathbf{x}_k\mathbf{x}_k\right\}$  *r*  $\left\{\mathbf{x}_k\mathbf{x}_k\mathbf{x}_k\right\}$  *r*  $\left\{\mathbf{x}_k\mathbf{x}_k\mathbf{x}_k\right\}$  *r*  $\left\{\mathbf{x}_k\mathbf{x}_k\mathbf{x}_k\mathbf{x}_k\$ Every parametrized family of crisp norms  $\{\|\cdot\|_e : e \in A\}$  on a crisp linear space X can be considered as a soft  $e$ |+||x|| = 0̃, iff  $e = 0$  and  $x = 0$  iff  $x_e = 0$ <br>  $\|\vec{r}\|$ , then<br>  $\| = |re| + \|r \cdot x\| = |r|(|r| + \|x\|) = |\tilde{r}|\|x_e\|$ <br>  $y \| \le |e| + |e'| + \|x\| + \|y\| = (|e| + \|x\|) + (|e'| + 1)\|$ <br>  $\{\|\cdot\|_e : e \in A\}$  on a crisp linear space X c *e* + ||*x*|| = 0 , iff *e* = 0 and *x* = 0 iff *x<sub>e</sub>* = 0 <br>  $\tilde{r}$ , then<br>  $\therefore |re| + ||r \cdot x|| = |r| (|r| + ||x||) = |\tilde{r}|| ||x_e||$ <br>  $\leq |e| + |e'| + ||x|| + ||y|| = (|e| + ||x||) + (|e'| + ||y||) = ||x_e|| + ||y_{e'}||$ .<br>  $\therefore ||_e : e ∈ A$ } on a crisp linear space *X* can norm on the soft linear space  $X$ . *X* .

Proof :

Journal of Inapi A4-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
(<br> **1.** Let  $x_c \in SV(X)$ , then  $|x_c| = c + |x| \ge 0$ <br>
Let  $x_c \in SV(X)$  then  $|x_c| = 0$  iff  $|e + |c| = 0$ , iff  $e = 0$  and  $x = 0$  iff  $x_c = 0$ <br>  $x_c \in SV(X)$  and fo *Fournal of Iraqi* Al-Khwarizmi (HKh) Volume:8 Issue:2 Year: 2024 - Pages: 44-68<br>
<sup>2</sup>:<br>
2. Let  $x_e \in SV(X)$ , then  $\left|x_e\right| = e|+||x|| \ge 0$ <br>
2. Let  $x_e \in SV(X)$ , then  $\left|x_e\right| = e|+||x|| = \tilde{0}$ , iff  $e = 0$  and  $x = 0$  iff  $x_e = \tilde{0}$ <br> Let X be the absolute soft linear space over a field F, A be a nonempty set of parameters. Let  $\{\|\cdot\|_{e} : e \in A\}$ :<br>
d as a soft<br>  $\left\{\left\|\cdot\right\|_{e} : e \in A\right\}$ <br>
us define a  $e \text{ is a soft}$ <br> $\cdot \parallel_e : e \in A$ }<br>define a be a family of crisp norms on the linear space X. Let  $x \in X$ , then  $x(e) \in X$ , for every  $e \in A$ . Let us define a *F*, *A* be a nonempty set of parameters. Let  $x \in X$ , then  $x(e) \in X$ , for every  $e \in A$ . Let u<br>*X*, for all  $e \in A$ . *x* ear space *X* can be considered as a soft<br> **onempty set of parameters.** Let { $\|\cdot\|_e : e \in A$ }<br> *x*(*e*) ∈ *X*, for every *e* ∈ *A*. Let us define a<br>
∈ *A*. neters. Let  $\{\|\cdot\|_e : e \in A\}$ <br> $e \in A$ . Let us define a function  $\|\cdot\|$ :  $X \to \Box$  (A)<sup>\*</sup> by  $\|x\|(e) = \|x(e)\|_e$  for all  $x \in X$ , for all  $\|\tilde{r} \cdot x_e\| = \|(r \cdot x)_{re}\| = |re| + \|r \cdot x\| = |r|$ <br>  $\in SV(X)$ , then<br>  $\| = \|(x + y)_{(e+e')} \| = |e+e'| + \|x + y\| \le |e| + |e'| + \|x\|$ .<br>
4)<br>
atrized family of crisp norms { $\| \cdot \|_e : e \in A$ } on a<br>
soft linear space X.<br>
be the absolute soft linear space  $\|\tilde{r} \cdot x_e\| = \|(r \cdot x)_{re}\| = |re| + \|r \cdot x\| = |r|(|r| + \|x\|) = |\tilde{r}|\|x_e\|$ <br>  $y_e \in SV(X)$ , then<br>  $y_e \in SV(X) = \|(x + y)_{(e + e')}\| = |e + e'| + \|x + y\| \le |e| + |e'| + \|x\| + \|y\| = (|e| + \|x\|) + (|e'| + \|y\|)$ <br>
(3.4)<br>
innetrized family of crisp norms {|| - ||<sub>a</sub> : *e*  $|e'| + ||x + y|| \le |e| + |e'| + ||x|| + ||y||$ <br>
sp norms  $\{||\cdot||_e : e \in A\}$  on a crime space over a field F, A b<br>
details integrate X. Let  $x \in X$ ,  $(e) = ||x(e)||_e$  for all  $x \in X$ , for *e* + *e'* | + || *x* + *y*||  $\leq$  |*e*|| + |*e'* | + || *x*|| + || *x*|| + || *x*|| = (|*e*||<br> *x* .<br> *x* . *x* .<br> *x* . *x* . *x* . *x* Id F, A be a nonempty set of parameters. Let  $\{\|\cdot\|_{e} : e \in A$ <br> *x*  $\in X$ , then  $x(e) \in X$ , for every  $e \in A$ . Let us define a  $x \in X$ , for all  $e \in A$ .  $\|(x + y)_{(e+e')} \| = |e+e'| + \|x + y\| \le |e| + |e'| + \|x\| +$ <br>
ized family of crisp norms  $\{\| \cdot \|_e : e \in A\}$  on a<br>
ft linear space X.<br>
e absolute soft linear space over a field F, A<br>
is a position of the space over a field F, A<br>
is a positi *zy*(*x*), then<br>  $= |(x + y)_{(x + x)}| = |e + e'| + |x + y| \le |e| + |e'| + |x| + |y| = (|e| + ||x|) + (|e'| + ||y|) = ||x_e|| + ||y_e||$ .<br>
4)<br>
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terized family of crisp norms  $(||\cdot||, :e \in A)$  on a crisp linear space *X* can be considered a ( ) ( )( ) ( ) ( )( )  $||x|| + ||y|| = (|e| + ||x||) + (|e'| + |\theta'|)$ <br>
on a crisp linear space *X* c<br> *F*, *A* be a nonempty set of<br> *e*  $X$ , then  $x(e) \in X$ , for ex<br> *K*, for all  $e \in A$ .<br> *f* soft norm.<br> *X*, then  $||x|| \ge 0$  for all  $x \in X$ <br>  $|x(e)||_e = \theta$  for all  $e \$ *ratizmi* (JIKh) Volume 8 Issue:2 Year: 2024 pages: 44-68<br>  $\|\cdot\|e| + |x| \ge 0$ <br> *i* if  $|e| + |x| \ge 0$ <br> *i* if  $\|\cdot\| = |\bar{e}| + |x| \ge 0$ <br>  $\|\cdot\|e\| = |x^2| + |x$ of final Al-Khwarizmi (HKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>  $Y(X)$ , then  $\left|x_1\right| = |\epsilon| - |\epsilon| \ge \hat{0}$ <br>  $Y(X)$ , then  $\left|x_2\right| = |\epsilon| - |\epsilon| \ge \hat{0}$ <br>
(b), then  $\left|x_2\right| = |\epsilon| + |\epsilon| \ge \hat{0}$ <br>
(d for every soft scalar  $\hat{\epsilon}$ , th *e* = 0 and  $x = 0$  iff  $x_e = \tilde{0}$ <br>  $r|\langle |r| + ||x|| \rangle = |\tilde{r}|| ||x_e||$ <br>  $|\cdot ||y|| = (|e| + ||x||) + (|e'|| + ||y||) = ||x_e|| + ||y_e||$ <br>
a crisp linear space  $X$  can be considere<br> *A* be a nonempty set of parameters. Let<br> *X*, then  $x(e) \in X$ , for every  $e$ Iraqi Al-Khwarizmi (JIKD) Volume.8 Issue.2 Year: 2024 pages: 44-68<br>
(X), then  $||x|| = |x|| + ||x|| = 0$ , iff  $e = 0$  and  $x = 0$  iff  $x_n = 0$ <br>
(for every soft scalar  $\hat{r}$ , then<br>  $||\hat{r} \cdot \mathbf{x}|| = |x|| + |x| = 0$ , iff  $e = 0$  and  $x = 0$ 

Then  $\|\cdot\|$  is a soft norm on *X* .

To verify it we now verify the conditions 1,2,3 and 4 for soft norm.

Then 
$$
\|\cdot\|
$$
 is a soft norm on *X*.  
To verify it we now verify the conditions 1,2,3 and 4 for soft norm.  
1. We have  $\|x\|(e) = \|x(e)\|_e \ge 0$  for all  $e \in A$ , for all  $x \in X$ , then  $\|x\| \ge 0$  for all  $x \in X$ 

2.Let ify it we now verify the conditions 1,2,3 and<br>have  $||x||(e) = ||x(e)||_e \ge 0$  for all  $e \in A$ , for a<br> $x \in X$ , then  $||x|| = \tilde{0}$  iff  $||x||(e) = \theta$  for all  $e \in A$ , then  $||x|| = 0$  iff  $||x||(e) = \theta$  for all  $e \in A$  i i the linear space *X*. Let  $x \in X$ , then  $\|x\|(e) = \|x(e)\|_e$  for all  $x \in X$ , for all  $e \in X$ <br>
e conditions 1,2,3 and 4 for soft norm.<br>
0 for all  $e \in A$ , for all  $x \in X$ , then  $\|x\|$ <br>  $x\|(e) = \theta$  for all  $e \in A$  iff  $\|x(e)\|_e = \$ 3 and 4 for soft norm.<br>
for all  $x \in X$ , then  $||x|| \ge 0$  for all  $x \in X$ <br>  $e \in A$  iff  $||x(e)||_e = \theta$  for all  $e \in A$  iff  $x(e) = e$ iff  $\|x(e)\|_e = \theta$  for all  $e \in A$  iff  $x(e) = \theta$  for all  $\equiv$  *X*, then *x*(*e*)  $\in$  *X*, fc, for all *e*  $\in$  *A*.<br>soft norm.<br>, then  $||x|| \ge 0$  for all *z*<br>(*e*) $||_e = θ$  for all *e*  $\in$  *A x* example  $x(e) \in X$ , for every  $e \in A$ .<br> *x*, for all  $e \in A$ .<br> *x* soft norm.<br> *x*, then  $||x|| \ge \tilde{0}$  for all  $x \in X$ <br>  $x(e)||_e = \theta$  for all  $e \in A$  iff  $x(e) = \theta$  for or all  $x \in X$ <br> $e \in A$  iff  $x(e) = \theta$  for all  $x(e) = \theta$  for all 1. We have  $||x||(e) = ||x(e)||_e \ge 0$  for all  $e \in A$ , for all  $e \in A$ , then  $||x|| = 0$  iff  $||x||(e) = \theta$  for all  $e \in A$  iff  $x = 0$ iff  $x=0$  $x \in X$ , then  $||x|| = \tilde{0}$  iff  $||x||(e) = \theta$  for all  $e \in A$ <br> *x* iff  $x = \tilde{0}$ <br> *x* and  $\tilde{r}$  soft scalar, then  $||\tilde{r} \cdot x||(e) = ||(\tilde{r} \cdot x||)$  $e \in A$ , so Then  $\|\cdot\|$  is a soft norm on *X*.<br> *To verify it we now verify the conditions* 1,2,3 and 4 for soft n<br>
1. We have  $\|x\|(e) = \|x(e)\|_e \ge 0$  for all  $e \in A$ , for all  $x \in X$ , the<br>
2.Let  $x \in X$ , then  $\|x\|=0$  iff  $\|x\|(e) = \theta$  f

3. Let and r soft scalar, then  $\|r \cdot x\| (e) = \| (r \cdot x)(e) \| = |r| \|x(e) \| = (|r| \|x\|)(e)$  for all  $e \in A$ , so

$$
\|\tilde{r} \cdot x\| = |\tilde{r}| \|x\|
$$
 for all  $x \in X$  and for every soft scalar  $\tilde{r}$ ,

*x*  $\in$  *X* and  $\tilde{r}$  soft scalar, then  $\|\tilde{r} \cdot x\|$   $(e) = \|\tilde{r} \cdot x\|$   $(e) = \|\tilde{r} \cdot x\|$   $(e) \cdot \|\tilde{r}\|$   $x(e) \cdot \|\tilde{r}\|$   $x(e) \cdot \|\tilde{r}\|$  for all  $x \in X$  and for every soft scalar  $\tilde{r}$ ,<br>  $x, y \in X$ , then  $(\|x\| + \|y\|)($ 4. Let  $x, y \in X$ , then  $(\|x\| + \|y\|)(e) = \|x\|(e) + \|y\|(e) = \|x(e)\| + \|y(e)\| \ge \|x(e) + y(e)\| = \|x + y\|(e)$  for all

Journal of Iraqi Al-Khwarizmi (JIKh) Volum
$$
e \in A, \text{ so } ||x + y|| \le ||x|| + ||y|| \text{ for all } x, y \in X
$$

$$
|| \cdot || \text{ is a soft norm on } X \text{ and consequently } (X, || \cdot ||) \text{ is a}
$$
**Theorem (3.5)**

 $\|\cdot\|$  is a soft norm on X and consequently  $(X, \|\cdot\|)$  is a soft normed space.

### **Theorem (3.5)**

Every crisp norm  $\|\cdot\|_X$  on a crisp linear space X can be extended to a soft norm on the soft linear space *X* . Proof : Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 202<br>
w|| for all  $x, y \in X$ <br>
nd consequently  $(X, \|\cdot\|)$  is a soft normed space.<br>
crisp linear space X can be extended to a soft norm<br>
blue soft vector space X using a nonempty set qi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
|y|| for all x, y e X<br>
and consequently  $(X, \|\cdot\|)$  is a soft normed space.<br>
a crisp linear space X can be extended to a soft norm on the soft linear space X olume:8 Issue:2 Year: 2024<br>
is a soft normed space.<br>
be extended to a soft norm of<br>
using a nonempty set of para<br>  $(e) = ||x(e)||_x$  for all  $x \in X$ , for<br>
an be easily proved that  $||\cdot||$  is Volume:8 Issue:2 Year: 2024 pages:<br>  $\|\cdot\|$  is a soft normed space.<br>
an be extended to a soft norm on the soft norm of the soft norm of the soft norm of  $X$  using a nonempty set of parameters<br>  $x \|(e) = \|x(e)\|_X$  for all  $x \in$ et of parameters  $A$ .<br>  $x \in X$ , for all  $e \in A$ .  $e$  sort linear space  $X$ .<br>
ters  $A$ .<br>  $e \in A$ .

 First we construct the absolute soft vector space *X* using a nonempty set of parameters *A* .

Let us define a function  $\|\cdot\|$ :  $SE(X) \to \Box (A)^*$  by  $\|x\|(e) = \|x(e)\|_X$  for all  $x \in X$ , for all

Then using the same procedure as theorem (5.3), it can be easily proved that  $\|\cdot\|$  is a soft norm on X.

This soft norm is generated using the crisp norm  $\|\cdot\|_X$  and it is said to be the soft norm generated by *X*  $\|\cdot\|_{X}$ . st we construct the absolute soft vector space<br>us define a function  $\|\cdot\|$ :  $SE(X) \rightarrow \square(A)^*$  by<br>n using the same procedure as theorem (5.3),<br>soft norm is generated using the crisp norm<br>.<br>**orem (3.6)**<br> $(X, \|\cdot\|, A)$  is a soft n theorem (5.3), it can be easily proved the crisp norm  $\|\cdot\|_x$  and it is said to be<br>
vace, then<br>  $x \|(e) = 0$  iff  $x(e) = \theta$ , for any  $x \in X$  at<br>
set, for each  $x \in X$  and  $e \in A$ *x*(*e*) = *θ*, for any *x* ∈ *X* and *e* ∈ *A*.<br>*x*(*e*) = *θ*, for any *x* ∈ *X* and *e* ∈ *A*.  $x \in X$  and  $e \in A$ .  $e \in A$ . First we construct the absolute soft vector space<br>
t us define a function  $\| \cdot \| : SE(X) \to \square (A)^*$  by<br>
en using the same procedure as theorem (5.3),<br>
is soft norm is generated using the crisp norm<br>  $\|_x$ .<br> **eorem (3.6)**<br>
t (X dure as theorem (5.3), it can be easily proved that  $\|\cdot\|$  is a soft nor<br>
dusing the crisp norm  $\|\cdot\|_x$  and it is said to be the soft norm gener<br>
rmed space, then<br>
, then  $\|x\|(e) = 0$  iff  $x(e) = \theta$ , for any  $x \in X$  and he soft norm generated by<br>  $d e \in A$ .<br>  $x \in X$ ,  $||x||_e = ||x|| (e)$ ,<br>  $x \in X$ .  $x \parallel_e = ||x||(e),$ <br> $x \parallel_e = ||x||(e),$ *x x*  $\left|\frac{1}{2}(x) + \frac{1}{2}\right| \left|\frac{1}{2}\right| \left|\frac{1}{2}\right| \left|\frac{1}{2}\right| \left|\frac{1}{2}\right| \left|\frac{1}{2}\right|\left|\frac{1}{2}\right|\left|\frac{1}{2}\right|\left|\frac{1}{2}\right|\left|\frac{1}{2}\right|\left|\frac{1}{2}\right|\left|\frac{1}{2}\right|\left|\frac{1}{2}\right|\left|\frac{1}{2}\right|\left|\frac{1}{2}\right|\left|\frac{1}{2}\right|\left|\frac{1}{2}\right|\left|\frac{1}{2}\right|\left|\frac{1}{2}\right|\left|\frac{1}{$ 

### **Theorem (3.6)**

Let  $(X, \|\cdot\|, A)$  is a soft normed space, then

1. for any (3.6)<br>  $\begin{cases} 3.6 \\ \text{or} \end{cases}$ , *A*) is a soft normed space, then<br>  $x \in X$  and  $e \in A$ , then  $||x||(e) = 0$  iff  $x(e) = e$ and oft normed space, then<br> $e \in A$ , then  $||x||(e) = 0$  iff  $x(e) = \theta$ , for any  $x(e) = 0$ , for any  $x(e) = 0$ . , then  $||x||(e) = 0$  iff  $x(e) = \theta$ , for any  $x \in X$  and  $x(e) = \theta$ , for any  $x \in X$  and  $e \in A$ .<br> $x \in X$  and  $e \in A$ any  $x \in X$  and  $e \in A$ .<br> $e \in A$  $e \in X$  and  $e \in A$ , then  $||x||(e) = 0$  iff  $x(e) = \theta$ <br> $x(e) = x$  is a singleton set, for each  $x \in X$  a<br> $e \in A$ , define  $||\cdot||_e : X \to \square^+$  be the function A, then  $||x||(e) = 0$  iff  $x(e) = \theta$ , for any<br>ingleton set, for each  $x \in X$  and  $e \in$ <br> $|\cdot||_e : X \to \square^+$  be the function such that<br> $x(e) = x$ . Then for each  $e \in A$ ,  $||\cdot||_e$  i

2.  $\{||x||(e): x(e) = x\}$  is a singleton set, for each  $x \in X$  and

3. for each , define  $\|\cdot\|_{e}: X \to \square^+$  be the function such that for each  $x \in X$ ,  $\|x\|_{e} = \|x\|(e)$ ,  $x(e) = x$  is a singleton set, for each *x* ∈ *X*<br>*h e* ∈ *A*, define  $|| \cdot ||_e : X \rightarrow ∎^+$  be the functic<br>*x* ∈ *X* such that *x*(*e*) = *x*. Then for each *e* ∈ *e*  $\in$  *k* and  $e \in A$ <br> *e*  $\in$  *k*  $\in$  *k*  $\|x\|_e = \|x\|(e)$ <br> *e*  $\in$  *k*  $\in$   $\| \cdot \|_e$  is a norm on *X*.<br> *k*(*e'*) = 1 *k* if  $e' = e$ ,  $\lambda(e') = 0$  if  $e' \neq e$ .<br> *e* = *e'*. We have  $\|\lambda x\| = |\lambda| \|x\|$ . for each  $x \in X$ ,  $||x||_e = ||x||(e)$ ,<br>
a norm on X.<br>  $= e$ ,  $\lambda(e') = 0$  if  $e' \neq e$ .<br>  $\lambda ||x|| = |\lambda|| ||x||$ .  $\|x\|_e = \|x\| (e),$ <br> $e' \neq e.$  $\exists X \text{ and } e \in A.$ <br>  $\text{ar each } x \in X, ||x||_e = ||x||(e),$ <br>  $\text{norm on } X.$ <br>  $\forall e, \lambda(e') = 0 \text{ if } e' \neq e.$ <br>  $\lambda x || = |\lambda|| ||x||.$ 

where  $x \in X$  such that  $x(e) = x$ . Then for each  $e \in A$ ,  $\|\cdot\|$  is a norm on *X* .

#### Proof :

1. Let us consider a soft scalar  $\lambda$  such that  $\lambda(e') = 1$ , if  $e' = e$ ,  $\lambda(e') = 0$  if  $e' \neq e$ at for each  $x \in X$ ,  $||x||_e = ||x|| (e)$ <br>is a norm on X.<br> $e' = e$ ,  $\lambda(e') = 0$  if  $e' \neq e$ .  $i' = e$ ,  $\lambda(e') = 0$  if  $e' \neq e$ .

Then  $(\lambda x)(e) = \theta$  for  $e' \neq e$ ,  $(\lambda x)(e) = x(e')$  for  $e = e'$ . We have  $\|\lambda\|$ (*e*):  $x(e) = x$  is a singleton set, for each  $x \in X$  and  $e \in X$ <br>
each  $e \in A$ , define  $\|\cdot\|_e : X \to \square^+$  be the function such that<br>  $x(e) = x$ . Then for each  $e \in A$ ,  $\|\cdot\|_e$  is<br>  $\therefore$ <br>
1. Let us consider a soft scalar  $\lambda$  suc at  $x(e) = x$ . Then for each  $e \in A$ ,  $\|\cdot\|_e$  is a norm on  $X$ .<br>
er a soft scalar  $\lambda$  such that  $\lambda(e') = 1$ , if  $e' = e$ ,  $\lambda(e') = 0$  if  $e'$ <br>  $e' \neq e$ ,  $(\lambda x)(e) = x(e')$  for  $e = e'$ . We have  $\|\lambda x\| = |\lambda| \|x\|$ . for normed space, then<br>  $\in A$ , then  $||x||(e) = 0$  iff  $x(e) = \theta$ , for any  $x \in X$ <br>
a singleton set, for each  $x \in X$  and  $e \in A$ <br>  $e ||\cdot||_e : X \to \square^+$  be the function such that for eat<br>
at  $x(e) = x$ . Then for each  $e \in A$ ,  $||\cdot||_e$  is at that  $x(e) = x$ . Then for each  $e \in A$ ,  $\vert$ <br>ider a soft scalar  $\lambda$  such that  $\lambda(e') = 1$ <br>or  $e' \neq e$ ,  $(\lambda x)(e) = x(e')$  for  $e = e'$ . W<br> $x \parallel (e) = 0$  iff  $\lambda x = \theta$ , iff  $x(e) = \theta$ .<br>have  $\Vert x \Vert = \Vert x - y + y \Vert \le \Vert x - y \Vert + \Vert y \Vert$ . *x*. Then for each  $e \in A$ ,  $\|\cdot\|_e$  is a norm on *X*.<br>
alar  $\lambda$  such that  $\lambda(e') = 1$ , if  $e' = e$ ,  $\lambda(e') = 0$  if  $\lambda(x)(e) = x(e')$  for  $e = e'$ . We have  $\|\lambda x\| = |\lambda| \|x\|$ .<br>  $\lambda x = \theta$ , iff  $x(e) = \theta$ .<br>  $\|\lambda x\| = \|x\| \|x\| + \|y\|$   $\Rightarrow \|x\| = \|y$ 

 This shows that iff  $\lambda x = \theta$ , iff

2. Let  $x, y \in X$ , we have *xx*)(*e*) = *θ* for *e'*  $\neq$  *e*,  $(\lambda x)(e) = x(e')$  for *e* = *d*<br>*zx*)(*e*) = *θ* for *e'*  $\neq$  *e*,  $(\lambda x)(e) = x(e')$  for *e* = *d*<br>*shows that*  $||x||(e) = 0$  *iff*  $\lambda x = \theta$ *, iff*  $x(e) = \theta$ *<br><i>xx*,  $y \in X$ , we have  $||x|| = ||x - y + y|| \le ||x - y|| +$ 

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\nSimilarly 
$$
||y|| - ||x|| \le ||x - y||
$$
. So  $||x|| - ||y|| \le ||x - y||$ . Now if  $x, y \in X$  such that  $x(e) = y(e)$  then  
\n $||x||(e) - ||y||(e)| \le ||x - y||(e) = 0$  (by 1) since  $(x - y)(e) = x(e) - y(e) = 0$ . i.e.  $||x||(e) = ||y||(e)$ ,  
\nwhich proves (2).  
\n3. Since for  $e \in A$ ,  $||x||(e) : x(e) = x$  is a singleton set, the function  $|| \cdot ||_e : X \to \square^+$  is well defined. Hence  
\nfrom soft norm axioms, it follows that  $|| \cdot ||_e$  is a norm on  $X$ .  
\n**Theorem (3.7) Decomposition Theorem**

which proves (2).

3. Since for  $e \in A$ ,  $\{||x||(e): x(e) = x\}$  is a singleton set, the function  $||\cdot||_e : X \to \square^+$  is well defined. Hence Year: 2024 pages: 44-68<br>
ch that  $x(e) = y(e)$  then<br>
. i.e.  $||x||(e) = ||y||(e)$ ,<br>  $\cdot ||_e : X \rightarrow \square^+$  is well defined. Hence from soft norm axioms, it follows that  $\|\cdot\|_e$  is a norm on X... ch proves (2).<br>
ince for  $e \in A$ ,  $\{||x||(e) : x(e) = x\}$  is a<br>
i soft norm axioms, it follows that  $||\cdot||$ <br>
orem (3.7) Decomposition Theorem<br>
(X,  $||\cdot||$ ) is a soft normed space satisfie<br>  $\therefore$  For  $x \in X$  and  $e \in A$  the set  $\{||x$ 

#### **Theorem (3.7) Decomposition Theorem**

Let  $(X, \|\cdot\|)$  is a soft normed space satisfies the following condition

 $N_5$ : For  $x \in X$  an *x X* and **e** a *k* is the set of  $\|\cdot\|_e$  is a norm on  $\pi$ .<br> **a** *k* is a *k* is a *k* is a singleto <br>  $e \in A$ , the set  $\{\|x\|(e) : x(e) = x\}$  is a singleto , the set  $\{\Vert x\Vert (e) : x(e) = x\}$  is a singleton set and if for each  $e \in A$ ,  $\Vert \cdot \Vert_e : X \to \Box$ varizmi (JIKh) Volume:8 Issue:2 Year: 2024<br>  $-\Vert y \Vert \le \Vert x - y \Vert$ . Now if  $x, y \in X$  such that  $x(e)$ <br>  $\Rightarrow$  ) since  $(x - y)(e) = x(e) - y(e) = 0$ . i.e.  $||x||(e)$ <br>  $\}$  is a singleton set, the function  $|| \cdot ||_e : X \rightarrow \Box$ <br>
at  $|| \cdot ||_e$  is a norm o  $e \in A$ ,  $\|\cdot\|_e : X \to \square^+$ <br>Then for each  $e \in A$  $+$   $$ ned. Hence<br>  $\cdot \parallel_e : X \to \Box^+$ <br>  $\cdot$  each  $e \in A$ , be a function such that for each  $x \in X$ ,  $||x||_e = ||x||(e)$ , where *x*  $x \in X$  is a singleton set, the function  $\|\cdot\|_{e} : X$ <br>*x* s that  $\|\cdot\|_{e}$  is a norm on *X*..<br>**Theorem**<br>ace satisfies the following condition<br>set  $\{\|x\|(e) : x(e) = x\}$  is a singleton set and i<br> $x \in X$ ,  $\|x\|_{e} = \|x\|(e)$ , **a** singleton set, the function  $\|\cdot\|_e : X \cdot$ <br>  $\|_e$  is a norm on X...<br> **1**<br>
ies the following condition<br>  $e) : x(e) = x\}$  is a singleton set and if  $x\|_e = \|x\|(e)$ , where  $x \in X$  such that probabilition<br>gleton set and if for each  $e \in A$ ,  $\|\cdot\|_e : X \to x \in X$  such that  $x(e) = x$ . Then for each  $e \in A$ such that  $x(e) = x$ . Then for each *f* for each  $e \in A$ ,  $\|\cdot\|_e : X \to \square^+$ <br> $x(e) = x$ . Then for each  $e \in A$ ,  $\rightarrow \Box$ <sup>+</sup><br> $e \in A$ ,  $\cdot \parallel_e$  is a norm on *X* . *EA*,  $\{||x||(e): x(e) = x\}$  is a singleton set, the function  $|| \cdot ||_e : X \rightarrow \mathbb{R}$ <br>
a axioms, it follows that  $|| \cdot ||_e$  is a norm on X...<br> **Decomposition Theorem**<br>
a soft normed space satisfies the following condition<br>
X and  $e \in$ for each  $e \in A$ ,  $\|\cdot\|_e : X \to \square^+$ <br>  $(e) = x$ . Then for each  $e \in A$ ,<br>  $e$  e crisp real number for<br>  $\alpha$  the condition  $N_5$  and<br>  $\alpha$  soft norm satisfying<br>  $d(x, y) = ||x - y||$ , for all

#### Proof :

Clearly  $\|\cdot\|_{e}: X \to \square^+$  is a rule that assign a vector of X to a nonnegative crisp real number for all *e* is a norm on *X*.<br>
of :<br>
Clearly  $\|\cdot\|_e : X \to \square^+$  is a rule that assign<br>  $e \in A$ . Now the well defined property of  $\|\cdot\|$ <br>
soft norm axioms gives the norm conditions . Now the well defined property of  $\|\cdot\|_e$ , for all  $e \in A$ of *X* to a nonnegative crisp real number for  $e \in A$  is follows from the condition  $N_5$  and for all  $e \in A$ . Thus the soft norm satisfying is follows from the condition  $N_5$  and the soft norm axioms gives the norm conditions of  $\|\cdot\|_e$ , for all  $e \in A$ . Thus the soft norm satisfying o a nonnegative crisp real number for<br>*s* follows from the condition  $N_5$  and<br> $e \in A$ . Thus the soft norm satisfying  $N<sub>5</sub>$  gives a parameterized family of crisp norms. of:<br>
Clearly  $\|\cdot\|_{e} : X \to \square^{+}$  is a rule that assign a<br>  $e \in A$ . Now the well defined property of  $\|\cdot\|_{e}$ <br>
soft norm axioms gives the norm conditions of<br>
gives a parameterized family of crisp norms.<br> **orem (3.8)**<br>
( degrade in the sampleton set and if for each  $e \in A$ ,  $|| \cdot ||_e : X \to \Box$ <br>
where  $x \in X$  such that  $x(e) = x$ . Then for each  $e \in A$ <br>
or of X to a nonnegative crisp real number for<br>
dull  $e \in A$  is follows from the condition  $N_5$  a condition<br>
mgleton set and if for each  $e \in A$ ,  $\|\cdot\|_e : X \to \square^+$ <br>  $x \in X$  such that  $x(e) = x$ . Then for each  $e \in A$ ,<br>
X to a nonnegative crisp real number for<br>  $\in A$  is follows from the condition  $N_5$  and<br>
call  $e \in A$ . Thus all  $e \in A$ . Now the well defined property of  $\|\cdot\|$  the soft norm axioms gives the norm conditions  $N_5$  gives a parameterized family of crisp norm<br>**Theorem (3.8)**<br>Let  $(X, \|\cdot\|, A)$  be a soft normed space. Let us  $x, y \in X$ *e* ∈ *A*, the set  $\left\|x\right\|(\epsilon) : x(\epsilon) = x\}$  is a singleton set and if for each  $\epsilon \in A$ ,  $\|\cdot\|_e : X \to \square^+$ <br>for each  $x \in X$ ,  $\|x\|_e = \|x\|(\epsilon)$ , where  $x \in X$  such that  $x(\epsilon) = x$ . Then for each  $\epsilon \in A$ ,<br> $\to \square^+$  is a rule that *f* leagl Al-Khwarizmi (JRh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>  $-\pi \int \cdot S \phi \left\| |x| - |y| \right\| \le |x - y|$ . Now if  $x, y \in X$  such that  $x(x) = y(x)$  then<br>  $y(x) = 0$  (by 1) since  $(x - y)(x) = x(x) - y(x) = 0$ , i.e.  $|x(x) = |x(x)|$ <br>  $y(x) = |x(x) - y(x)|$ *d*[(*e*) = 0(*by* 1) since  $(x - y)(e) = x(e) - y(e) = 0$ ; i.e.,  $|x| (v) = |y| (v)$ .<br>  $d_1^2(v) : x(e) = x$  is a singleton set, the function  $|z| : x \rightarrow 0^+$  is well defined. Hence<br>  $d_1^2(v) : x(e) = x$  is a singleton set, the function  $|z| : x \rightarrow 0^+$ *x z x y y z x y y z* ( ) ( ) *d*<sub>x</sub> : For  $x \in X$  and  $e \in A$ , the set  $\{||x||(e): x(e) = x\}$  is a singleton set and if for each  $e \in A$ ,  $||x||_e \cdot X \rightarrow 1$ <sup>+</sup><br>ex a function such that for each  $x \in X$ ,  $|x|_e = |x|(e)$ , where  $x \in X$  such that  $x(e) = x$ . Then for each

#### **Theorem (3.8)**

Let be a soft normed space. Let us define  $d: X \times X \to \Box (A)^*$  by  $d(x, y) = ||x - y||$ , for all . Then d is a soft metric on X. *X* . **i**  $\left\{\n\begin{aligned}\n\cdot \| \cdot \| \cdot A & \text{be a soft normed space.} \downarrow \\
\cdot \| \cdot \| \cdot A & \text{be a soft normed space.} \downarrow \\
X \cdot \text{Then } d \text{ is a soft metric on } X \text{ and } X \text{ is a soft metric on } X.\n\end{aligned}\n\right\}$ *x*,  $\|\cdot\|$ , *A*) be a soft normed space. Let us define<br> *x*. Then *d* is a soft metric on *X*.<br> *x*,  $y \in X$ , then  $d(x, y) = \|x - y\| \ge 0$ <br> *x*,  $y \in X$ , then  $d(x, y) = 0 \iff \|x - y\| = 0 \iff$ <br>  $\|x - y\| = 0 \iff$ 

Proof :

1. Let  $x, y \in X$ , then

2. Let , then

3. Let 
$$
x, y \in X
$$
, then  $d(x, y) = ||x - y|| = ||y - x|| = d(y, x)$ 

$$
x, y \in X
$$
. Then *d* is a soft metric on *X*.  
\nProof:  
\n1. Let *x*, *y* ∈ *X*, then *d*(*x*, *y*) =  $||x - y|| \ge 0$   
\n2. Let *x*, *y* ∈ *X*, then *d*(*x*, *y*) = 0  $\Leftrightarrow$   $||x - y|| = 0$   $\Leftrightarrow$  *x* − *y* = 0  $\Leftrightarrow$  *x* = *y*  
\n3. Let *x*, *y* ∈ *X*, then *d*(*x*, *y*) =  $||x - y|| = ||y - x|| = d(y, x)$   
\n4. Let *x*, *y*,  $\tilde{z} \in X$ , then  $||x - \tilde{z}|| = ||(x - y) + (y - \tilde{z})|| \le ||x - y|| + ||y - \tilde{z}||$ , so *d*(*x*,  $\tilde{z}$ ) ≤ *d*(*x*, *y*) + *d*(*y*,  $\tilde{z}$ ) *d* is a soft metric on *X*, *d* is said to be the soft metric induced by the soft norm || · ||. From the a

is a soft metric on X, d is said to be the soft metric induced by the soft norm  $\|\cdot\|$ . From the above theorem it also follows that every soft normed space is also a soft metric space.

### **Theorem (3.9) Translation invariance**

A soft metric d induced by a soft norm  $\|\cdot\|$  on a normed linear space  $(X, \|\cdot\|)$  satisfies

Journal of Iraqi Al-Khwarizmi (JKh) Volume:8 Issue:2 Year: 2024 pages: 44-68  
\n**Theorem (3.9) Translation invariance**  
\nA soft metric *d* induced by a soft norm 
$$
||\cdot||
$$
 on a normed linear space  $(X_1||\cdot||)$  satisfies  
\n1.  $d(x + \bar{z}, y + \hat{z}) = d(x, y)$ , for all  $x, y, \bar{z} \in X$   
\n2.  $d(\bar{r} \cdot x, \bar{r} \cdot y) = |\bar{r}| d(x, y)$ , for all  $x, y \in X$  and for every soft scalar  $\bar{r}$ .  
\nProof:  
\nWe have,  
\n $d(x + \bar{z}, y + \bar{z}) = |(x + \bar{z}) - (y + \bar{z})|| = |x - y|| = d(x, y)$  and  
\n $d(\bar{r} \cdot x, \bar{r} \cdot y) = |[\bar{r} \cdot (x - \bar{y})] - |\bar{r} \cdot (x - y)|| - |\bar{r}| ||x - y| - |\bar{r}| d(x, y)$   
\n**Theorem (3.10)**  
\nLet  $d : X \times X \rightarrow \mathbb{E}$  (A)' be a soft metric. *X* is a soft normed space iff the following conditions :  
\n1.  $d(x + \bar{z}, y + \bar{z}) = d(x, y)$ , for all  $x, y, \bar{z} \in X$   
\n2.  $d(\bar{r} \cdot x, \bar{r} \cdot y) = |\bar{r}| d(x, y)$ , for all  $x, y \in X$  and for every soft scalar  $\bar{r}$ .  
\nsatisfied.  
\n**Proof :**  
\nIf  $d(x, y) = ||x - y||$ , from theorem(3.9), we have then  
\n $d(x + \bar{z}, y + \bar{z}) = d(x, \bar{y})$  and  $d(\bar{r} \cdot x, \bar{r} \cdot y) = |\bar{r}| d(x, y)$   
\nSuppose that the conditions of the theorem are satisfied.  
\nTaking  $||x| = d(x, \bar{0})$  for every  $x \in X$  we have  
\n1.Let  $x \in X$ , then  $||x| = \bar{0} \iff d(x, \bar{0}) = \bar{0} \iff x = \bar{0}$   
\n3. Let  $x \in$ 

Proof :

We have,

$$
d(x+\tilde{z}, y+\tilde{z}) = ||(x+\tilde{z})-(y+\tilde{z})|| = ||x-y|| = d(x, y) \text{ and}
$$
  

$$
d(\tilde{r} \cdot x, \tilde{r} \cdot y) = ||\tilde{r} \cdot x - \tilde{r} \cdot y|| = ||\tilde{r} \cdot (x-y)|| = |\tilde{r}||x-y|| = |\tilde{r}|d(x, y)
$$

#### **Theorem (3.10)**

Let  $d: X \times X \to \Box$  (A)<sup>\*</sup> be a soft metric. X is a soft normed space iff the following conditions :

2. 
$$
d(\vec{r} \cdot x, \vec{r} \cdot y) = |x| \cdot d(x, y)
$$
, for all  $x, y \in X$  and for every soft scalar  $\vec{r}$ .  
\nProof:  
\nWe have,  
\n $d(x + \vec{z}, y + \vec{z}) = |(x + \vec{z}) - (y + \vec{z})|| = ||x - y|| = d(x, y)$  and  
\n $d(\vec{r} \cdot x, \vec{r} \cdot y) = ||\vec{r} \cdot x - \vec{r} \cdot y|| = ||\vec{r} \cdot (x - y)|| = |\vec{r}|| ||x - y|| = |\vec{r}| d(x, y)$   
\n**Theorem (3.10)**  
\nLet  $d : X \times X \rightarrow \Box (A)^*$  be a soft metric.  $X$  is a soft normed space iff the following conditions:  
\n1.  $d(x + \vec{z}, y + \vec{z}) = d(x, y)$ , for all  $x, y, \vec{z} \in X$   
\n2.  $d(\vec{r} \cdot x, \vec{r} \cdot y) = |\vec{r}| d(x, y)$ , for all  $x, y \in X$  and for every soft scalar  $\vec{r}$ .  
\nsatisfied.  
\nProof:  
\nIf  $d(x, y) = ||x - y||$ , from theorem(3.9), we have then  
\n $d(x + \vec{z}, y + \vec{z}) = d(x, y)$  and  $d(\vec{r} \cdot x, \vec{r} \cdot y) = |\vec{r}| d(x, y)$   
\nSuppose that the conditions of the theorem are satisfied.  
\nTaking  $||x|| = d(x, \vec{0})$  for every  $x \in X$  we have  
\n1. Let  $x \in X$ , then  $||x|| = d(x, \vec{0}) \ge \vec{0}$   
\n2. Let  $x \in X$ , then  $||x|| = \vec{0} \Leftrightarrow d(x, \vec{0}) = \vec{0} \Leftrightarrow x = \vec{0}$ 

satisfied.

Proof :

If  $d(x, y) = ||x - y||$ , from theorem(3.9), we have then

and *x X*

Suppose that the conditions of the theorem are satisfied .

Taking  $||x|| = d(x, 0)$  for every  $x \in X$  we have

- 1.Let  $x \in X$ , then  $d(x+z, y+z) = d(x, y)$  as<br>
se that the conditions of the theorem are sations  $||x|| = d(x, 0)$  for every  $x \in X$  we have<br>  $x \in X$ , then  $||x|| = d(x, 0) \ge 0$ 2. Let se that the conditions of the theorem are satis<br>  $||x|| = d(x, 0)$  for every  $x \in X$  we have<br>  $x \in X$ , then  $||x|| = d(x, 0) \ge 0$ <br>  $x \in X$ , then  $||x|| = 0 \iff d(x, 0) = 0 \iff x$ , then  $x \in X$ , then  $||x|| = d(x, 0) \ge 0$ <br>  $x \in X$ , then  $||x|| = 0 \Leftrightarrow d(x, 0) = 0 \Leftrightarrow$ <br>  $x \in X$  and for every soft scalar  $\tilde{r}$ , then  $x \in X$ , then  $||x|| = a(x, 0) \ge 0$ <br>  $x \in X$ , then  $||x|| = \tilde{0} \iff d(x, \tilde{0}) = \tilde{0} \iff x =$ <br>  $x \in X$  and for every soft scalar  $\tilde{r}$ , then<br>  $||\tilde{r} \cdot x|| = d(\tilde{r} \cdot x, \tilde{0}) = d(\tilde{r} \cdot x, \tilde{r})$ <br>  $x, y \in X$ , then
- 3. Let  $x \in X$  and for every soft scalar r, then

$$
\|\tilde{r} \cdot x\| = d(\tilde{r} \cdot x, \tilde{0}) = d(\tilde{r} \cdot x, \tilde{r} \cdot \tilde{0}) = |\tilde{r}| d(x, \tilde{0}) = |\tilde{r}| \|x\|
$$

4. Let  $x, y \in X$ , then

$$
\|x + y\| = d(x + y, \tilde{0}) = d(x, -y) \le d(x, \tilde{0}) + d(\tilde{0}, -y) = \|x\| + |-1\|y\| = \|x\| + \|y\|
$$

#### **Definition (3.11)**

*x x* y  $\|\cdot\| = d(x + y, 0) = d(x, -y) \leq d(x, 0) + d(0, -y) = \|x\| + \|1\| \|y\| = \|x\| + \|y\|$ <br>
(*x + y*,  $\hat{0} = d(x, -y) \leq d(x, 0) + d(0, -y) = \|x\| + \|1\| \|y\| = \|x\| + \|y\|$ <br>
(*x*) Let  $(X, \|\cdot\|)$  be a soft normed space and  $(Y, A)$  be a non-null member of Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024<br>  $||x + y|| = d(x + y, \tilde{0}) = d(x, -y) \le d(x, \tilde{0}) + d(\tilde{0}, -y) = ||x|| + |-1||y|| =$ <br> **inition (3.11)**<br>  $(X, ||\cdot||)$  be a soft normed space and  $(Y, A)$  be a non-null member of  $S(X)$ .<br>  $|||y|| = ||x|| + ||y||$ <br>*S(X)*. Then the function  $Y$ . . Then the function Journal of Iraqi Al-Khwariz<br>  $||x + y|| = d(x + y, \tilde{0}) = d(x$ <br>
nition (3.11)<br>  $X, ||\cdot||$  be a soft normed space and<br>  $: SE(Y) \rightarrow \Box (A)^*$  given by  $||x||_y = ||$ <br>
norm  $||\cdot||_y$  is known as the relative<br>
d a normed subspace or simply a su<br>
nitio *Journal of Iraqi Al-Khwarizmi (JIKh) Volu*<br>  $||x + y|| = d(x + y, 0) = d(x, -y) \le d(x, 0) +$ <br> **finition (3.11)**<br>  $\therefore (X, ||\cdot||)$  be a soft normed space and  $(Y, A)$  be a nor-<br>  $\forall y : SE(Y) \rightarrow \Box (A)^*$  given by  $||x||_y = ||x||$  for all  $x \in Y$ <br>
is norm Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>  $||x + y|| = d(x + y, \tilde{0}) = d(x, -y) \leq d(x, \tilde{0}) + d(\tilde{0}, -y) = ||x|| + |-1||y|| = ||x|| + ||y||$ <br> **efinition (3.11)**<br>
et  $(X, ||\cdot||)$  be a soft normed space and  $(Y, A)$  be a n hwarizmi (JIKh) Volume:8 Issue:2 Y<br>  $y = d(x, -y) \leq d(x, 0) + d(0, -y) = ||x|| +$ <br> *x* e and  $(Y, A)$  be a non-null member of<br>  $x||_Y = ||x||$  for all  $x \in Y$  is a soft norm of<br>
velative norm induced on  $Y$  by  $||\cdot||$ . for all a non-null member of  $S(X)$ . Then the functor  $x \in Y$  is a soft norm on  $Y$ . is a soft norm on *Y* . **nition (3.11)**<br>  $[X, || \cdot ||]$  be a soft normed space and  $(Y, A)$  be<br>  $: SE(Y) \rightarrow \Box (A)^*$  given by  $||x||_y = ||x||$  for all<br>
norm  $|| \cdot ||_y$  is known as the relative norm incord a normed subspace or simply a subspace of<br> **nition (3.12)**<br>

This norm  $\|\cdot\|_Y$  is known as the relative norm induced on Y by  $\|\cdot\|$ . The soft normed space  $(Y, \|\cdot\|_Y, A)$  is 8<br>
( $Y$ ,  $\left\| \cdot \right\|_Y$ , A) is ion $(Y, \left\| \cdot \right\|_{Y}, A)$  is called a normed subspace or simply a subspace of the soft normed space  $(X, \|\cdot\|, A)$ .  $\begin{aligned} 1\|y\| &= \|x\| + \|y\| \ S(X) \,. \end{aligned}$  Then the function  $Y$ .<br>  $e$  soft normed space  $(Y, \|\cdot\|_Y, A)$  is  $(X, \|\cdot\|, A)$ .

### **Definition (3.12)**

Let  $(X, \|\cdot\|, A)$  be a soft normed space and  $r \ge 0$  be a soft real number. We define the followings;

be a soft normed space and 
$$
\tilde{r} \ge \tilde{0}
$$
 be a soft real number. We define the fo  
\n
$$
\beta(a, \tilde{r}) = \{x : \|x - a\| < \tilde{r}\} \subset SE(X), \ \overline{\beta}(a, \tilde{r}) = \{x : \|x - a\| \le \tilde{r}\} \subset SE(X) \text{ and }
$$
\n
$$
S(a, \tilde{r}) = \{x : \|x - a\| = \tilde{r}\} \subset SE(X)
$$
\nand  $S(a, \tilde{r})$  are respectively called an open ball, a closed ball and a sph  
\n
$$
S(\beta(a, \tilde{r}))
$$
, 
$$
SS(\overline{\beta}(a, \tilde{r}))
$$
 and 
$$
SS(S(a, \tilde{r}))
$$
 are respectively called a soft op

firmal of Iraqi AI-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>  $|x + y|| = d(x + y, \hat{0}) = d(x, -y) \leq d(x, \hat{0}) + d(\hat{0}, -y) = ||x|| + ||-1||y|| = ||x|| + ||y||$ <br>
1)<br>
1)<br>
asoft normed space and  $(Y, A)$  be a non-null member of  $S(X)$ . Then the *Journal of Traqi* Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>  $||x + y|| = d(x + y, \delta) = d(x, -y) \leq d(x, \delta) + d(\delta, -y) = ||x|| + ||-1||y|| = ||x|| + ||y||$ <br> **Definition (3.11)**<br> **Solution (3.11)**<br>  $||\cdot||_r$  is known as the relative norm and are respectively called an open ball, a closed ball and a sphere with centre at *a* and radius r.  $SS(\beta(a,r))$ ,  $SS(\beta(a,r))$  and  $SS(S(a,r))$  are respectively called a soft open ball, a soft closed *r*  $\pi$ <br> *r*  $\pi$  **r** and subspace or simply a subspace of the soft norme<br>
(3.12)<br> *f*  $\alpha, \tilde{r}$  = { $x$  :  $\|x-a\| < \tilde{r}$ }  $\subset$   $SE(X)$ ,  $\overline{\beta}(a, \tilde{r}) = \{x : \|B\| > 0$ <br>  $S(a, \tilde{r}) = \{x : \|x-a\| = \tilde{r}\} \subset SE$ <br>
(*a,*  $\tilde{r}$ ) and  $\hat{S}$  ≥  $\tilde{O}$  be a soft real number. We  $(\tilde{X})$ ,  $\overline{\beta}(a, \tilde{r}) = \{x : ||x - a|| \le \tilde{r}\}$   $\subset$ <br> $[x : ||x - a|| = \tilde{r}\}$   $\subset$   $SE(X)$ <br>y called an open ball, a closed b.<br> $SS(S(a, \tilde{r}))$  are respectively cal dius  $\tilde{r}$ . ball and a soft sphere with centre at  $x$  and radius  $r$ . *r* .  $\hat{f} = \{x : ||x - a|| \le \tilde{r}\} \subset SE(X)$  and<br>  $\hat{f} \subset SE(X)$ <br>
pen ball, a closed ball and a sphere with central<br>
are respectively called a soft open ball, a soft<br>  $(X, ||\cdot||, A)$  is said to be convergent in  $(X, ||\cdot||)$ <br>  $\rightarrow \infty$ . This mean

#### **Definition (3.13)**

A sequence of soft elements  $S(\beta(a,r))$  and  $SS(S(a,r))$  are i<br>intre at x and radius  $\tilde{r}$ .<br> ${x_n}$  in a soft normed space  $(X,$ in a soft normed space  $(X, \|\cdot\|, A)$  is said to be convergent in  $(X, \|\cdot\|, A)$  if th centre at *a*<br>
, a soft closed<br>  $(X, \|\cdot\|, A)$  if<br>
sen arbitrarily, there is a soft element *x*<sub>*x*</sup>, *x*<sub>*x*</sub> *x x* and radius  $\tilde{r}$ .<br> *x* and radius  $\tilde{r}$ .<br> *x* and radius  $\tilde{r}$ .<br> *x* and *x* and radius  $\tilde{r}$ .<br> *x* and *x* and radius  $\tilde{r}$ .<br> *x* and *x* and *x* and *x* and *x* and *x* and *x* </sub> such that  $||x_n - x|| \to 0$  as  $n \to \infty$ . This means for every  $\varepsilon > 0$ , chosen arbitrarily,  $\subset SE(X)$ ,  $\overline{\beta}(a, \tilde{r}) = \{x : ||x - a|| \leq \tilde{r}\} \subset SE(X)$  and<br>  $\tilde{r}$   $= \{x : ||x - a|| = \tilde{r}\} \subset SE(X)$ <br>
tively called an open ball, a closed ball and a sphere with centre at<br>
and  $SS(S(a, \tilde{r}))$  are respectively called a soft open ball, there exists a natural number  $B(\overline{\beta}(a, r))$  and  $SS(S(a, r))$  are resp<br>tire at x and radius  $\overline{r}$ .<br> $x_n$  in a soft normed space  $(X, \|\cdot\|)$ ,<br>uch that  $||x_n - x|| \to 0$  as  $n \to \infty$ . T<br> $k = k(\varepsilon)$ , such that  $\overline{0} \le ||x_n - x|| < \varepsilon$ <br> $n \to \infty$  or by , such that  $\overline{\beta}(a,\tilde{r}) = \{x : ||x - a|| \leq \tilde{r}\} \subset SE(X)$  and<br>  $-a||=\tilde{r}\} \subset SE(X)$ <br>
ed an open ball, a closed ball and a sphere with centre at *a*<br>  $(a,\tilde{r})$  are respectively called a soft open ball, a soft closed<br>  $\tilde{r}$ .<br>
space  $(X, ||\cdot||, A$ be convergent in  $(X, \|\cdot\|, A)$  if<br>every  $\varepsilon > 0$ , chosen arbitrarily,<br> $n > k$ . i.e.,  $n > k$   $x \in \beta(x, \varepsilon)$ . . i.e., *n* externally a soft closed<br> *n* ball, a soft closed<br> *n* in  $(X, \|\cdot\|, A)$  if<br> *n*, chosen arbitrarily,<br>  $n > k$   $x \in \beta(x, \varepsilon)$ . We denote this by  $x_n \to x$  as  $n \to \infty$  or by Elements  $\{x_n\}$  in a soft normed space  $(X, \|\cdot\|, A)$  is said to be c<br>
nt  $x \in X$  such that  $||x_n - x|| \to 0$  as  $n \to \infty$ . This means for eve<br>
1 number  $k = k(\varepsilon)$ , such that  $0 \le ||x_n - x|| < \varepsilon$ , whenever  $n >$ <br>  $x_n \to x$  as  $n \to \infty$  and a soft sphere with centre at<br> **inition** (3.13)<br>
equence of soft elements { $x_n$ } in<br>
e is a soft element  $x \in X$  such th<br>
e exists a natural number  $k = k$  (<br>
denote this by  $x_n \to x$  as  $n \to$ <br>  $x_n = x$ , x is said to be the l k. i.e.,  $n > k$   $x \in \beta(x, \varepsilon)$ .<br>( $\Box$ ,  $\Vert \cdot \Vert$ ) or  $(\Box$ ,  $\Vert \cdot \Vert$ , A) be<br>rameters. Let  $Y_A \subset \Box$  such  $(X, \| \cdot \|, A)$  if<br>
bsen arbitrarily,<br>  $x \in \beta(x, \varepsilon)$ .<br>  $(\Box, \| \cdot \|, A)$  be<br>  $\forall t \in Y_A \subset \Box$  such<br>
of *Y* where

 $\lim_{n\to\infty} x_n = x$ , x is said to be the limit of the sequence  $x_n$  as  $n \to \infty$ .

#### **Example (3.14)**

Let us consider the set of all real numbers endowed with the usual norm  $\|\cdot\|$ . Let  $(\Box, \|\cdot\|)$  or  $(\Box, \|\cdot\|, A)$  be the soft norm generated by the crisp norm  $\|\cdot\|$ , where A is the nonempty set of parameters. Let  $Y_A \subset \Box$  such that  $Y(e) = (0,1]$  in the real line, for all *Y*: exists a natural number  $k = k(\varepsilon)$ , such that  $||x_n - \varepsilon|$  exists a natural number  $k = k(\varepsilon)$ , such denote this by  $x_n \to x$  as  $n \to \infty$  or both  $x_n = x$ , x is said to be the limit of the **mple (3.14)**<br>as consider the set Figure 2.1 and  $\|\cdot\|$ . Let  $\Box$ <br> *e*  $\in$  *A*. Let us choose a sequence  $\{x_n\}$  of soft  $\Box$ <br> *e*  $\in$  *A*. Let us choose a sequence  $\{x_n\}$  of soft . Let us choose a sequence  $\|\cdot\|$ . Let  $(\Box, \|\cdot\|)$  or  $(\Box, \|\cdot\|, A)$ <br>set of parameters. Let  $Y_A \subset \Box$  su<br> $\{x_n\}$  of soft elements of  $Y_A$  when of soft elements of  $Y_A$  where  $\tilde{z}_{n}(e) = \frac{1}{n}$  for all  $n \in \mathbb{Z}$ , for all  $e \in A$ .  $\lim_{n \to \infty} x_n = x$ , *x* is said to be the<br> **Example (3.14)**<br>
et us consider the set of all reverse to the last  $Y(e) = (0,1]$  in the real line<br>  $x_n(e) = \frac{1}{n}$  for all  $n \in \mathbb{Z}$ , for all *n*  $f(x) = \frac{1}{n}$  for all  $n \in \mathbb{Z}$ , for all  $e \in A$ . Then there is  $x \in Y_A$  such that  $x_n \to x$  in  $(Y, \|\cdot\|_Y, A)$ . However the I numbers endowed with the usual norm  $\|\cdot\|$ . Let  $(\square, \|\cdot\|)$  or  $(\square, \text{sign norm})\|\cdot\|$ , where *A* is the nonempty set of parameters. Let *Y* for all  $e \in A$ . Let us choose a sequence  $\{x_n\}$  of soft elements of  $e \in A$ . Then t *xm*  $\|\cdot\|$ . Let  $(\Box, \|\cdot\|)$  or  $(\Box, \|\cdot\|, A)$  be<br>pty set of parameters. Let  $Y_A \subset \Box$  such<br>ce  $\{x_n\}$  of soft elements of  $Y_A$  where<br> $x_n \to x$  in  $(Y, \|\cdot\|_Y, A)$ . However the  $\forall x \in \mathbb{R}^n, n \geq k$   $x \in p(x, \varepsilon)$ .<br>
t  $(\Box, \Vert \cdot \Vert)$  or  $(\Box, \Vert \cdot \Vert, A)$  be arameters. Let  $Y_A \subset \Box$  such soft elements of  $Y_A$  where  $(Y, \Vert \cdot \Vert_Y, A)$ . However the *K*. I.e.,  $n > k$   $x \in p(x, \varepsilon)$ .<br>  $(\Box, \Vert \cdot \Vert)$  or  $(\Box, \Vert \cdot \Vert, A)$  be<br>
arameters. Let  $Y_A \subset \Box$  such<br>
oft elements of  $Y_A$  where<br>  $[Y, \Vert \cdot \Vert_Y, A)$ . However the

sequence **Journal of Iraqi Al-Khwa**<br>{ $y_n$ } of soft elements of  $Y_A$ of soft elements of  $Y_A$  where  $y_n(e) = \frac{1}{2}$  for all  $n \in \mathbb{Z}$ , for all  $e \in A$ . IKh) Volume:8 Issue:2 Year: 2024 pa<br>  $y_n(e) = \frac{1}{2}$  for all  $n \in \mathbb{Z}$ , for all  $e \in A$ . for all  $n \in \mathbb{Z}$ , for all  $e \in A$ . is convergent in 924 pages:  $44-68$ <br>*e*  $\in$  *A*. is convergent in Journal of Iraqi Al-Khwarizmi (<br>sequence  $\{y_n\}$  of soft elements of  $Y_A$  where<br> $(Y, \|\cdot\|, A)$  and converges to  $\frac{1}{2}$ .<br>Theorem (3.15) and converges to  $\frac{1}{2}$ . 1 2<sup>2</sup>

### **Theorem (3.15)**

Limit of a sequence in a soft normed space, if exists is unique.

### Proof :

 If possible let there exists a sequence form, if exists is unique.<br>{ $x_n$ } of soft elements in a soft not<br>Then there is at least one a  $\epsilon \in A$ . of soft elements in a soft normed space  $(X, \|\cdot\|, A)$  such onvergent in<br>  $(X, \| \cdot \|, A)$  such<br>  $-y \| (e) \neq 0$ . We that  $\lim_{n \to \infty} x_n = x$ ,  $\lim_{n \to \infty} x_n = y$ , where  $x \neq y$ . Then ther 4) and converges to  $\frac{1}{2}$ .<br> **n** (3.15)<br>
a sequence in a soft normed spa<br>
ossible let there exists a sequence  $x_n = x$ ,  $\lim_{n \to \infty} x_n = y$ , where  $x \neq$ <br>
a positive real number  $\varepsilon_e$  satisfy  $\lim_{n \to \infty} x_n = x$ ,  $\lim_{n \to \infty} x_n = y$ , where  $x \neq y$ . Then there is at least erges to  $\frac{1}{2}$ .<br>in a soft normed space, if exists<br>nere exists a sequence { $x_n$ } of so<br> $x_n = y$ , where  $x \neq y$ . Then thereal number  $\varepsilon_e$  satisfying  $0 < \varepsilon_e$  $\lim_{x \to \infty} x_n = y$ , where  $x \neq y$ . Then there is *x* and *x* a *x* is unique.<br> **1** we are  $\{x_n\}$  of soft elements in a so  $x \neq y$ . Then there is at least one expansion . Then there is at least one soft normed space  $(X, \|\cdot\|, A)$  such<br>  $e \in A$  such that  $||x - y|| (e) \neq 0$ . We such that convergent in<br>  $e(X, ||\cdot||, A)$  such<br>  $x - y || (e) \neq 0$ . We consider a positive real number  $\varepsilon_e$  satisfying  $0 < \varepsilon_e < \frac{1}{2} ||x - y|| (e)$ . JIKh) Volume:8 Issue:2 Year: 2024 p<br>  $y_n(e) = \frac{1}{2}$  for all  $n \in \mathbb{Z}$ , for all  $e \in A$ .<br>
f exists is unique.<br>
f exists is unique.<br>  $x_n$ } of soft elements in a soft normed sp<br>
hen there is at least one  $e \in A$  such that<br>  $e \rightarrow \mathbf{e}$   $\mathbf{v}$   $\mathbf{v}$ Kh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>  $v_n(e) = \frac{1}{2}$  for all  $n \in \mathbb{Z}$ , for all  $e \in A$ , is convergent in<br>
exists is unique.<br>
<br>
<br>
} of soft elements in a soft normed space  $(X, \|\cdot\|, A)$  such<br>
nn there is at least Let there exists a sequence  $\{x_n\}$  of soft elements in a soft normed space  $(X, \|\cdot\|, A)$  such<br>  $\lim_{n \to \infty} x_n = y$ , where  $x \neq y$ . Then there is at least one  $e \in A$  such that  $||x - y||(e) \neq 0$ . We<br>
tive real number  $\varepsilon_e$  sat It elements in a soft normed space  $(X, \|\cdot\|, A)$  s<br>
e is at least one  $e \in A$  such that  $||x - y||(e) \neq 0$ .<br>  $\frac{1}{2}||x - y||(e)$ .<br>  $k_1 = k_1(\varepsilon)$ ,  $k_2 = k_2(\varepsilon)$  such that  $n > k_1$ <br>  $k_2 \in \varepsilon_e$ , in particular. Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
sequence  $\{y_n\}$  of soft elements of  $Y_k$  where  $y_n(e) = \frac{1}{2}$  for all *n* ∈ l, , for all *e* ∈ A, is convergent in<br>  $(x, | \cdot |, A)$  and conver *e* of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br> *of* dements of  $Y_x$  where  $y_n(e) = \frac{1}{2}$  for all  $n \in \mathbb{N}$ , for all  $e \in A$ , is convergent in<br> *verges* to  $\frac{1}{2}$ .<br>
in a soft normed space, i Journal of Iraqi Al-Khwarizmi (JIKh) Volume: 8 Issue: 2 Year: 2024 pages: 44-68<br>
cc [y<sub>n</sub>] of soft elements of  $Y_x$  where  $y_x(e) = \frac{1}{2}$  for all  $\pi \in \mathbb{N}$ , for all  $e \in A$ , is convergent<br>
(A) and converges to  $\frac{1}{2}$ . Chwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br> *f*  $Y_a$  where  $y_a(e) = \frac{1}{2}$  for all  $n \in \mathbb{D}$ , for all  $e \in A$ , is convergent in<br>
read space, if exists is unique.<br>
sequence  $\{x_*\}$  of soft elements in a so xumal of fraqi Al-Khwarizmi (HKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
of soft elements of  $Y_k$  where  $y_k(e) = \frac{1}{2}$  for all  $n \in \mathbb{D}$ , for all  $e \in A$ , is convergent in<br>
2 converges to  $\frac{1}{2}$ .<br>
5)<br>
5)<br>
1 anoce Somptime 1 and  $x_n = x$ ,  $\lim_{n \to \infty} x_n = y$ , where  $x \neq$ <br>
a positive real number  $\varepsilon_e$  satisf<br>
with  $\varepsilon(e) = \varepsilon_e$ . Since  $x_n \to x$ <br>
anding to  $\varepsilon > 0$ , there exist nature  $\beta(x, \varepsilon) \Rightarrow \|x_n - x\| < \varepsilon \Rightarrow$ <br>  $\Rightarrow k_2 \Rightarrow x_n \in \beta(y, \varepsilon) \Rightarrow \|x$ If possible let there exists a sequence { $x_n$ } of soft elements in a soft normed<br>  $\lim_{n\to\infty} x_n = x$ ,  $\lim_{n\to\infty} x_n = y$ , where  $x \neq y$ . Then there is at least one  $e \in A$  such t<br>
sider a positive real number  $\varepsilon_e$  satisfying d space  $(X, \|\cdot\|, A)$  such<br>
that  $||x - y||(e) \neq 0$ . We<br>
that  $n > k_1$ <br>
cular.<br>  $\|\cdot\|x_n - y\|(e) < 2\varepsilon_e$ <br>  $\{\|x_n - x_m\| : n, m \in \mathbb{Z}\}$  of soft real<br>  $n \in \mathbb{Z}$ 

Let  $\varepsilon > 0$  with  $\varepsilon(e) = \varepsilon_e$ . Since  $x_n \to$ 

Corresponding to  $\varepsilon > 0$ , there exist natural numbers  $k_1 = k_1(\varepsilon)$ ,  $k_2 = k_2(\varepsilon)$  such that  $\Rightarrow x_n \in \beta(x, \varepsilon) \Rightarrow ||x_n - x|| < \varepsilon \Rightarrow ||x_n - x|| (e) < \varepsilon_e$ , in particular.

Also,  $n > k$ ,  $\Rightarrow x_n \in \beta(y, \varepsilon) \Rightarrow ||x_n - y|| < \varepsilon \Rightarrow ||x_n - y|| (e) < \varepsilon$ , in particular.

Hence for all  $n > k = \max\{k_1, k_2\}$ ,  $\Rightarrow$   $||x - y|| < \varepsilon \Rightarrow ||x - y||(e) \le ||x_n - x||(e) + ||x_n - y||(e) < 2\varepsilon_e$ 

### **Definition (3.16)**

So,  $\varepsilon_e > \frac{1}{2} ||x - y||(e)$ . This contradicts our hypothesis. Hence the result follows.<br> **Definition (3.16)**<br>
A sequence  $\{x_n\}$  of soft elements in  $(X, ||\cdot||, A)$  is said to be bounded if the set A sequence  $\{x_n\}$  of soft elements in  $(X, \|\cdot\|, A)$  is said to be bounded if the set  $\{\|x_n - x_m\| : n, m \in \Box\}$  of soft real  $\|\mathbf{x}_n - \mathbf{x}\| \le \varepsilon \ \Rightarrow \ \|\mathbf{x}_n - \mathbf{y}\| \le \varepsilon$ , in particular.<br>  $\|\mathbf{x}_n - \mathbf{y}\| \le \varepsilon \ \Rightarrow \ \|\mathbf{x}_n - \mathbf{y}\| \le \varepsilon \le \varepsilon$ , in particularity is our hypothesis. Hence the result follows.<br>  $(\mathbf{X}, \|\cdot\|, \mathbf{A})$  is said to be bo numbers is bounded, i.e., the there exist  $k > 0$  such that  $||x_n - x_m|| \le k$  for all *x* - *y*  $\|(e) \le \varepsilon_e$ , in particular.<br> *x* - *y*  $\|(e) \le \|x_n - x\|(e) + \|x_n - y\|(e) < 2\varepsilon_e$ <br>
nce the result follows.<br>
be bounded if the set  $\{\|x_n - x_m\| : n, m \in \square\}$  of soft real<br>  $x_n - x_m \| \le k$  for all  $n, m \in \square$  $(e) + ||x_n - y|| (e) < 2\varepsilon_e$ <br>ws.<br>et  $\{||x_n - x_m|| : n, m \in \square \}$  of soft real<br> $n, m \in \square$  $\|x_m\|$ : *n*, *m*  $\in$   $\Box$  } of soft real<br> *n*, *m*  $\geq$  *k*, i.e.,

### **Definition (3.17)**

A sequence  $\{x_n\}$  of soft elements in a soft normed space  $(X, \|\cdot\|, A)$  is said to be a Cauchy sequence in bounded, i.e., the there exist  $k > 0$ <br>3.17)<br>{ $x_n$ } of soft elements in a soft no<br>conding to every  $s > 0$ , there exist the result follows.<br>
be bounded if the set  $\{\Vert x_n - x_m \Vert : n, m \in \Box \}$  of<br>  $x_n - x_m \Vert \le k$  for all  $n, m \in \Box$ <br>
(X,  $\Vert \cdot \Vert$ , A) is said to be a Cauchy sequence in<br>
the that  $\Vert x_n - x_m \Vert \le k$ , for all  $n, m \ge k$ , i.e., X if corresponding to every  $\varepsilon > 0$ , there exist  $k \in \mathbb{Z}$  such that  $||x_n - x_m|| \le k$ , for all  $n, m \ge k$ , i.e., result follows.<br> **nded if the set**  $\{\Vert x_n - x_m \Vert : n, m \in \Box \}$  of soft real<br>  $\Vert \leq k$  for all  $n, m \in \Box$ <br> *x*, *A*) is said to be a Cauchy sequence in<br>  $x_n - x_m \Vert \leq k$ , for all  $n, m \geq k$ , i.e., *x*  $c_{\epsilon} > \frac{1}{2} ||x - y||(\epsilon)$ . This contradicts our hypothesis. Hence the rest<br>**Definition (3.16)**<br>A sequence { $x_n$ } of soft elements in  $(X, ||\cdot||, A)$  is said to be bounded<br>umbers is bounded, i.e., the there exist  $k > 0$  such of soft elements in  $(X, \|\cdot\|, A)$  is said to be bounded<br>
2d, i.e., the there exist  $k > 0$  such that  $||x_n - x_m|| \le k$ <br> *f* soft elements in a soft normed space  $(X, \|\cdot\|, A)$  is<br>
1g to every  $\varepsilon > 0$ , there exist  $k \in \square$  such tha

### **Theorem (3.18)**

Every convergent sequence in a soft normed linear space is Cauchy and every Cauchy sequence is bounded.

Proof :

 Let  ${x_n}$  be a convergent sequence of  ${x_n}$ be a convergent sequence of soft elements with limit  $x$  (say) in  $(X, \|\cdot\|)$ 

124 pages: 44-68<br>( $X$ ,  $\|\cdot\|$ )<br> $\|\cdot\|$ Then corresponding to each  $\varepsilon > 0$ , there exists  $k \in \mathbb{Z}$  such that  $x_n \in \beta(x, \frac{\varepsilon}{2})$  i.e.,  $||x_n - x|| \leq \frac{\varepsilon}{2}$  for all First 2024 pages: 44-68<br>
(ay) in  $(X, \|\cdot\|)$ <br>
( $(x, \frac{\varepsilon}{2})$  i.e.,  $||x_n - x|| \leq \frac{\varepsilon}{2}$  for all<br>
ce  $\{x_n\}$  is ue:2 Year: 2024 pages: 44-68<br>
int *x* (say) in  $(X, \|\cdot\|)$ <br>  $x_n \in \beta(x, \frac{\varepsilon}{2})$  i.e.,  $||x_n - x|| \leq \frac{\varepsilon}{2}$  for<br>  $\therefore$  Hence  $\{x_n\}$  is 2  $\overline{\phantom{a}}$ pages: 44-68<br>  $\cdot$  |  $\cdot$  |  $\cdot$  |  $\leq \frac{\varepsilon}{2}$  for all ges: 44-68<br> $-x \leq \frac{\varepsilon}{2}$  for all Let  $\{x_n\}$  be a convergent sequence of soft elements with limit  $x$  (say) in  $(X, \|\cdot\|)$ <br>
Then corresponding to each  $\varepsilon > 0$ , there exists  $k \in \square$  such that  $x_n \in \beta$  $(x, \frac{\varepsilon}{2})$  i.e.,  $\|x_n - x\| \leq \frac{\varepsilon}{2}$  for all<br>  $n$ Then for  $n, m \ge k$ ,  $||x_n - x_m|| \le ||x_n - x|| + ||x - x_m|| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$ . Hence  $\{x_n\}$  is<br>chy sequence.<br>let  $\{x_n\}$  be a Cauchy sequence of soft elements in  $(X, ||\cdot||)$ . Then there exis *nnal* of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages:<br> *n* a convergent sequence of soft elements with limit  $x$  (say) in  $(X, \|\cdot\|)$ <br>
ding to each  $\varepsilon > 0$ , there exists  $k \in \square$  such that  $x_n \in \beta(x, \frac{\varepsilon}{$ *x*<sup>*x*</sup> *x*<sub>*x*</sub> *x*<sub>*x*</sub> *x*<sub>*x*</sub> *x x*<sup>*x*</sup> *x x x*<sup>*x*</sup> *x x*<sup>*x*</sup> *x x*<sup>*x*</sup> *x x*<sup>*x*</sup> *x*<sup>*x*</sup> Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
t sequence of soft elements with limit  $x$  (say) in  $(X, \|\cdot\|)$ <br>  $\varepsilon > 0$ , there exists  $k \in \square$  such that  $x_n \in \beta(x, \frac{\varepsilon}{2})$  i.e.,  $||x_n - x|| \leq \frac{\varepsilon}{2}$  for all ch that  $x_n \in \beta(x, \frac{\varepsilon}{2})$  i.e.,  $||x_n - x|| \leq \frac{\varepsilon}{2}$ <br> $\frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$ . Hence  $\{x_n\}$  is<br> $(X, ||\cdot||)$ . Then there exists  $k \in \mathbb{Z}$  such  $x\{\|x_n - y_m\|(e) : 1 \leq n, m \leq k\}$  for all  $e \in$ *x x*<sub>*n*</sub> *y* be a convergent sequence of soft elements w<br> *x* **z** *z k x*<sub>*n*</sub> be a convergent sequence of soft elements w<br> *x x x x k n n <i>m x k* ,  $||x_n - x_m|| \le ||x_n - x|| + ||x - x_m|| < \frac{\varepsilon}{2}$ <br> *x x x n* to each  $\varepsilon > 0$ , there exists  $k \in \mathbb{Z}$  such<br>  $\geq k$ ,  $||x_n - x_m|| \leq ||x_n - x|| + ||x - x_m|| < \frac{\varepsilon}{2}$ <br>
auchy sequence of soft elements in (*i*,  $n, m \geq k$ . Take *M* with  $M(e) = \max\{n, m \geq k, ||x_n - y_m|| \leq ||x_n - y_k|| + ||x_k - y_k||$ *MKh)* Volume:8 Issue:2 Year: 2024 pages: 44-68<br> *ft* elements with limit *x* (say) in  $(X, \|\cdot\|)$ <br>  $\leq k \in \square$  such that  $x_n \in \beta(x, \frac{\varepsilon}{2})$  i.e.,  $\|x_n - x\| \leq \frac{\varepsilon}{2}$  for all<br>  $\|x - x_m\| \leq \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$ . Henc *m m n m k m <i>k n n n n n e e k n n n n x e <i>n n x n n x e <i>g* (*x*, *n n n x e <i>g* (*x*, *n n n x e e y n n x n a x e g* (*x*, *x*<sub>*x*</sub> be a convergent sequence of soft elements with limit *x* (say) in  $(X, \|\cdot\|)$ <br>orresponding to each  $\varepsilon > 0$ , there exists  $k \in \|\cdot\|$  such that  $x_n \in \beta(x, \frac{\varepsilon}{2})$  i.e.,  $\|x_n - x\|$ <br>then for  $n, m \ge k$ ,  $\|x_n - x_m\| \le \|x$ 

a Cauchy sequence.

Next let  $\{x_n\}$  be a Cauchy sequence of soft elements in  $(X, \|\cdot\|)$ . Then there exists  $k \in \square$  such that , for all  $n,m \ge k$ . Take M with  $M(e) = \max\{\Vert x_n - y_m \Vert (e): 1 \le n, m \le k\}$  for all *e*  $\in$  *A* . Then for responding to each  $\varepsilon > 0$ , there exists  $\kappa \in$ <br>
en for  $n, m \ge k$ ,  $||x_n - x_m|| \le ||x_n - x|| + ||x - x||$ <br>
sequence.<br>  $[x_n]$  be a Cauchy sequence of soft elemen<br>  $\langle x_n \rangle$  be a Cauchy sequence of soft elemen<br>  $\langle x_n \rangle$  be a Cauchy sequenc and and the same of soft elements in  $(X, \|\cdot\|)$ . Then there e<br> *n* axe *M* with *M*(*e*) = max{ $\|x_n - y_m\|$ (*e*) : 1 ≤ *n*, *n*<br>  $\|x_n - y_m\|$  ≤  $\|x_n - y_k\|$  +  $\|x_k - y_m\|$  < *M* + 1 .<br> *n*, *m* ∈ □ and consequently the sequence  $m \ge k$ . Take *M* with  $M(e) = \max\{\left\|x_n\right\}$ <br>  $m \ge k$ ,  $\left\|x_n - y_m\right\| \le \left\|x_n - y_k\right\| + \left\|x_k - y_m\right\|$ <br>
for all  $n, m \in \square$  and consequently the<br>  $Y(e) \ne \phi$  for all  $e \in A$ , in a soft norme<br>
1 number *k* such that  $\left\|x\right\| \le k$  for In there exists  $k \in \Box$  such that<br>  $\{ :1 \le n, m \le k \}$  for all  $e \in A$ .<br>  $\Box$ <br>  $\Box$   $(X, \Vert \cdot \Vert, A)$  is said to be bounded

Thus,  $||x_n - y_m|| < M + 1$  for all  $n, m \in \square$  and consequently the sequence is bounded.

### **Definition (3.19)**

A soft subset  $Y_A$  with  $Y(e) \neq \emptyset$  for all  $e \in A$ , in a soft normed space  $(X, \|\cdot\|, A)$  is said to be bounded and consequently the sequence is bounded.<br>  $e \in A$ , in a soft normed space  $(X, \|\cdot\|, A)$  is<br>
that  $\|x\| \le k$  for all  $x \in Y_A$ . if there exists a soft real number k such that  $||x|| \le k$  for all x the sequence is bounded.<br> *A*  $\text{Area}$  *x*  $\in$  *Y<sub>A</sub>*.<br> *X*  $\in$  *Y<sub>A</sub>*. Therefore  $X_m \parallel \cdots \parallel X_m \parallel \cdots \parallel X_m \parallel$ <br>  $\cdots \parallel X_m \parallel \cdots \parallel X_m \parallel$ <br>  $\cdots \parallel X_m \parallel \cdots \parallel X_m \parallel$ <br>  $\cdots \parallel X_m \parallel \cdots \parallel X_m \parallel$ <br>  $\cdots \parallel X_m \parallel \cdots \parallel X_m \parallel$ <br>  $\cdots \parallel X_m \parallel \cdots \parallel X_m \parallel$ <br>  $\cdots \parallel X_m \parallel \cdots \parallel X_m \parallel$ <br>  $\cdots \parallel X_m \parallel \cdots \parallel X_m \parallel$ <br>  $\cdots \parallel X_m \parallel \cdots \parallel X_m \parallel$ <br>  $\cdots \parallel X_m \$ oft subset  $Y_A$  with  $Y(e) \neq \phi$  for all  $e \in A$ , in<br>ere exists a soft real number k such that  $||x|| \le$ <br>**nition (3.20)**<br>oft normed space  $(X, ||\cdot||, A)$  is said to be con<br>element of X i.e., every complete soft norm<br>**orem (3.21)** *n*, *m* ∈ *□* and consequently the sequence is bounded.<br>
for all  $e \in A$ , in a soft normed space  $(X, ||\cdot||, A)$  is said to be bounded<br>  $\cdot k$  such that  $||x|| \le k$  for all  $x \in Y_A$ .<br>
is said to be complete if every Cauchy sequen for all  $e \in A$ , in a soft normed space  $(X, \|\cdot\|, A)$  is said to be bounded  $x$  *k* such that  $\|x\| \le k$  for all  $x \in Y_A$ .<br>
is said to be complete if every Cauchy sequence in *X* converges to amplete soft normed space is cal

### **Definition (3.20)**

A soft normed space  $(X, \|\cdot\|, A)$  is said to be complete if every Cauchy sequence in X converges to a soft element of X i.e., every complete soft normed space is called a soft Banach's Space. *x* it normed space  $(X, \|\cdot\|, A)$  is said to be complete if every Cau lement of *X* i.e., every complete soft normed space is called **rem (3.21)**<br> $X, \|\cdot\|, A$  be a soft normed space. Then<br> $x_n \to x$  and  $y_n \to y$  then  $x_n + y_n \to x +$ *n*  $(X, \|\cdot\|, A)$  is said to be complete if every Cauchy sequence<br>i.e., every complete soft normed space is called a soft Banach<br>a soft normed space. Then<br> $y_n \to y$  then  $x_n + y_n \to x + y$ .<br> $\lambda_n \to \lambda$  then  $\lambda_n \cdot x_n \to \lambda \cdot x$ ., where i.e., every complete soft normed space is called a soft Banach<br> *n* a soft normed space. Then<br>  $y_n \to y$  then  $x_n + y_n \to x + y$ .<br>  $\lambda_n \to \lambda$  then  $\lambda_n \cdot x_n \to \lambda \cdot x$ ., where  $\{\lambda_n\}$  is a sequence of soft<br> *n* are Cauchy sequences i

### **Theorem (3.21)**

Let  $(X, \|\cdot\|, A)$  be a soft normed space. Then  $\|\cdot\|$ , *A*) be a soft normed space. The<br>  $\lambda_n \to x$  and  $y_n \to y$  then  $x_n + y_n \to x$ <br>  $\lambda_n \to x$  and  $\lambda_n \to \lambda$  then  $\lambda_n \cdot x_n \to \lambda \cdot x_n$  and  $\{y_n\}$  are Cauchy sequences in<br>  $\{x_n + y_n\}$  and  $\{\lambda_n \cdot x_n\}$  are also Cauchy

1. If  $x_n \to x$  and  $y_n \to y$  then  $x_n + y_n \to y$ .<br>2. If  $x_n \to x$  and  $\lambda_n \to \lambda$  then  $\lambda_n \cdot x_n \to \lambda$ <br>3. If  $\{x_n\}$  and  $\{y_n\}$  are Cauchy sequences

2. If  $x_n \to x$  and  $\lambda_n \to \lambda$  then  $\lambda_n \cdot x_n \to \lambda \cdot x$ , where Element of *X* i.e., every complete soft normed space is called<br> **rem** (3.21)<br> *X*,  $\|\cdot\|$ , *A*) be a soft normed space. Then<br>  $x_n \to x$  and  $y_n \to y$  then  $x_n + y_n \to x + y$ .<br>  $x_n \to x$  and  $\lambda_n \to \lambda$  then  $\lambda_n \cdot x_n \to \lambda \cdot x$ ., where  $\{\lambda_n\}$  is a sequence of soft scalars is a sequence of soft scalars. e { $\lambda_n$ } is a sequence of soft scalard  $\lambda_n$ } is a Cauchy sequence of some oft normed space. Then<br>  $\rightarrow$  y then  $x_n + y_n \rightarrow x + y$ .<br>  $\rightarrow \lambda$  then  $\lambda_n \cdot x_n \rightarrow \lambda \cdot x$ ., where { $\lambda$ <br>
re Cauchy sequences in X and { $\lambda_n$ }<br>
{ $\lambda_n \cdot x_n$ } are also Cauchy sequences

3. If  $\{x_n\}$  and  $\{y_n\}$  are Cauchy sequences in X and  $\{\lambda_n\}$  is a Cauchy sequence of is a Cauchy sequence of soft scalars,

then  $\{x_n + y_n\}$  and  $\{\lambda_n \cdot x_n\}$  are also Cauchy sequences in X.

Proof :

1. Since  $x_n \to x$  and  $y_n \to y$ , for  $\varepsilon > 0$ , there exist a positive integers *x<sub>n</sub>*  $\rightarrow$  *x* and  $y_n \rightarrow y$ , for  $\varepsilon > 0$ , there exist a positive integers  $k_1, k_2$  such that  $n > k_1$  and  $||y_n - y|| < \frac{\varepsilon}{2}$  for all  $n > k_2$ . Let  $k = \max\{k_1, k_2\}$ , then both 924 pages: 44-68<br> $k_1, k_2$  such that such that 2<sup>2</sup> Journal of Iraqi Al-1<br> *x<sub>n</sub>* - *x* in *x<sub>n</sub>* - *x* and *y<sub>n</sub>* - <br> *x<sub>n</sub>* - *x*  $\|x\| < \frac{\varepsilon}{2}$  for all *n* > *k*<sub>1</sub> and  $\|$ <br>
elations hold for *n* ≥ *k*. Then  $\frac{\varepsilon}{2}$  for all  $n > k_1$  and  $\|y_n -$ Journal of Iraqi Al-Khwarizmi (JIKh) Volu<br>
1. Since  $x_n \to x$  and  $y_n \to y$ , for  $\varepsilon > 0$ , there exi<br>  $-x \leq \frac{\varepsilon}{2}$  for all  $n > k_1$  and  $||y_n - y|| < \frac{\varepsilon}{2}$  for all  $n > k$ <br>
tions hold for  $n \geq k$ . Then 1 of Iraqi Al-Khwarizmi (JIKh) Vol<br>  $x$  and  $y_n \to y$ , for  $\varepsilon > 0$ , there ex<br>  $n > k_1$  and  $||y_n - y|| < \frac{\varepsilon}{2}$  for all  $n >$ <br>  $\ge k$ . Then and  $2^{2}$ Khwarizmi (JIKh) Volume:8 Is:<br> *y*, for  $\varepsilon > 0$ , there exist a posi<br> *y*<sub>n</sub> - *y*|| <  $\frac{\varepsilon}{2}$  for all *n* > *k*<sub>2</sub>. Let *l*  $\frac{\varepsilon}{2}$  for all  $n > k_2$ . Let  $k = n$ warizmi (JIKh) Volume:8 Issue:2 Year: 2024 page:<br>
, for  $\varepsilon > 0$ , there exist a positive integers  $k_1, k_2$  such<br>  $-y \le \frac{\varepsilon}{2}$  for all  $n > k_2$ . Let  $k = \max\{k_1, k_2\}$ , then bo Volume:8 Issue:2 Year: 2024 pag<br>
re exist a positive integers  $k_1, k_2$  sue<br>  $n > k_2$ . Let  $k = \max\{k_1, k_2\}$ , then b . Let  $k = \max\{k_1, k_2\}$ , then both the above ssue:2 Year: 2024 pages: 44-68<br>
sitive integers  $k_1, k_2$  such that<br>  $k = \max\{k_1, k_2\}$ , then both the above<br>  $||y_1 - y|| \le \frac{\varepsilon}{\epsilon} + \frac{\varepsilon}{n} = \varepsilon$  for  $n \ge k$ . relations hold for  $n \geq k$ . Then  $\rightarrow$  *x* and  $y_n \rightarrow y$ , for  $\varepsilon > 0$ , there  $\vert n > k_1$  and  $\Vert y_n - y \Vert < \frac{\varepsilon}{2}$  for all  $n > n \ge k$ . Then urnal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>  $\pi x \to x$  and  $y_n \to y$ , for  $\varepsilon > \hat{0}$ , there exist a positive integers  $k_1, k_2$  such that<br>
call  $n > k_1$  and  $||y_n - y|| < \frac{\varepsilon}{2}$  for all  $n > k_2$ . That  $x + h$  the above  $n \geq k$ . **1.** Since  $x_n \to x$  and  $y_n \to y$ , for  $\varepsilon > 0$ , there exist a positive integers  $k_1, k_2$  such that  $||x_n - x|| < \frac{\varepsilon}{2}$  for all  $n > k_1$  and  $||y_n - y|| < \frac{\varepsilon}{2}$  for all  $n > k_2$ . Let  $k = \max\{k_1, k_2\}$ , then both the above relat 1. Since  $x_n \to x$  and  $y_n \to y$ , for  $\varepsilon > 0$ , there e<br>  $x_n - x \le \frac{\varepsilon}{2}$  for all  $n > k_1$  and  $\|y_n - y\| < \frac{\varepsilon}{2}$  for all  $n >$ <br>
elations hold for  $n \ge k$ . Then<br>  $\|(x_n + y_n) - (x + y)\| = \|(x_n - x) + (y_n - y)\| \le$ <br>  $\Rightarrow$   $x_n + y_n \to x + y$ .<br>
Since *n k x x <sup>n</sup>*

$$
\|(x_n + y_n) - (x + y)\| = \|(x_n - x) + (y_n - y)\| \le \|x_n - x\| + \|y_n - y\| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon \text{ for } n \ge k
$$

2. Since  $\|x_n - x\|^2$ <br>  $\|f(x_n + y_n) - (x + y)\| = \|f(x_n - x) + (y_n - y)\| \le \|x_n - x\| + \|y_n - y\| <$ <br>  $y_n \to x + y$ .<br>  $x_n \to x$  and  $\lambda_n \to \lambda$  we get, for  $\varepsilon > 0$ , there exist a positive i<br>  $\varepsilon$  for all  $n \ge k$ and  $k_1$  and  $||y_n - y|| \le \frac{1}{2}$  for all  $n > k_2$ . Let  $k = \max\{k_1, k_2\}$ , then<br>  $k$ . Then<br>  $(x + y)|| = ||(x_n - x) + (y_n - y)|| \le ||x_n - x|| + ||y_n - y|| \le \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$  for<br>  $\lambda_n \to \lambda$  we get, for  $\varepsilon > 0$ , there exist a positive integers k su we get, for  $\varepsilon > 0$ , there exist a positive integers k such that  $\|x_n - x\| < \varepsilon$  for all  $n \ge k$ *y*.<br> *n*  $\lambda_n \rightarrow \lambda$  we get, for  $\varepsilon > 0$ , then<br>  $n \ge k$ h that<br> $n \geq k$ and  $\lambda_n \to \lambda$  we get, for  $\varepsilon > 0$ , , the<br>  $n \ge k$ <br>  $\|x_n - x\| + \|x\| < \varepsilon + \|x\|$ , for al<br>  $\{\|x_n\|\}$  is bounded.

Now, 
$$
||x_n|| = ||x_n - x + x|| \le ||x_n - x|| + ||x|| < \varepsilon + ||x||
$$
, for all  $n \ge k \implies ||x_n|| < \varepsilon + ||x||$  for all  $n \ge k$ 

Thus the sequence  $\{\Vert x_n \Vert\}$  is bounded.

Journal of Iraqi AI-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68  
\n1. Since 
$$
x_n \rightarrow x
$$
 and  $y_n \rightarrow y$ , for  $\varepsilon > \bar{0}$ , there exist a positive integers  $k_1, k_2$  such that  
\n
$$
||x_n - x|| \leq \frac{\varepsilon}{2}
$$
 for all  $n > k_1$  and 
$$
||y_n - y|| \leq \frac{\varepsilon}{2}
$$
 for all  $n > k_2$ . Let  $k = \max\{k_1, k_2\}$ , then both the above  
\nrelations hold for  $n \geq k$ . Then  
\n
$$
||(x_n + y_n) - (x + y)|| = |[(x_n - x) + (y_n - y)]| \leq ||x_n - x|| + ||y_n - y|| \leq \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon
$$
 for  $n \geq k$ .  
\n⇒  $x_n + y_n \rightarrow x + y$ .  
\n2. Since  $x_n \rightarrow x$  and  $\lambda_n \rightarrow \lambda$  we get, for  $\varepsilon > \bar{0}$ , , there exist a positive integers  $k$  such that  
\n
$$
||x_n - x|| < \varepsilon
$$
 for all  $n \geq k$   
\nNow, 
$$
||x_n|| = ||x_n - x + x|| \leq ||x_n - x|| + ||x|| < \varepsilon + ||x||
$$
, for all  $n \geq k$  ⇒  $||x_n| < \varepsilon + ||x||$  for all  $n \geq k$   
\nThus the sequence  $(||x_n||)$  is bounded.  
\nNow,  
\n
$$
||\lambda_n - x_n - \lambda \cdot x|| = ||\lambda_n - \lambda - x_n + \lambda \cdot x_n - \lambda \cdot x|| = ||(\lambda_n - \lambda) \cdot x_n + \lambda \cdot (x_n - x)| \leq ||(\lambda_n - \lambda) \cdot x_n|| + ||\lambda \cdot (x_n - x)||
$$
  
\nSince  $x_n \rightarrow x$  and  $\lambda_n \rightarrow \lambda$  we get,  $|\lambda_n - \lambda| \rightarrow \bar{0}$  and  $||x_n - x|| \rightarrow \bar{0}$  as  $n \rightarrow \infty$ .  
\nNow using above we get, 
$$
||\lambda_n - x_n - \lambda \cdot x|| \Rightarrow |\
$$

3. Let If and be Cauchy sequences in  $X$ , then for  $\varepsilon > 0$ , there exist a positive integers Now using above we<br>3. Let If  $\{x_n\}$  and  $\{y_n, k_1, k_2 \text{ such that } ||x_n - z|| \}$ such that 2<sup>2</sup>  $\frac{\varepsilon}{2}$  for all  $n \geq k_1$  and  $||y_n$ and 2<sup>2</sup>  $\frac{\varepsilon}{2}$  for all  $n \geq k_2$ *x x n m*

Let  $k = \max\{k_1, k_2\}$ , then both the above relations hold for

Now, 
$$
||(x_n + y_n) - (x_m + y_m)|| = ||(x_n - x_m) + (y_n - y_m)|| \le ||x_n - x_m|| + ||y_n - y_m|| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon
$$
 for  $n, m \ge k$ .

is a Cauchy sequences in *X* .

Since  $\{x_n\}$  is a Cauchy sequences in X, for  $\varepsilon > 0$ , there exist a positive integers k such that  $||x_n + y_n) - (x_m + y_m)|| = ||(x_n - x_n + y_n)||$ <br>is a Cauchy sequences <br> $\{x_n\}$  is a Cauchy sequences in X<br> $||x_n|| \leq \varepsilon$  for all  $n, m \geq k$  $\|x_n - x_m\| < \varepsilon$  for all  $n, m \ge k$ . then both the above relations hold to<br>  $(x_m + y_m) \le ||x_m - x_m + (y_n - y_m)|| \le ||x||$ <br>
Cauchy sequences in *X*.<br>
hy sequences in *X*, for  $\varepsilon > 0$ , there  $n, m \ge k$ .

Taking in particular of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issu $n = m + 1$ ,  $||x_{m+1}|| < \varepsilon$  for all  $n, m \ge k$ , so  $\{||x, \}$ <br>d too. for all  $n, m \geq k$ , so  $\{||x_n||\}$  is bounded. *h*) Volume:8 Issue:2 Year: 2024 pag<br>  $n, m \ge k$ , so  $\{\Vert x_n \Vert\}$  is bounded. g in particular  $n = m + 1$ ,  $||x_{m+1}||$ <br>{ $\lambda_n$ } is bounded too.

Now  $\{\lambda_n\}$  is bounded too.

Journal of Iraqi AI-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68  
\nTaking in particular 
$$
n = m + 1
$$
,  $|x_{m+1}| \le \varepsilon$  for all  $n, m \ge k$ , so  $||x_{m}||$  is bounded.  
\nNow  $(\lambda_{\tau})$  is bounded too.  
\nThen,  $||\lambda_{\tau} \cdot x_{n} - \lambda_{\tau} \cdot x_{m}|| = ||\lambda_{\tau} \cdot (x_{n} - \lambda_{\tau} \cdot x_{m} + \lambda_{\tau} \cdot x_{m} - \lambda_{\tau} \cdot x_{m}|| = ||\lambda_{\tau} \cdot (x_{n} - x_{m}) + (\lambda_{\tau} - \lambda_{m}) \cdot x_{m}||$   
\n $||\lambda_{\tau} \cdot x_{n} - \lambda_{m} \cdot x_{m}|| \le ||\lambda_{\tau}|| \cdot x_{m} - \lambda_{\tau} \cdot x_{m} + \lambda_{\tau} \cdot x_{m} - \lambda_{m} \cdot x_{m}|| = ||\lambda_{\tau} \cdot (x_{n} - x_{m}) + (\lambda_{\tau} - \lambda_{m}) \cdot x_{m}||$   
\n $||\lambda_{\tau} \cdot x_{n} - \lambda_{m} \cdot x_{m}|| \le ||\lambda_{\tau}|| \cdot x_{m} - \lambda_{\tau} \cdot ||\lambda_{\tau}|| \cdot ||\lambda_{\tau}||$ , then  $||\lambda_{\tau}||$   
\n $||\lambda_{\tau}||$  are also Cauchy sequences in  $X$ .  
\n**PROOF**  
\nIf  $\lambda_{\tau} \ge \delta$  and subspace in a soft normed space  $(X, ||\cdot||, \Lambda)$ , then the closure of  $M_A$ ,  $M_A$  is also a soft  
\nsubspace.  
\nProof:  
\nLet  $x, y \in \overline{M_A}$ , we must show that any linear combination of  $x, y$  belongs to  $\overline{M_A}$ .  
\nSince  $x, y \in \overline{M_A}$ , corresponding to  $z > \delta$ , there exists soft elements  $x_1, y_2 \in \overline{M_A}$  such that  
\n $||x - x_1|| < |x - x_1| + |y - y_1| < |x - x_1| + |y - y_1| + |x - x_1| + |y - y_1| + |x$ 

 $\Rightarrow$  { $\lambda_n \cdot x_n$ } are also Cauchy sequences in X. *X* .

### **Theorem (3.22)**

If  $M_A$  is a soft subspace in a soft normed space  $(X, \|\cdot\|, A)$ , then the closure of  $M_A$ ,  $M_A$  is also a soft subspace. ⇒ { $\lambda_n \cdot x_n$ } are also Cauchy sequences in *X*.<br> **Theorem (3.22)**<br>
If *M<sub>A</sub>* is a soft subspace in a soft normed space  $(X, \|\cdot\|, A)$ <br>
subspace.<br>
Proof :<br>
Let *x*, *y*∈ $\overline{M_A}$ , we must show that any linear comb<br>
Since *x x*, *y* belongs to  $\overline{M}$ **em (3.22)**<br>
is a soft subspace in a soft normed space  $(X, |\cos \theta|)$ <br> *Ce.*<br>
Let  $x, y \in M_A$ , we must show that any linear conductions  $x, y \in M_A$ , corresponding to  $\varepsilon > 0$ , there exists  $\|x - x_1\| < \varepsilon$ , is also a soft<br>  $x, y$  belongs to  $\overline{M}_A$ ,  $\overline{M}_A$  is also a soft<br>  $x_1, y_2 \in \overline{M}_A$  such that *x*  $\mathbb{R}^d$  and  $(X, \|\cdot\|, A)$ , then the closure of  $M_A$ ,  $\overline{M_A}$  is also a soft <br>*x* any linear combination of  $x, y$  belongs to  $\overline{M_A}$ .<br>*x* there exists soft elements  $x_1, y_2 \in \overline{M_A}$  such that <br> $x - x_1 \leq \varepsilon$ ,

Proof :

Let  $x, y \in M_A$ , we must show that any linear combination of belongs to  $M_A$ .  $M_A$ .

, corresponding to  $\varepsilon > 0$ , there exists soft elements  $x_1, y_2 \in M_A$  such that

$$
\|x - x_1\| < \varepsilon \,, \quad \|y - y_1\| < \varepsilon
$$

For soft scalars  $\alpha, \beta > 0$ ,

$$
\left\|(\alpha \cdot x + \beta \cdot y) - (\alpha \cdot x_1 + \beta \cdot y_1)\right\| = \left\|\alpha \cdot (x - x_1) + \beta \cdot (y - y_1)\right\| \leq |\alpha| \left\|x - x_1\right\| + |\beta| \left\|y - y_1\right\| \leq \varepsilon (|\alpha| + |\beta|) = \varepsilon' \text{ (say)},
$$

The above inequality shows that  $\alpha x_1 + \beta y_1$  belongs to the open ball  $\beta(\alpha x + \beta y_1, \vec{\epsilon})$ . As  $\alpha x_1 + \beta y_1$ both what any linear combination of x, y belongs to<br>  $\varepsilon > \tilde{0}$ , there exists soft elements  $x_1, y_2 \in \overline{M_A}$  such<br>  $||x - x_1|| < \varepsilon$ ,  $||y - y_1|| < \varepsilon$ <br>  $\alpha \cdot (x - x_1) + \beta \cdot (y - y_1)|| \le | \alpha | ||x - x_1|| + |\beta || |y - y_1||$ <br>  $\alpha x_1 + \beta y_1$  belon closure of  $M_A$ ,  $\overline{M_A}$  is also a soft<br>  $\therefore$ , y belongs to  $\overline{M_A}$ .<br>  $\therefore$ , y <sub>2</sub> $\in \overline{M_A}$  such that<br>  $+ |\beta| \|y - y_1\| < \varepsilon (|\alpha| + |\beta|) = \varepsilon'$  (say),<br>  $\beta(\alpha x + \beta y, \varepsilon')$ . As  $\alpha x_1 + \beta y_1$ <br>
a soft subspace of X.  $\left|\beta\right|$  =  $\varepsilon'$  (say),<br>  $\alpha x_1 + \beta y_1$ and  $\varepsilon' > 0$  are arbitrary, it follows that  $\alpha x + \beta y \in \overline{M_A}$ . Hence  $\overline{M_A}$  is a soft subspace of X. at any linear combination of x, y belongs to  $\overline{M}_A$ .<br>  $\tilde{J}$ , there exists soft elements  $x_1, y_2 \in \overline{M}_A$  such that<br>  $||x - x_1|| < \varepsilon$ ,  $||y - y_1|| < \varepsilon$ <br>  $-x_1) + \beta \cdot (y - y_1)|| \le |a|| ||x - x_1|| + |\beta|| |y - y_1|| < \varepsilon (|\alpha| + |\beta|) = \varepsilon'$  (sa *x* + *p* · *y*) – (*a* · *x*<sub>1</sub> + *p* · *y*<sub>1</sub>)||= ||*a* · (*x* –<br>above inequality shows that  $\alpha x_1 + \beta x_2$ <br> $\epsilon' > 0$  are arbitrary, it follows that  $\alpha$ <br>**nition (3.23)**<br>ft linear space *X* is said to be of fin<br>vectors in the open ball  $\beta(\alpha x + \beta y, \varepsilon')$ . As  $\alpha x_1 + \beta y_1$ <br>
the open ball  $\beta(\alpha x + \beta y, \varepsilon')$ . As  $\alpha x_1 + \beta y_1$ <br>
[ence  $\overline{M}_A$  is a soft subspace of  $X$ .<br>
if there is a finite set of linearly independent<br>
ctors in a soft linear sp

### **Definition (3.23)**

A soft linear space *X* is said to be of finite dimensional if there is a finite set of linearly independent soft vectors in  $X$  which also generates  $X$ . *X* .

### Theorem(3.24)

Let  $x_1, x_2, \dots, x_n$  be a linearly independent set of soft vectors in a soft linear space X. Then there is a soft real number  $c > 0$  such that for every set of soft scalars  $\lambda_1, \lambda_2, \dots, \lambda_n$  we have

$$
\|\lambda_1\cdot x_1+\lambda_2\cdot x_2+\cdots+\lambda_n\cdot x_n\|\geq \tilde{c}(|\lambda_1|+|\lambda_2|+\cdots+|\lambda_n|)
$$

Proof :

The theorem will be proved if we can prove

orem will be proved if we can prove  
\n
$$
\|\lambda_1 \cdot x_1 + \lambda_2 \cdot x_2 + \dots + \lambda_n \cdot x_n \| (e) \ge \tilde{c}(\lambda_1 | + |\lambda_2| + \dots + |\lambda_n|)(e) \text{ for all } e \in A
$$

i.e.,  $\|\lambda_1(e) \cdot x_1(e) + \lambda_2(e) \cdot x_2(e) + \cdots + \lambda_n(e) \cdot x_n(e)\|_e \geq (c(e)(|\lambda_1(e)| + |\lambda_2(e)| + \cdots + |\lambda_n(e)|))$  for all  $e \in A$  .

Now,  $x_1, x_2, \dots, x_n$  being soft vectors in  $X$ ,  $x_1(e), x_2(e), \dots, x_n(e)$  are vectors in X and  $\lambda_1, \lambda_2, \dots, \lambda_n$ *x* Journal of Iraqi Al-Khwarizm<br> *x* 1 2  $\pi$  4  $\pi$  2  $\pi$  4  $\pi$  2  $\pi$  4  $\pi$  5  $\pi$  3  $\pi$  3  $\pi$  3  $\pi$ *x<sub>n</sub>*  $\| (JIKh) \text{ Volume:8 Issue:2 Year: } 2024 \text{ pages: } 44$ <br> *x<sub>n</sub>*  $\| (e) \ge \tilde{c}(\lambda_1 | + |\lambda_2| + \cdots + |\lambda_n|)(e) \text{ for all } e \in A$ <br>  $\langle x, y \rangle \cdot x_n(e) \| \ge \langle \tilde{c}(e)(\lambda_1(e) | + |\lambda_2(e) | + \cdots + |\lambda_n(e)|) \rangle \text{ for } X \text{ , } x_1(e), x_2(e), \cdots, x_n(e) \text{ are vectors in } X \text{ and } \lambda \text{ are scalars.}$ <br>
space  $(X, \$ 14-68<br>
or all  $e \in A$ .<br>  $\lambda_1, \lambda_2, \dots, \lambda_n$ <br>
at the above being soft scalars  $\lambda_1(e), \lambda_2(e), \dots, \lambda_n(e)$  are scalars.  $\ge \tilde{c}(\vert \lambda_1 \vert + \vert \lambda_2 \vert + \cdots + \vert \lambda_n \vert)(e)$  for  $\Vert e \ge \tilde{c}(e)(\vert \lambda_1(e) \vert + \vert \lambda_2(e) \vert + \cdots)$ <br>(e),  $x_2(e), \cdots, x_n(e)$  are vectors.<br>ars.<br>(X,  $\Vert \cdot \Vert_e$ ) we get a real num  $\lambda_2(e) \cdot x_2(e) + \cdots + \lambda_n(e) \cdot x_n(e) \Big|_e \geq \tilde{c}(e)$ <br>being soft vectors in  $X$ ,  $x_1(e), x_2(e)$ <br> $\lambda_1(e), \lambda_2(e), \cdots, \lambda_n(e)$  are scalars.<br>operty of normed linear space  $(X, \|\cdot\|_e)$ <br> $\tilde{c}(e) = c_e$ , for all  $e \in A$ . rs in  $X$ ,  $x_1(e), x_2(e), \dots, x_n(e)$  are vectors<br>  $a_n(e)$  are scalars.<br>
linear space  $(X, \|\cdot\|_e)$  we get a real number<br>  $e \in A$ .

al of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
m will be proved if we can prove<br>  $\|\cdot x_1 + \lambda_2 \cdot x_2 + \cdots + \lambda_n \cdot x_n\| (e) \ge \tilde{c}(\lambda_1 | + |\lambda_2| + \cdots + |\lambda_n|)(e)$  for all  $e \in A$ <br>  $\lambda_2(e) \cdot x_2(e) + \cdots + \lambda_n(e) \cdot x_n(e)$ Then using the property of normed linear space  $(X, \|\cdot\|_e)$  we get a real number  $c_e$ , such that the above relation holds for  $c(e) = c_e$ , for all or all  $e \in A$ .<br>
(*A*) with finite parameter set *A* is consistent norm as defined in Example (*5*)

#### **Theorem (3.25)**

nal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
em will be proved if we can prove<br>  $\lambda_1 \cdot x_1 + \lambda_2 \cdot x_2 + \cdots + \lambda_n \cdot x_n |e) \ge \tilde{c}(|\lambda_1| + |\lambda_2| + \cdots + |\lambda_n|)(e)$  for all  $e \in A$ <br>  $+\lambda_2(e) \cdot x_2(e) + \cdots + \lambda_n(e) \$ Every Cauchy sequence in  $\Box$  (A) with finite parameter set A is convergent, i.e., the set of all soft real numbers with its usual modulus soft norm as defined in Example (5.2) with finite parameter set *A* , is a soft Banach space. Every Cauchy sequence in  $\Box$  (*A*) with finite parameter set *A* is convergent, i.e., the set of<br>
numbers with its usual modulus soft norm as defined in Example (5.2) with finite paramete<br>
soft Banach space.<br>
Proof:<br>
Let

### Proof :

Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
1 The theorem will be proved if we can prove<br>  $\left|\lambda_1 \cdot x_1 + \lambda_2 \cdot x_2 + \cdots + \lambda_n \cdot x_n \right| (\epsilon) \geq \tilde{c}(\left|\lambda_1 \right| + \left|\lambda_2 \right|) + \cdots + \left|\lambda_n \right|)(\epsilon)$  for all Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
<sup>2</sup><br>
<sup>2</sup>:<br>
<sup>2</sup>:<br>
<sup>2</sup>: *Parameters will be proved if we can prove***<br>**  $\left| \lambda_2 \cdot x_2 + \cdots + \lambda_n \cdot x_n \right| (e) \ge c(\left| \lambda_1 \right| + \left| \lambda_2 \right| + \cdots + \left| \lambda_n \right|) (e)$  Let with its usual modulus soft norn<br>ach space.<br> $\{x_n\}$  be any arbitrary Cauchy se<br>uch that  $|x_n - x_m| < \varepsilon$  for all *n m* be any arbitrary Cauchy sequence in  $\Box$  (A). Then corresponding to every  $\varepsilon > 0$ , there exist operty of normed linear space  $(X, \|\cdot\|_e)$  we get a real nu<br>  $\tilde{c}(e) = c_e$ , for all  $e \in A$ .<br>
quence in  $\Box$  (*A*) with finite parameter set *A* is converge<br>
usual modulus soft norm as defined in Example (5.2) wi<br>
e.<br>
any *n* finite parameter set *A* is converged<br> *n* form as defined in Example (5.2) wit<br> *y* sequence in  $\Box$  (*A*). Then correspone<br> *n*,  $m \ge k$ , i.e.,  $|x_n - x_m|(e) < \varepsilon(e)|$  for a<br> *n*. Then  $\{x_n(e)\}$  is a Cauchy sequence , i.e.,  $|x_n - x_m|(e) < \varepsilon(e)$  for all  $n, m \ge k$ , i.e.  $\geq (\tilde{c}(e)(|\lambda_1(e)| + |\lambda_2(e)| + \cdots + |\lambda_n(e)|))$  for all  $e \in A$ .<br> *x*<sub>2</sub>(*e*),  $\cdots$ , *x<sub>n</sub>*(*e*) are vectors in *X* and  $\lambda_1, \lambda_2, \cdots, \lambda_n$ <br> *n*,  $\|\cdot\|_e$ ) we get a real number  $c_e$ , such that the above<br>
eter set *A* is convergent i.e., the set of all soft real<br>
inite parameter set *A*, is a<br>  $\log$  to every  $\varepsilon > 0$ , there exist<br>  $n, m \ge k$ , i.e.<br>
ordinary real numbers  $\square$  $| \lambda_1 \cdot x_1 + \lambda_2 \cdot x_2 + \cdots + \lambda_n \cdot x_n | (e) \ge c(\lambda_1| + |\lambda_2| + \cdots + |\lambda_n|) (e)$  for all  $e \in A$ <br> *x* e..  $| \lambda_1(e) \cdot x_1(e) + \lambda_2(e) \cdot x_2(e) + \cdots + \lambda_n(e) \cdot x_n(e)|$   $\ge \left( \tilde{c}(e) (\lambda_1(e)| + |\lambda_2(e)| + \cdots + |\lambda_n(e)|) \right)$  for all  $e$ .<br> *xow*,  $x_1, x_2, \cdots, x_n$  be for all  $n, m \ge k$ . Then  $\{x_n(e)\}\$ is a Cauchy sequence of ordinary real numbers  $\Box$ <br> *e* Completeness of  $\Box$  and finiteness of *A*, it follows that  $\{x_n(e)\}\$  is convergent<br>  $x_n(e) \rightarrow x_e$ , for each *e* ∈ *A*.<br> *e* ment *x* d (*A*) with finite parameter set *A* is convergent, i.e.<br>
lus soft norm as defined in Example (5.2) with finit<br>  $y$  Cauchy sequence in  $\Box$  (*A*). Then corresponding to<br>  $|$  for all  $n, m \ge k$ , i.e.,  $|x_n - x_m|(e) \le \varepsilon(e)$  for a for each  $e \in A$ . By the Completeness of  $\Box$  and finiteness of A, it follows that  $\{x_n(e)\}\$ is convergent *f*<sub>*x<sub>n</sub>*</sub>} be any arbitrary Cauchy sequence in *□*<br>*e e h* that  $|x_n - x_m| < \varepsilon$  *f* for all *n*, *m* ≥ *k*, i.e.,  $|x_n - x_m| < \varepsilon$  *e (e) f* or all *n*, *m* ≥ *k*. Then {*x<sub>n</sub>*(*e)*} is<br>*e* ∈ *A*. By the Completenes to every  $\varepsilon > 0$ , there exist<br>  $m \ge k$ , i.e.<br>
dinary real numbers  $\Box$ <br>
{ $x_n(e)$ } is convergent for each  $e \in A$ . Let  $x_n(e) \to x_e$ , for each *e*  $\left| \int_{x_n}^{\infty} f(x_n) \right| \leq \frac{2}{\pi} \int_{x_n}^{\infty} f(x_n) \, dx$  *e A e A e A e A e A e A e A e e A e e A e e A e e e A e e e A e e e A e e e A e e n* (*A*) with limited in  $\Box$  (*A*) with limited in modulus soft norm as arbitrary Cauchy seque  $x_m$   $\leq \varepsilon$  | for all *n*, *m* ≥ *k* for all *n*, *e k i e x*<sub>*n*</sub> (*A*) with limite parameter set *A* is convergent, i.e., the <br>*i*al modulus soft norm as defined in Example (5.2) with finite parabitrary Cauchy sequence in  $\Box$  (*A*). Then corresponding to ev quence in  $\Box$  (*A*). Then corresponding<br>  $\lambda \ge k$ , i.e.,  $|x_n - x_m|(e) < \varepsilon(e)|$  for all *n*<br>
hen  $\{x_n(e)\}$  is a Cauchy sequence of o<br>  $\Box$  and finiteness of *A*, it follows that<br>  $e \in A$ .<br>  $x(e) = x_e$ , for each  $e \in A$ . Then *x* is Let  $\{x_n\}$  be any arbitrary Cauchy sequence in  $\Box(A)$ .  $1 \le \Box$  such that  $\left|x_n - x_m\right| < \varepsilon \mid$  for all  $n, m \ge k$ , i.e.,  $\left|x_n - x_m\right|$  $\left|x_n(e) - x_m(e)\right| < \varepsilon(e) \mid$  for all  $n, m \ge k$ . Then  $\{x_n(e)\}$  is a Car for each  $e \in A$ . By urbitrary Cauchy sequence in  $\Box$  (<br>  $x_m \leq \varepsilon$  | for all  $n, m \geq k$ , i.e.,  $x_n$ <br>
for all  $n, m \geq k$ . Then  $\{x_n(e)\}$  is<br>  $\Box$  Completeness of  $\Box$  and finiten<br>  $x(e) \rightarrow x_e$ , for each  $e \in A$ .<br>
nent  $x$  defined by  $x(e) = x_e$ , for *m*  $\left[\text{Cov}(A) \right]$  *y* +  $\lambda_2(e)$  -*x*<sub>2</sub>(*e*) +  $\cdots$  +  $\lambda_a(e)$  -  $x_a(e)$   $\Big|_{e} \geq \frac{1}{2}c(e)\Big|\lambda_2(e)\Big| + \frac{1}{2}2(e)\Big| + \cdots$ <br> *x<sub>x</sub>* being soft vectors in  $X$ ,  $X_1(e), X_2(e), \cdots, X_n(e)$  are vectors  $X_1(x), X_2(x), \cdots, X_n(e)$  are vectors  $\lambda_1(e), \lambda_2(e), \cdots,$ *A*·(c) *n*·*n*(c)+*n*·**-***n*, *n*(e) *x*<sub></sub>*n*(c) *n*, *z*(c) *n*(*x*<sub>1</sub>*n*(c), *x*<sub>*n*</sub>(c))<sup>*n*</sup> *n* .*x*<sub>*n*</sub>(c), *n* .*x*<sub>*n*</sub>(c), *n* .*x*<sub>*n*</sub>(c), *n* .*x*<sub>*n*</sub>(c) *n* .*x*<sub>*n*</sub>(c) *n* .*x*<sub>*n*</sub>(c) *n* .*x*<sub>*n*</sub>(*n*)

Consider the soft element x defined by  $x(e) = x_e$ , for each  $e \in A$ . Then x is a soft real number and it follows that the sequence  $\{x_n\}$  of soft real numbers is convergent and it converges to the soft real le Completeness of  $□$  and finiteness of  $x_n(e) \rightarrow x_e$ , for each  $e \in A$ .<br>
ment *x* defined by *x*(*e*) = *x<sub>e</sub>*, for each ence {*x<sub>n</sub>*} of soft real numbers is conv (*A*) is a soft Banach space. number x. Hence  $\Box$  (A) is a soft Banach space.

### **Theorem (3.26)**

Every finite dimensional soft normed linear space over a finite parameter set *A* is complete. Proof :

Let X be a finite dimensional soft normed linear space over a finite parameter set A. Let  $\{y_m\}$  be any  ${y_m}$  be any<br>*Suppose* arbitrary Cauchy sequence in X . We show that  $\{y_m\}$  converges to Exercise the parameter set<br>  ${y_m}$  converges to some sof<br>  ${x_n}$  be a basis for X. converges to some soft element  $y \in X$ . Suppose blete.<br> *y* E *X* . Suppose<br> *y* E *X* . Suppose<br>
as a unique that the dimension of X is n, and let  $\{x_1, x_2, \dots, x_n\}$  be a basis for X . Then each  $y_m$  has a unique  $x$   $x$   $(x e) = x_e$ , for each  $e \in A$ . Then  $x$  is a so<br>real numbers is convergent and it converg<br>h space.<br>inear space over a finite parameter set  $A$  is<br>normed linear space over a finite parameter<br>show that  $\{y_m\}$  converge

Because **Journal of Iraqi Al-Khwa**<br>{ $y_m$ } is a Cauchy sequence,<br> $\leq \varepsilon$  for  $m r > k$ . From then is a Cauchy sequence, for  $\varepsilon > 0$  arbitrary there exist a positive integer k such that *Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 l*<br> *m y<sub>m</sub>*  $y_r$  *j s a Cauchy sequence, for*  $\varepsilon > 0$  *arbitrary there*  $y_m - y_r$  $\leq \varepsilon$  *for*  $m, r > k$ *. From theorem(4.5.23), it follows there*  $x > |y_m - y_r|$  *= \left\| \sum\_{j=1}* for al of Iraqi Al-Khwarizmi (JIKh) Volum<br> *Cauchy sequence, for*  $\varepsilon > 0$  arbitrary<br> *m, r > k*. From theorem(4.5.23), it fol . From theorem(4.5.23), it follows that there exists  $c > 0$  such that me:8 Issue:2 Year: 2024 pages: 44-6<br>there exist a positive integer k such there exist a positive integer k such that<br>ows that there exists  $\tilde{c} > \tilde{0}$  such that<br> $m, r > k$ .

Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issu  
\nBecause 
$$
\{y_m\}
$$
 is a Cauchy sequence, for  $\varepsilon > \tilde{0}$  arbitrary there exist  
\n
$$
\|y_m - y_r\| < \varepsilon
$$
 for  $m, r > k$ . From theorem(4.5.23), it follows that t  
\n
$$
\varepsilon > \|y_m - y_r\| = \left\| \sum_{j=1}^n (\lambda_j^{(m)} - \lambda_j^{(r)}) x_j \right\| \ge \tilde{c} \sum_{j=1}^n |\lambda_j^{(m)} - \lambda_j^{(r)}|
$$
, for  $m, r > k$ .  
\nConsequently,  $|\lambda_j^{(m)} - \lambda_j^{(r)}| \le \tilde{c} \sum_{j=1}^n |\lambda_j^{(m)} - \lambda_j^{(r)}| < \frac{\varepsilon}{c}$   
\nshows that each of the *n* sequences  $\lambda_j^{(m)} = {\lambda_j^{(n)}, \lambda_j^{(2)}, \lambda_n^{(3)}, \cdots}$ ,  $j =$   
\nis finite, converges to  $\lambda_j$ , (say),  $j = 1, 2, \cdots, n$ .  
\nWe now define the soft element  $y = \lambda_1 \cdot x_1 + \lambda_2 \cdot x_2 + \cdots + \lambda_n \cdot x_n$  w  
\nMoreover, since  $\lambda_j^{(m)} \rightarrow \lambda_j$  as  $m \rightarrow \infty$  and  $j = 1, 2, \cdots, n$ ; we have  
\n
$$
\|y_m - y\| = \left\| \sum_{j=1}^n (\lambda_j^{(m)} - \lambda_j) x_j \right\| \ge \tilde{c} \sum_{j=1}^n |\lambda_j^{(m)} - \lambda_j| \|x_j\| \rightarrow \tilde{0}
$$
as  $m \rightarrow \infty$ .  
\n**4. Soft Banach Algebra**  
\n**Definition (4.1)**  
\nA soft algebra  $F_A$  of *X* over *F* is called a soft Banach algebra if  
\nrespect to a soft norm that satisfies the inequality  $\|x_1 y\| \le \|x\| \|y\|$  and

Consequently,  $1^+$   $1^ 1^ C^$  $n \mid (m)$   $(n) \mid C$ *j*  $c\sum|\lambda_j^{(m)}-\lambda_j^{(r)}|<\frac{c}{r}$ *c*

**Iournal of Iraqi Al-Khwarizmi (JIKh)** Volume:8 Issue:2 Year: 2024 pages: 44-68<br> **P**ocause  $\{y_m\}$  is a Cauchy sequence, for  $\epsilon > \tilde{0}$  arbitrary there exist a positive integer k such that<br>  $\left|y_m - y_r\right| \le \epsilon$  for  $m, r > k$ Journal of Iraqi AL-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
ccause  $\{y_n\}$  is a Cauchy sequence, for  $z > 0$  arbitrary there exist a positive integer k such that<br>  $\left\|y_m - y_x\right\| \le \delta$  for  $m, r > k$ . From t rnal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
a Cauchy sequence, for  $\varepsilon > 0$  arbitrary there exist a positive integer k such that<br>  $m, r > k$ . From theorem(4.5.23), it follows that there exists of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
auchy sequence, for  $\varepsilon > 0$  arbitrary there exist a positive integer k such that<br>  $r > k$ . From theorem(4.5.23), it follows that there exists  $\tilde{c} >$ shows that each of the *n* sequences  $\lambda_j^{(m)} = {\lambda_j^{(1)}, \lambda_j^{(2)}, \lambda_n^{(3)}, \cdots}$ ,  $j = 1, 2, \cdots, n$  is Cauchy in  $\Box$  (*A*) and rizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages:<br>
or  $\varepsilon > 0$  arbitrary there exist a positive integer k s<br>
em(4.5.23), it follows that there exists  $\tilde{c} > 0$  such t<br>  $\left| \lambda_j^{(m)} - \lambda_j^{(r)} \right|$ , for  $m, r > k$ .<br>  $\left| \lambda_j^{(m)} - \lambda$ arizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
for  $\varepsilon > 0$  arbitrary there exist a positive integer k such that<br> *j*<sub>2</sub> $\left| \lambda_j^{(m)} - \lambda_j^{(r)} \right|$ , for *m*, *r* > k .<br>  $-\lambda_j^{(r)} \left| \langle \lambda_j^{(m)} - \lambda_j^{(r)} \rangle \right|$ , for *m*, *r*  $(A)$  and  $A$ *A* is finite, converges to  $\lambda_j$ , (say),  $j = 1, 2, \dots, n$ . ce, for  $\varepsilon > \tilde{0}$  arbitrary there exist a positi<br>
heorem(4.5.23), it follows that there exist<br>  $\geq \tilde{c} \sum_{j=1}^{n} \left| \lambda_j^{(m)} - \lambda_j^{(r)} \right|$ , for  $m, r > k$ .<br>  $\left| \lambda_j^{(m)} - \lambda_j^{(r)} \right| < \frac{\varepsilon}{c}$ <br>
es  $\lambda_j^{(m)} = \left\{ \lambda_j^{(1)}, \lambda_j^{(2)},$ *y x x x* 1 1 2 2 *n n* the *n* sequences  $\lambda_j^{(m)} = \{\lambda_j^{(m)} = 0\}$ <br>
(to  $\lambda_j$ , (say),  $j = 1, 2, \dots, n$ <br>
(soft element  $y = \lambda_1 \cdot x_1 +$ <br>  $\lambda_j^{(m)} \rightarrow \lambda_j$  as  $m \rightarrow \infty$  and  $j$  $\left|\sum_{i=1}^{m} (\lambda_i^{(m)} - \lambda_j^{(r)}) x_j \right| \geq c \sum_{j=1}^{n} \left| \lambda_j^{(m)} - \lambda_j^{(r)} \right|, \text{ for } m, r > k.$ <br>  $\left| \lambda_j^{(m)} - \lambda_j^{(r)} \right| \leq c \sum_{j=1}^{n} \left| \lambda_j^{(m)} - \lambda_j^{(r)} \right| < \frac{\varepsilon}{c}$ <br>
of the *n* sequences  $\lambda_j^{(m)} = {\lambda_j^{(m)}, \lambda_j^{(2)}, \lambda_n^{(3)}, \cdots}, j = 1, 2, \cdots, n$  $-\lambda_j^{(r)}\Big|$ , for  $m, r > k$ .<br>  $\leq \frac{\varepsilon}{c}$ <br>  $\lambda_j^{(1)}, \lambda_j^{(2)}, \lambda_n^{(3)}, \cdots$ },  $j = 1, 2, \cdots, n$  is Cauchy<br>  $n$ .<br>  $\lambda_2 \cdot x_2 + \cdots + \lambda_n \cdot x_n$  which is clearly a so:<br>  $j = 1, 2, \cdots, n$ ; we have<br>  $\Big\|x_j\Big\| \to \tilde{0}$  as  $m \to \infty$ . i.e.  $y$ *m*  $\cdot \cdot$ , *n* is Cauchy in  $\Box$  (*A*) and *A*<br>is clearly a soft element of *X*.<br> $y_m \to y$  as  $m \to \infty$ .

We now define the soft element  $y = \lambda_1 \cdot x_1 + \lambda_2 \cdot x_2 + \cdots + \lambda_n \cdot x_n$  which is clearly a soft element of X. Moreover, since as  $m \to \infty$  and  $j = 1, 2, \dots, n$ ; we have

$$
\left\|y_m - y\right\| = \left\|\sum_{j=1}^n (\lambda_j^{(m)} - \lambda_j) x_j\right\| \ge \tilde{c} \sum_{j=1}^n \left|\lambda_j^{(m)} - \lambda_j\right| \left\|x_j\right\| \to \tilde{0} \text{ as } m \to \infty \text{ i.e. } y_m \to y \text{ as } m \to \infty.
$$

#### **4. Soft Banach Algebras**

#### **Definition (4.1)**

ournal of Iraqi Al-Khwarizmi (JIKh)<br>
is a Cauchy sequence, for  $\varepsilon > 0$  arbit<br>
for  $m, r > k$ . From theorem(4.5.23), i<br>  $= \left\| \sum_{j=1}^{n} (\lambda_j^{(m)} - \lambda_j^{(r)}) x_j \right\| \ge \tilde{c} \sum_{j=1}^{n} |\lambda_j^{(m)} - \lambda_j^{(r)}|$ <br>  $\cdot |\lambda_j^{(m)} - \lambda_j^{(r)}| \le \tilde{c} \sum_{$ urnal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024<br>
s a Cauchy sequence, for  $\varepsilon > \tilde{0}$  arbitrary there exist a positive integ<br>
or  $m, r > k$ . From theorem(4.5.23), it follows that there exists  $\tilde{c} > \tilde{0}$ <br> *Journal of Iraqi Al-Khwarizmi (JIKh) Volur*<br>
cause  $\{y_m\}$  is a Cauchy sequence, for  $\varepsilon > \tilde{0}$  arbitrary t<br>  $\|y_m - y_r\| \le \varepsilon$  for  $m, r > k$ . From theorem(4.5.23), it follows  $\|y_m - y_r\| = \left\|\sum_{j=1}^n (\lambda_j^{(m)} - \lambda_j^{(r)})x_j\right\|$ Journal of Iraqi Al-Khwarizm<br> *j* is a Cauchy sequence, for  $\varepsilon$ <br>  $\therefore$  for  $m, r > k$ . From theorem(<br>  $\left| = \left\| \sum_{j=1}^{n} (\lambda_j^{(m)} - \lambda_j^{(r)}) x_j \right\| \geq \tilde{c} \sum_{j=1}^{n} \left| \lambda_j^{(m)} - \lambda_j^{(r)} \right|$ <br>  $y, \left| \lambda_j^{(m)} - \lambda_j^{(r)} \right| \leq \tilde{c} \sum_{j$ *Journal of Iraqi Al-Khwarizmi (JIKh)* Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
recase  $\{y_n\}$  is a Cauchy sequence, for  $z > \hat{0}$  arbitrary there exist a positive integer  $k$  such that<br>  $y_n - y_r \Big| \leq \hat{c}$  for  $m, r > k$ . Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
use  $\{y_n\}$  is a Cauchy sequence, for  $z > \hat{0}$  arbitrary there exit a positive integer k such that<br>  $y_n - y_n\Big| = \left|\sum_{j=1}^{n} (\lambda_j^{(n)} - \lambda_j^{(n)})x_j\$ A soft algebra  $F_A$  of X over F is called a soft Banach algebra if  $F_A$  is a soft Banach space with respect to a soft norm that satisfies the inequality  $||xy|| \le ||x|| ||y||$  and if  $F_A$  contains an identity  $\tilde{e}$  such that  $x\tilde{e} = \tilde{e}x = x$  with  $||\tilde{e}|| = 1$ .<br> **Theorem (4.2)**<br>  $F_A$  is a soft Banach algebra iff  $F(e)$  is a  $+ \cdots + \lambda_n \cdot x_n$  which is clearly a soft e<br> *i*, *n*; we have<br>
→ Ô as  $m \rightarrow \infty$ . i.e.  $y_m \rightarrow y$  as  $m \rightarrow \infty$ <br>
mach algebra if  $F_A$  is a soft Banach s<br>  $xy \le ||x|| ||y||$  and if  $F_A$  contains an ide that  $xe = ex = x$  with  $||e|| = 1$ . better, since  $\lambda_j^{(m)} \to \lambda_j$  as  $m \to \infty$  and  $j = 1, 2, \dots, n$ ; wordenly  $\|x_j\| = \left\| \sum_{j=1}^n (\lambda_j^{(m)} - \lambda_j) x_j \right\| \ge \tilde{c} \sum_{j=1}^n |\lambda_j^{(m)} - \lambda_j| \|x_j\| \to 0$  as  $m$ -<br>**oft Banach Algebras**<br>**nition (4.1)**<br>ft algebra  $F_A$  of  $X$  over  $F(e)$  is a Banach algebra  $F(e)$  is a Banach algebra. oft norm that satisfies<br>  $= x$  with  $||\tilde{e}|| = \tilde{1}$ .<br>
2)<br>
Banach algebra iff  $F(x)$ <br>
s from the definition c<br>
space  $(X, ||\cdot||)$  is soft c<br>
(e) for each  $x \in X$ , w<br>
3) bet to a soft norm that satisfies the in<br>  $x \tilde{e} = \tilde{e} x = x$  with  $||\tilde{e}|| = \tilde{1}$ .<br> **orem (4.2)**<br>
is a soft Banach algebra iff  $F(e)$  is a<br>
f:<br>
f:<br>
follows from the definition of soft<br>
normed space  $(X, ||\cdot||)$  is soft com

#### **Theorem (4.2)**

*F<sup>A</sup>* is a soft Banach algebra iff is a Banach algebra for all

#### Proof :

follows from the definition of soft algebra and the following theorem .

soft normed space  $(X, \|\cdot\|)$  is soft complete iff  $(X, \|\cdot\|_e)$  is complete for all  $e \in A$  where  $\|\cdot\|_e$  defined h algebra iff  $F(e)$  is a Banach algebra<br>the definition of soft algebra and the<br> $(X, \|\cdot\|)$  is soft complete iff  $(X, \|\cdot\|_e)$ <br>r each  $x \in X$ , where  $x \in X$  such that. ch algebra for all  $e \in A$ .<br>
a and the following theorem<br>  $(X, ||\cdot||_e)$  is complete for all<br>
uch that  $x(e) = x$  $e \in A$  where  $\|\cdot\|_e$  defined as  $||x||_e = ||x||(e)$  for each  $x \in X$ , where  $x \in X$  such that finition of soft algebra and the following theorem .<br>
is soft complete iff  $(X, ||\cdot||_e)$  is complete for all  $e \in A$  w<br>  $x \in X$ , where  $x \in X$  such that  $x(e) = x$ a for all *e* ∈ *A*.<br>
following theorem .<br>
is complete for all *e* ∈ *A* where  $\|\cdot\|_e$ <br>  $x(e) = x$ *x* in of soft algebra and the following theorem.<br>
ft complete iff  $(X, \|\cdot\|_{e})$  is complete for all  $e \in A$  where  $\|\cdot\|_{e}$ ,<br>  $x_n \to x$  and  $y_n \to \tilde{y}$  then  $x_n y_n \to x \tilde{y}$ .<br>  $x_n \to x$  and  $y_n \to \tilde{y}$  then  $x_n y_n \to x \tilde{y}$ . *n*  $\theta$  *n*  $\theta$  *n*  $\theta$  *n*  $\theta$  *n* $\theta$  **<b>***n*  $\theta$  *n*  $\theta$  *n* for all  $e \in A$ .<br>
bllowing theorem .<br> *s* complete for all  $e \in A$  where  $||\cdot||_e$  defined<br>  $(e) = x$ <br>  $x_n y_n \to x \tilde{y}$ .<br> *s*.

#### **Theorem (4.3)**

In a soft Banach algebra  $F_A$ , if  $x_n \to x$  and  $y_n$ .

i.e., multiplication in a soft Banach algebra is continuous.

#### Proof :

Since  $x_n \to x$  and  $y_n \to y$  in  $F_A$ . So  $x_n(e) \to x(e)$  and  $y_n(e) \to y(e)$  for all  $e \in A$  in pages: 44-68<br>  $e \in A$  in  $(F(e), ||\cdot||_e)$ *e*

*Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Y*<br>  $x_n \to x$  and  $y_n \to y$  in  $F_A$ . So  $x_n(e) \to x(e)$  and  $y_n(e) \to y(e)$ <br>
ce  $F(e)$  is Banach algebra for all  $e \in A$  (by theorem 6.2) and of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 p<br>  $y_n \to y$  in  $F_A$ . So  $x_n(e) \to x(e)$  and  $y_n(e) \to y(e)$  for all  $e \in A$ <br>
anach algebra for all  $e \in A$  (by theorem 6.2) and in Banach al mi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>  $x_n(e) \rightarrow x(e)$  and  $y_n(e) \rightarrow y(e)$  for all  $e \in A$  in  $(F(e), \|\cdot\|_e)$ <br>  $1 e \in A$  (by theorem 6.2) and in Banach algebra<br>  $e) \rightarrow x(e)y(e)$  for all  $e \in A$ , which proves that  $x_n y_n \rightarrow xy$ . Sume: 2 Year: 2024 pages: 44<br>
(*e*)  $\rightarrow$  y(*e*) for all *e*  $\in$  *A* in (*F*)<br>
6.2) and in Banach algebra<br> *e*  $\in$  *A*, which proves that  $x_n$ 8 Issue: 2 Year: 2024 pages: 44-68<br> *y<sub>n</sub>*(*e*)  $\rightarrow$  *y*(*e*) for all *e*  $\in$  *A* in (*F*(*e*), $\|\cdot\|_e$ )<br> *m* 6.2) and in Banach algebra<br>
all *e*  $\in$  *A*, which proves that  $x_n y_n \rightarrow xy$ . 44-68<br>  $(F(e), \| \cdot \|_e)$ 4-68<br>*F*(*e*),  $\|\cdot\|_e$ ) Now since  $F(e)$  is Banach algebra for all  $\rightarrow x$  and  $y_n \rightarrow y$  in  $F_A$ . So  $x_n(e) \rightarrow x(e)$  and  $y_n(e) \rightarrow y(e)$  for all  $e \in A$  in  $F(e)$  is Banach algebra for all  $e \in A$  (by theorem 6.2) and in Banach algebra on is continuous so,  $x_n(e)y_n(e) \rightarrow x(e)y(e)$  for all  $e \in A$ , which proves multiplication is continuous so,  $x_n(e)y_n(e) \rightarrow x(e)y(e)$  for all hwarizmi (JIKh) Volume:8 Issue:2 Year<br> *n*<sub>A</sub>. So  $x_n(e) \rightarrow x(e)$  and  $y_n(e) \rightarrow y(e)$ :<br> *n*<sub>n</sub> $(e)y_n(e) \rightarrow x(e)y(e)$  for all  $e \in A$ , whisp Banach algebras on a crisp linear space  $\tilde{X}$ . *Xhwarizmi (JIKh)* Volume:8 Issue:2 Year: 2024 pages: 44-68<br> *F<sub>A</sub>* . So  $x_n(e) \rightarrow x(e)$  and  $y_n(e) \rightarrow y(e)$  for all  $e \in A$  in  $(F(e), \|\cdot\|_e)$ <br>
ora for all  $e \in A$  (by theorem 6.2) and in Banach algebra<br>  $x_n(e)y_n(e) \rightarrow x(e)y(e)$  for all  $e \$ *e*) → *y*(*e*) for all *e* ∈ *A* in (*F*(*e*), $|| \cdot ||_e$ )<br>5.2) and in Banach algebra<br>*e* ∈ *A*, which proves that  $x_n y_n \to xy$ . , which proves that  $( F(e), \| \cdot \|_e )$ <br> $x_n y_n \to xy$ .  $x_n \to x$  and  $y_n \to y$  in  $F_A$ . So  $x_n(e)$  –<br>
nce  $F(e)$  is Banach algebra for all  $e \in A$ <br>
cation is continuous so,  $x_n(e)y_n(e) \to x$ <br>
m (4.4)<br>
arameterized family of crisp Banach alg<br>
algebra on the soft vector space  $\hat{X}$ .<br>  $\{\$ *e*  $\rightarrow$  *x* and  $y_n \rightarrow y$  in  $F_A$ . So  $x_n(e) \rightarrow x(e)$  and  $y_n(e) \rightarrow$ <br> *e*  $F(e)$  is Banach algebra for all  $e \in A$  (by theorem 6.2) *i*<br> *e (4.4)*<br>
ameterized family of crisp Banach algebras on a crisp line<br>
gebra on the soft ve

#### **Theorem (4.4)**

Every parameterized family of crisp Banach algebras on a crisp linear space *X* can be considered as a soft Banach algebra on the soft vector space  $\tilde{X}$ . *X* ̃. ves that  $x_n y_n \to xy$ .<br>
an be considered as a soft<br>  $(X, ||\cdot||_e)$ <br>  $||x||(e) = ||x(e)||$  for all  $x \in X$ 

### Proof :

Let  $\{\|\cdot\|_{e} : e \in A\}$  be a family of crisp norms on the linear space X such that  $(X, \|\cdot\|_{e})$ *e*

are Banach algebra for first vector space *X*.<br> **a** family of crisp norms on the linear space<br>  $e \in A$ . Now let us define a function  $\|\cdot\|$ : *X* → . Now let us define a function  $\|\cdot\|: X \to \Box (A)^*$  by  $\|x\|(e) = \|x(e)\|_e$  for all  $x \in X$ , *z*) → *y*(*e*) for all  $e \in A$  in  $(F(e), \|\cdot\|_e)$ <br>
.2) and in Banach algebra<br>  $e \in A$ , which proves that  $x_n y_n \to xy$ .<br>
linear space *X* can be considered as a soft<br>
ace *X* such that  $(X, \|\cdot\|_e)$ <br>  $\therefore X \to \square (A)^*$  by  $\|x\|(e) = \|$  $g(e) \rightarrow y(e)$  for all  $e \in A$  in  $(F(e), || \cdot ||_e)$ <br>
6.2) and in Banach algebra<br>
1  $e \in A$ , which proves that  $x_n y_n \rightarrow xy$ .<br>
sp linear space *X* can be considered as a soft<br>
space *X* such that  $(X, || \cdot ||_e)$ <br>  $\cdot ||: X \rightarrow ∎ (A)^*$  by  $||x|| (e) =$ A in  $(F(e), ||\cdot||_e)$ <br>gebra<br>that  $x_n y_n \to xy$ .<br>be considered as a soft<br> $||\cdot||_e$ )<br> $(e) = ||x(e)||_e$  for all  $x \in X$ ,<br> $y||$  for all  $x, y \in X$  and *z*  $\in$  *A* in  $(F(e), \| \cdot \|_{e})$ <br>algebra<br>es that  $x_n y_n \rightarrow xy$ .<br>n be considered as a soft<br> $(X, \| \cdot \|_{e})$ <br> $x \| (e) = \| x(e) \|_{e}$  for all  $x \in X$ ,<br> $x \| \| y \|$  for all  $x, y \in X$  and  $x \in X$ , for all  $e \in A$ . Then  $\|\cdot\|$  is a soft norm on X. *e*  $\{ \| \cdot \|_e : e \in A \}$  be a family of crisp norms on the linnach algebra for  $e \in A$ . Now let us define a funct  $e \in A$ . Then  $\| \cdot \|$  is a soft norm on *X*.<br> **o** show that  $(X, \| \cdot \|)$  is a soft Banach algebra we h  $\|$  is com  $x \text{ can be considered as a soft}$ <br>  $x \text{ that } (X, \|\cdot\|_e)$ <br>  $x \text{ by } \|x\|(e) = \|x(e)\|_e \text{ for all } x \in X,$ <br>  $x \text{ by } \|x\| \|y\| \text{ for all } x, y \in X \text{ and}$  $\int_{e}$  for all  $x \in X$ ,<br> $x, y \in X$  and Let  $\{\|\cdot\|_e : e \in A\}$  be a family of cri<br>are Banach algebra for  $e \in A$ . Now le<br>for all  $e \in A$ . Then  $\|\cdot\|$  is a soft norm o<br>Now to show that  $(X, \|\cdot\|)$  is a soft Ba<br> $(X, \|\cdot\|)$  is complete.<br>Now  $\|xy\|_{(e)} = \|x(e)y(e)\| \le \|x(e)\| \|y$  $x(e)$  for all  $x \in X$ ,<br>all  $x, y \in X$  and<br> $xy \leq |x| ||y||$ .<br>umber k such

Now to show that  $(X, \|\cdot\|)$  is a soft Banach algebra we have to show that  $||xy|| \le ||x|| ||y||$  for all  $x, y \in X$  and  $(X, \|\cdot\|)$  is complete. show that  $||xy|| \le ||x|| ||y||$  for all  $x, y \in X$  and<br> $e \in A$ , which shows that  $||xy|| \le ||x|| ||y||$ .

$$
(X, \|\cdot\|) \text{ is complete.}
$$
  
\nNow  $\|xy\|(e) = \|x(e)y(e)\|_e \le \|x(e)\|_e \|y(e)\|_e \le \|x\|(e)\|y\|(e)$  for all  $e \in A$ , which shows that  $\|xy\| \le \|x\| \|y\|$ .  
\nNow let  $\{x_n\}$  be a Cauchy sequence in X. Then for any  $\varepsilon > 0$  there exists a soft natural number k such

Journal of Iraqi Al-Khwarizmi (JIKh) Volume: 8 Issue: 2 Year:<br>  $\rightarrow x$  and  $y_n \rightarrow y$  in  $F_A$ . So  $x_n(e) \rightarrow x(e)$  and  $y_n(e) \rightarrow y(e)$  for  $F(e)$  is Banach algebra for all  $e \in A$  (by theorem 6.2) and in Bition is continuous so,  $x_n(e)y_n(e)$ *eqi Al-Khwarizmi (JIKh) Volur*<br>  $\rightarrow$  *y* in  $F_A$ . So  $x_n(e) \rightarrow x(e)$  and<br>  $\rightarrow$  *e* e *A* (by the us so,  $x_n(e)y_n(e) \rightarrow x(e)y(e)$  f<br>  $\rightarrow$  *e* e *A* (by the us so,  $x_n(e)y_n(e) \rightarrow x(e)y(e)$  f<br>  $\rightarrow$  *e* e *x f* (*e*) *e*  $\rightarrow$  *x f* (*e*)  $\$ Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue.2 Year: 2024 pages: 44-68<br>
<br> *x x* - > *x* and  $y_x \rightarrow y$  in  $F_x$ . So  $x_x(e) \rightarrow x(e)$  and  $y_x(e) \rightarrow y(e)$  for all  $e \in A$  in  $(F(e), \|\cdot\|)$ <br>
ince  $F(e)$  is Banach algebra for all  $e \in$ Now let be a Cauchy sequence in X . Then for any  $\varepsilon > 0$  there exists a soft natural number k such that  $||x_{n+p} - x_n|| (e) < \frac{\varepsilon}{\epsilon}$  for all  $n \ge k(e)$ , for all  $\epsilon$ on the soft vector space  $\overrightarrow{X}$ .<br>  $\in A$  be a family of crisp norms on<br>  $\exists$ bra for  $e \in A$ . Now let us define a<br>
len  $\|\cdot\|$  is a soft norm on  $X$ .<br>
aat  $(X, \|\cdot\|)$  is a soft Banach algebra<br>
plete.<br>  $\|x(e)y(e)\|_{e} \leq \|x(e)\|_{e$ *x*  $\|e\|_e : e \in A\}$  be a family of crisp nearboth algebra on the soft vector space  $X$ <br>  $\vdots$ <br>  $\|e\|_e : e \in A\}$  be a family of crisp nearbon and algebra for  $e \in A$ . Now let us  $\|e \in A$ . Then  $\| \cdot \|$  is a soft norm on  $X$ arameterized family of crisp Banach algebras on a crisp linear space X can be con<br>algebra on the soft vector space X.<br>  $\left\{\left\|\cdot\right\|_c : e \in A\right\}$  be a family of crisp norms on the linear space X such that  $(X, \|\cdot\|_c)$ <br>
ach a soft Banach algebra we have to sl<br>  $\mathcal{L}(e) \leq ||x|| (e)||y|| (e)$  for all  $\epsilon$ <br>
uence in *X* . Then for any  $\varepsilon > 0$  the<br>  $n \geq k(e)$ , for all  $e \in A$ , then  $||x_{n+p}(\epsilon)$ <br>
in  $(X, ||\cdot||_{e})$  for all  $e \in A$ . ,for all  $e \leq ||e||y||(e)$  for all  $e \in A$ , which shows that  $||e||$ .<br>
en for any  $\varepsilon > 0$  there exists a soft natural nu<br>  $e \in A$ , then  $||x_{n+p}(e) - x_n(e)|| < \frac{\varepsilon}{2}(e)$  for all  $e$ , then  $||x_{n+p}(e) - x_n(e)|| \leq \frac{\varepsilon}{e}(e)$  for all  $e \in A$ , i.e. 5.2) and in Banach algebra<br>  $e \in A$ , which proves that  $x_n y_n \to xy$ .<br>
b linear space X can be considered as a soft<br>
back  $X$  such that  $(X, \|\cdot\|_e)$ <br>  $\|X \to \Box (A)^*$  by  $\|x\|(e) = \|x(e)\|_e$  for all  $x \in A$ ,<br>
show that  $\|xy\| \le \|x\| \|y$ *x* end in Banach algebra<br> *x* all  $e \in A$ , which proves that  $x_n y_n \rightarrow xy$ .<br> *x* erisp linear space *X* can be considered as a sor<br> *x* ar space *X* such that  $(X, \|\cdot\|_e)$ <br>  $\|\cdot\|: X \rightarrow \square (A)^*$  by  $\|x\|(e) = \|x(e)\|_e$  for all *x*<br> 1 6.2) and in Banach algebra<br>  $\mathbb{I} e \in A$ , which proves that  $x_n y_n \to xy$ .<br>
isp linear space  $X$  can be considered as a soft<br>
space  $X$  such that  $(X, \|\cdot\|_e)$ <br>  $\|\cdot\| \colon X \to \square (A)^*$  by  $\|x\| (e) = \|x(e)\|_e$  for all  $x \in X$ ,<br>
to s  $\|xy\| \le \|x\| \|y\|.$ <br>
number k such<br>  $e \in A$ , i.e.  $(X, \|\cdot\|)$  is complete.<br>
Now  $\|xy\|(e) = \|x(e)y(e)\|_{e} \le \|x(e)\|_{e} \|y(e)\|_{e} \le \|x\|(e) \|y\|(e)$  for a<br>
Now let  $\{x_n\}$  be a Cauchy sequence in  $X$ . Then for any  $\varepsilon > 0$ <br>
that  $\|x_{n+p} - x_n\|(e) < \frac{\varepsilon}{2}$  for all  $n \ge k(e)$ , for all  $e \in A$ is a Cauchy sequence in  $(X, \|\cdot\|_e)$  for all  $e \in A$ .  $||y||(e)$  for an  $e \in A$ , which shows that  $||xy||$  -<br>or any  $\varepsilon > 0$  there exists a soft natural number<br>4, then  $||x_{n+p}(e) - x_n(e)|| < \frac{\varepsilon}{2}(e)$  for all  $e \in A$ ,<br> $e \in A$ .  $xy \parallel (e) = \parallel x(e)y(e) \parallel_e \leq \parallel x(e) \parallel$ <br>
et { $x_n$ } be a Cauchy sequence<br>  $x_{n+p} - x_n \parallel (e) < \frac{\varepsilon}{2}$  for all  $n \geq n$ <br>
} is a Cauchy sequence in (<br>  $(X, \parallel \cdot \parallel_e)$  are Banach algebra<br>
Hence there must exist sor Then for any  $\varepsilon > 0$  there exists a soft natural<br>all  $e \in A$ , then  $||x_{n+p}(e) - x_n(e)|| < \frac{\varepsilon}{2}(e)$  for all<br>for all  $e \in A$ .<br> $e \in A$ , so there exist  $x_e$  such that  $x_n(e) \to x_e$ shows that  $||xy|| \le ||x|| ||$ <br>oft natural number k sun<br> $\sum_{r=0}^{\infty} (e)$  for all  $e \in A$ , i.e.<br> $\sum_{r=0}^{\infty} (e) \rightarrow x_e$  algebra for a<br>for all  $e \in A$ . *<sup>e</sup> x e x* (*e*) $\|e\|_{e} \le \|x\| (e) \|y\| (e)$  for all  $e \in A$ , wh<br>  $x \in X$ . Then for any  $\varepsilon > 0$  there exists<br>  $x \in X$ , then  $\|x_{n+p}(e) - x_n(e) \|x\|_{e}$  for all  $e \in A$ .<br>  $x \in X$  and  $e \in A$ , so there exist  $x_e$  such that<br>  $k_e(> k(e))$  such that  $\|\cdot X \to \Box(A)^*$  by  $\|x\| (e) = \|x(e)\|_e$  for a<br>
o show that  $\|xy\| \le \|x\| \|y\|$  for all  $x, y \in X$ <br>
all  $e \in A$ , which shows that  $\|xy\| \le \|x\| \|y\|$ <br>
o there exists a soft natural number  $k$  such<br>
there exists a soft natural number *n*  $\| \cdot \| : X \to \Box (A)^*$  by  $\| x \| (e) = \| x(e) \|_e$  for  $\| \cdot \| : X \to \Box (A)^*$  by  $\| x \| \| y \|$  for all  $x, y \in X$ <br> *n* all  $e \in A$ , which shows that  $\| xy \| \le \| x \| \| y$ <br>  $> \tilde{0}$  there exists a soft natural number  $k$  su<br>  $x_{n+p}(e) - x_n(e) \| \le \frac{\varepsilon}{2$  $X \rightarrow \Box (A)^*$  by  $||x||(e) = ||x(e)||_e$  for all  $x \in X$ ,<br>
how that  $||xy|| \le ||x|| ||y||$  for all  $x, y \in X$  and<br>  $e \in A$ , which shows that  $||xy|| \le ||x|| ||y||$ .<br>
here exists a soft natural number  $k$  such<br>  $e \in A$ , which shows that  $||xy|| \le ||x|| ||y||$ .<br>  $e$ (coursal of Fraqi Al-Khwarizmi (IIKh) Volume:8 Issue:2 Year: 2024 page<br>
(c) is Barnach algebra for all  $e \in A$  (by theorem 6,2) and in Barnach algebra is continuous so,  $x_1(e) \rightarrow x(e)$  and  $y_1(e) \rightarrow y(e)$  for all  $e \in A$ <br>
(c) i *x<sub>n</sub>*  $\rightarrow$  *x* and *y<sub>n</sub>*  $\rightarrow$  *y* in *F<sub>A</sub>* . So *x<sub>n</sub>*(*e*)  $\rightarrow$  *x*(*e*) and *y<sub>n</sub>*(*e*)  $\rightarrow$  *y*(*e*) cece *F*(*e*) is Banach algebra for all  $e \in A$  (by theorem 6.2) and i<br>eation is continuous so, *x<sub>n</sub>*(*e*) *y<sub>n</sub>*( in  $F_A$ . So  $x_n(e) \rightarrow x(e)$  and  $y_n(e) \rightarrow y(e)$  for<br>
gebra for all  $e \in A$  (by theorem 6.2) and in Ba<br>
gebra for all  $e \in A$  (by theorem 6.2) and in Ba<br>
f crisp Banach algebras on a crisp linear space<br>
ctor space  $\overline{X}$ .<br>
ily of *z x x x x z <i>x x* 

Since  $(X, \|\cdot\|_e)$  are Banach algebra for all  $e \in A$ , so there exist  $x_e$  such that  $x_n(e) \to x_e$  algebra for all *e*  $\left\{x_n(e)\right\}$  is a Cauchy sequence in  $(X, \|\cdot\|_e)$  for  $\left\{x_n(e)\right\}$  is a Cauchy sequence in  $(X, \|\cdot\|_e)$  for  $\text{Since } (X, \|\cdot\|_e)$  are Banach algebra for all  $e \in A$ . Hence there must exist some  $k_e > k$ . Hence there must exist some  $k_e$  (>  $k(e)$ ) such that  $||x_n(e) - x_e|| < \frac{\varepsilon}{2}(e)$  for all  $e \in A$ . *e* and  $e \in A$ , i.e.<br> $x_e$  algebra for all  $e \in A$ .  $\frac{c}{2}(e)$  for all  $e \in A$ , i.e.<br> $x_n(e) \rightarrow x_e$  algebra for all<br>for all  $e \in A$ .<br> $n \ge k(e)$ , for all  $e \in A$ ,<br> $k \in \mathbb{N}$  is a soft Banach  $e \in A$ , is a Cauchy sequence in  $(X, \|\cdot\|_e)$  for all  $e \in A$ .<br>  $X, \|\cdot\|_e$  are Banach algebra for all  $e \in A$ , so there exist  $x_e$  such th<br>
Hence there must exist some  $k_e(> k(e))$  such that  $||x_n(e) - x_e|| < \frac{\varepsilon}{2}$ <br>  $\int_{R}^{\infty}$ <br>  $x - x||($ t  $x_n(e) \to x_e$  algebra for all<br> *e*) for all  $e \in A$ .<br>
Il  $n \ge k(e)$ , for all  $e \in A$ ,<br>  $(X, \|\cdot\|)$  is a soft Banach

Now 
$$
||x_n - x||(e) = ||x_n(e) - x_e||_e < ||x_n(e) - x_{k_e}(e)|| + ||x_{k_e} - x_e(e)||_e < \varepsilon(e)
$$
 for all  $n \ge k(e)$ , for all  $e \in A$ ,

where  $x(e) = x_e$ . This shows that  $(X, \|\cdot\|)$  is a soft Banach space. Hence  $(X, \|\cdot\|)$  is a soft Banach algebra.

## **Definition (4.5)**

Let  $F_A$  be a soft algebra of X over F. A soft element  $x \in F_A$  is said to be invertible if it has inverse *A* Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br> **Definition (4.5)**<br>
Let  $F_A$  be a soft algebra of  $X$  over  $F$ . A soft element  $x \in F_A$  is said to be invertible if it has inverse<br>
in  $F_A$ mi (JIKh) Volume:8 Issue:2 Year: 2024 pa<br>*A* soft element  $x \in F_A$  is said to be invertity<br> $y \in F_A$  such that  $xy = yx = e^x$  and the y is cassid to be non-invertible soft element of  $F_A$ . such that  $xy = yx = e$  and the y is called the inverse ie:8 Issue:2 Year: 2024 pages: 44-68<br>*x* ∈ *F<sub>A</sub>* is said to be invertible if it has inverse<br>*xy* = *yx* =  $\tilde{e}$  and the *y* is called the inverse<br>vertible soft element of *F<sub>A</sub>*. of  $\tilde{x}$ , denoted by  $x^{-1}$ . Otherwise  $x^{-1}$  is said to be non-invertible soft element of  $F_A$ . **Definition (4.5)**<br>
Let  $F_A$  be a soft algebra of  $X$  over  $F$ . A soft element<br>  $n F_A$ , i.e. if there exists a soft element  $y \in F_A$  such that<br>
of  $\tilde{x}$ , denoted by  $x^{-1}$ . Otherwise  $x^{-1}$  is said to be non-<br>
Remark<br>
Cle *y ye y xz yx z ez z* ( ) ( ) . 2:2 Year: 2024 pages: 44-68<br>
said to be invertible if it has inverse<br>  $= \tilde{e}$  and the y is called the inverse<br>
oft element of  $F_A$ .<br>
the inverse is unique. because if<br>  $(xy)^{-1} = y^{-1} x^{-1}$ .<br>  $y^{-1} x^{-1} (xy) = \tilde{e}$ . invertible 11 it has inverse<br>
e y is called the inverse<br>
at of  $F_A$ .<br>
is unique, because if Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Is<br> **4.5**)<br>
soft algebra of *X* over F. A soft element  $x \in F_A$ <br>
f there exists a soft element  $y \in F_A$  such that  $xy = y$ <br>
d by  $x^{-1}$ . Otherwise  $x^{-+}$  is said to be non-invertib Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year:<br> **nition** (4.5)<br>  $F_A$  be a soft algebra of X over F. A soft element  $x \in F_A$  is said to be<br>  $F_A$  i.e. if there exists a soft element  $y \in F_A$  such that  $xy = yx = e^x$  Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
(4.5)<br>
soft algebra of X over F. A soft element  $x \in F_A$  is said to be invertible if it has inverse<br>
fif there exists a soft element  $y \in F_A$  su e:2 Year: 2024 pages: 44-08<br>
s said to be invertible if it has inverse<br>  $=\tilde{e}$  and the y is called the inverse<br>
soft element of  $F_A$ .<br>
the inverse is unique. because if<br>  $(xy)^{-1} = y^{-1}x^{-1}$ .<br>  $(y^{-1}x^{-1})(xy) = \tilde{e}$ .<br>
id to be .

## Remark

Clearly  $e$  is invertible. If  $x$  is invertible, then we can verify that the inverse is unique. because if  $vx = \tilde{e} = x\tilde{z}$ . Then  $y = y\tilde{e} = y(x\tilde{z}) = (yx)\tilde{z} = \tilde{e}z = \tilde{z}$ . it element of  $F_A$ .<br>  $y^{1} = y^1 x^1$ .<br>  $y^{1} = y^1 x^1$ .<br>  $y^{1} = y^1 x^1$ . enoted by  $x$ . Otherwise  $x$  is said to be non-invertible soft element of  $F_A$ .<br>  $\hat{y}$   $\hat{e}$  is invertible. If  $x$  is invertible, then we can verify that the inverse is unique. because if<br>  $\hat{x}$   $\hat{x}$  Then  $y = y\hat{e}$ 

Further, if x and y are both invertible then xy is invertible and  $(xy)^{-1} = y^{-1}x^{-1}$ .

For  $(xy)(y-x) = x(yy)$   $x^{-1} = x(e)x^{-1} = xx^{-1} = e$  and similarly

### **Definition (4.6)**

Let  $(G,*)$  be a group and  $F_A$  be a soft set over G. Then  $F_A$  is said to be a soft group over G if  $F(e)$  is her, if x and y are both invertible then xy<br>  $(xy)(y^{-1}x^{-1}) = x(yy^{-1})x^{-1} = x(e^{-1})x^{-1} = x^{-1}$ <br> **antion (4.6)**<br>  $(G, *)$  be a group and  $F_A$  be a soft set over C<br>
group of  $(G, *)$  for all  $e \in A$ .  $F(e)$  is a subgroup of  $(G,*)$  for all  $(x) = x(yy^{-1})x^{-1} = x(e^{-x})e^{-x} = 0$ <br>
(*G*,\*) for all  $e \in A$ .<br>
(*G*,\*) for all  $e \in A$ . (*G*,\*) be a group and  $F_A$  be a soft set over<br>bgroup of (*G*,\*) for all  $e \in A$ .<br>**orem (4.7)**<br>(*G*,\*) be a group and  $F_A$  be a soft set over said to be a soft group over *G* if  $F(e)$  is<br> $x, y \in F_A$ over *G*. Then  $F_A$  is said to be a soft group<br>
over *G*. If for any *x*,  $y \in F_A$ <br>  $x^* y \in F_A$  2.  $x^{-1} \in F_A$ ,<br>  $x(e)^{-1}$ . Then  $F_A$  is a soft group over *G*. *A A x* is said to be a soft group over *G* if *A*  $x^{-1} \in F_A$ ,<br> $x^{-1} \in F_A$ , ertible then *xy* is invertible and  $(xy)$ <br>  $x(e^{-x})x^{-1} = xx^{-1} = e^{-x}$  and similarly  $(y^{-1}x)$ <br>
a soft set over G. Then  $F_A$  is said to b<br>
..<br>
a soft set over G. If for any *x*,  $y \in F_A$ <br>
1.  $x * y \in F_A$  2.  $x^{-1} \in F_A$ ,<br>  $x^{-1}(e) = (x(e))^{-$ 

### **Theorem (4.7)**

Let be a group and  $F_A$  be a soft set over G. If for any  $x, y \in F_A$ 

1. 
$$
x * y \in F_A
$$
 2.  $x^{-1} \in F_A$ ,

where  $x * y(e) = x(e) * y(e)$  and  $x^{-1}(e) = (x(e))^{-1}$ . Then  $F_A$  is a soft group over *G* .

Proof :

Proof is obvious.

### **Remark**

This shows that in a soft algebra, the soft set generated by the all invertible elements is a soft group with respect to the composition defined as in theorem.

### **Definition (4.8)**

A series 1 *n* of soit elements *n*=1  $x_n$  of soft elements is said t  $\sum_{n=1}^{\infty} x_n$  of soft elements is said to be soft convergent if the partial sum of the series 1  $=\sum_{n=1}^k x_n$  is if group<br> $k = \sum_{n=1}^{k} x_n$  is *n*=1 oft group<br> $\tilde{s}_k = \sum_{n=1}^k x_n$  is soft convergent.

### **Theorem (4.9)**

Let *F<sup>A</sup>* be a soft Banach algebra. If *<sup>A</sup> x F* satisfies  $||x|| \le 1$ , then  $(e-x)$  is invertible an sue: 2 Year: 2024 pages: 44-68<br> $(\tilde{e} - x)$  is invertible and Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44.<br> **eorem (4.9)**<br>  $F_A$  be a soft Banach algebra. If  $x \in F_A$  satisfies  $||x|| \le \tilde{1}$ , then  $(\tilde{e} - x)$  is invertible and<br>  $-x)^{-1} = \tilde{e} + \sum_{n=1}^{\infty}$ 

Journal of Iraqi Al-Khwariz  
\n**Theorem (4.9)**  
\nLet 
$$
F_A
$$
 be a soft Banach algebra. If  $x \in F$   
\n $(\tilde{e} - x)^{-1} = \tilde{e} + \sum_{n=1}^{\infty} x^n$ .  
\nProof :  
\nSince  $F_A$  is soft algebra, so we have

Proof :

Journal of Iraqi Al-Khwarizmi<br> **heorem (4.9)**<br>  $et F_A$  be a soft Banach algebra. If  $x \in F_A$  sa<br>  $\tilde{e} - x$ )<sup>-1</sup> =  $\tilde{e} + \sum_{n=1}^{\infty} x^n$ .<br>
Proof :<br>
Since  $F_A$  is soft algebra, so we have Since  $F_A$  is soft algebra, so we have  $||x^j|| \le ||x||^j$  for any j tisfies  $||x|| \le \tilde{1}$ , then  $(\tilde{e} - x)$  is invertion-<br> $x^j ||\le ||x||^j$  for any positive integer  $j$ <br>the sequence of partial sum  $\tilde{s}_k = \sum^k j$ for any positive integer  $j$ , so that the infinite series 1 (1995) (1996) (1996) (1996) *n*  $n=1$   $n=1$ *x*|| is soft convergent becau  $\sum_{n=1}^{\infty} ||x||^n$  is soft convergent because. So the sequence of partial sum 1<sup>1</sup> *k* ger *j*, so that the infinite<br> $k = \sum_{n=1}^{k} x_n$  is a soft Cauchy  $=\sum_{n=1}$   $x_n$  is a soft Cauchy invertible and<br>
ger *j*, so that the infinite<br>  $\tilde{s}_k = \sum_{n=1}^k x_n$  is a soft Cauchy sequence since  $\|\sum_{n=1}^{k+p} x^n\| \leq \sum_{n=1}^{k+p} \|x\|^n$ . s soft algebra, so we have  $\begin{vmatrix} x^n \\ \vdots \\ x^{k+p} \\ \sum_{n=k}^{k+p} x^n \\ \vdots \\ x^{k+p} \\ \vdots \\ x^{k+p} \\ \end{vmatrix}$ s soft algebra, so we have<br>soft convergent because. S<br> $\sum_{n=k}^{k+p} x^n \le \sum_{n=k}^{k+p} ||x||^n$ .<br>complete so  $\sum_{n=0}^{\infty} x^n$  is soft of Iraqi Al-Khwarizmi (JIKh) Voluated and algebra. If  $x \in F_A$  satisfies  $||x|| \le$ <br>  $\therefore$ <br>
oft algebra, so we have  $||x^j|| \le ||x||^j$  for convergent because. So the sequen  $\|x^n\| \le \sum_{n=k}^{k+p} ||x||^n$ .<br>  $\therefore$ <br>  $\|x\| \le \sum_{n=1}^{\infty} x$ soft algebra, so we have  $||x^i|| \le ||x||^i$  for any position convergent because. So the sequence of parallel  $||x^i|| \le ||x||^i$  for any position  $\sum_{i=k}^{+\infty} x^n ||\langle \sum_{n=k}^{k+\infty} ||x||^n$ . mal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
Banach algebra. If  $x \in F_A$  satisfies  $||x|| \leq 1$ , then  $(\bar{e} - x)$  is invertible and<br>  $\begin{vmatrix} x^s \\ x^s \end{vmatrix}$ .<br>
Sooft algebra, so we have  $||x'|| \leq ||x||'$ sitive integer  $j$ , so that the infinite<br>
tial sum  $\tilde{s}_k = \sum_{n=1}^k x_n$  is a soft Cauchy<br>  $\tilde{s} = \tilde{e} + \sum_{n=1}^\infty x^n$ . tive integer  $j$ , so that the infinite<br>al sum  $\tilde{s}_k = \sum_{n=1}^k x_n$  is a soft Cauchy<br> $= \tilde{e} + \sum_{n=1}^\infty x^n$ . have  $||x^j|| \le ||x||^j$  for any positive integer *j*, so that the<br>
use. So the sequence of partial sum  $\tilde{s}_k = \sum_{n=1}^k x_n$  is a sof<br>
s soft convergent. Now let  $\tilde{s} = \tilde{e} + \sum_{n=1}^\infty x^n$ .<br>  $\tilde{s} = (\tilde{e} - x)^{-1}$ .<br>  $\tilde{s} = (\tilde{$ warizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>  $x \in F_A$  satisfies  $||x|| \leq 1$ , then  $(\tilde{e} - x)$  is invertible and<br>  $xe$  have  $||x'|| \leq ||x||'$  for any positive integer  $j$ , so that the infinite<br>
cause. So the sequence o 1 of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-6<br>
ach algebra. If  $x \in F_x$  satisfies  $||x|| \le 1$ , then  $(\tilde{e} - x)$  is invertible and<br>
the digebra, so we have  $||x'|| \le ||x||'$  for any positive integer  $|f|$ , (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br> *n*tisfies  $||x|| \le 1$ , then  $(\bar{e} - x)$  is invertible and<br> *n*  $x' || \le ||x||'$  for any positive integer *j*, so that the infinite<br>
the sequence of partial sum  $\bar{s}_k = \sum_{n=1}^{k} x_n$ of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
ach algebra. If  $x \in F_x$  satisfies  $||x|| \le 1$ , then  $(\hat{e} - x)$  is invertible and<br>  $\pi$  halgebra, so we have  $||x'|| \le |x||$  for any positive integer f j, so f Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
th algebra. If  $x \in F_n$  satisfies  $||x|| \le 1$ , then  $(\tilde{e} - x)$  is invertible and<br>
algebra, if  $x \in F_n$  satisfies  $||x|| \le 1$ , then  $(\tilde{e} - x)$  is invertible ansies  $||x|| \le 1$ , then  $(e - x)$  is invertible and<br>  $||x^i|| \le ||x||^i$  for any positive integer  $j$ , so that the infinite<br>
o the sequence of partial sum  $\overline{s}_k = \sum_{n=1}^k x_n$  is a soft Cauchy<br>
convergent. Now let  $\overline{s} = \hat{e} + \sum_{$ t Cauchy<br>  $(\tilde{e} - x)^n$   $\tag{2.3}$  $x_n^{x_n}$  is a soft Cauchy<br>  $x^{n+1}$ <br>  $x^{n+1}$ <br>  $x^{n+2} = e^x + \sum_{n=1}^{\infty} (e^x - x)^n$ .<br>  $\geq ||x||$ . is a soft Cauchy<br>  $= \tilde{e} + \sum_{n=1}^{\infty} (\tilde{e} - x)^n.$   $\| \cdot \|$ 

Since  $F_A$  is soft complete so 1 and 1  $n$ ,  $\alpha$ *n*=1 x<sup>"</sup> is soft convergent. Now  $\sum_{n=1}^{\infty} x^n$  is soft convergent. Now let  $\tilde{s} = \tilde{e} + \sum_{n=1}^{\infty} x^n$ . 1 *n n*=1  $\infty$ 

Now it is only we have to show that  $\tilde{s} = (\tilde{e} - x)^{-1}$ . 1

We have

$$
(\tilde{e} - x)(\tilde{e} + x + x^2 + \dots + x^n) = (\tilde{e} + x + x^2 + \dots + x^n)(\tilde{e} - x) = \tilde{e} - x^{n+1}
$$

Now again since  $||x|| \leq \tilde{1}$  so  $x^{n+1} \to 0$  as  $n \to \infty$ . Therefore letting  $n \to \infty$  in and remembering that multiplication in F is continuous we get,  $(c-x)s = s$ *s* is soft complete so  $\sum_{n=1}^{\infty} x^n$  is soft convergent. Now let<br> *s* only we have to show that  $\tilde{s} = (\tilde{e} - x)^{-1}$ .<br>  $(\tilde{e} - x)(\tilde{e} + x + x^2 + \dots + x^n) = (\tilde{e} + x + x^2 + \dots)$ <br>
in since  $||x|| \le \tilde{1}$  so  $x^{n+1} \to \tilde{0}$  as  $n \$  $(\tilde{e} + x + x^2 + \dots + x^n)(\tilde{e} - x) = \tilde{e} - x^{n+1}$ <br>
Therefore letting  $n \to \infty$  in and remember<br>  $\tilde{\delta} = \tilde{s}(\tilde{c} - x) = \tilde{c}$ <br>
1.<br>  $\tilde{e} - x \le |\tilde{s}|$ , Then  $x^{-1}$  exists and  $x^{-1} = \tilde{e} + \sum_{n=1}^{\infty}$ Igam since  $||x|| \le 1$  so  $x \to 0$  as  $n \to \infty$ . Therefore letting  $n \to \infty$  in and remembering that<br>
ilication in *F* is continuous we get,  $(\tilde{c} - x)\tilde{s} = \tilde{s}(\tilde{c} - x) = \tilde{c}$ <br>
at  $\tilde{s} = (\tilde{e} - x)^{-1}$ . This proves the propo  $\sum_{n=1}^{\infty} x^n$  is soft convergent. 1<br>
ow that  $\tilde{s} = (\tilde{e} - x)^{-1}$ .<br>  $+x + x^2 + \dots + x^n = (\tilde{e} + x + x^{n+1})$ <br>  $\tilde{e} = (\tilde{e} + x + x^{n+1})$ <br>  $\tilde{e} = (\tilde{e} - x)^n \tilde{s} = \tilde{s}(\tilde{c} - x)$ <br>
oves the proposition.<br>
bra. If  $x \in F_A$  and  $\|\tilde{e} \sum_{n=1}^{\infty} x^n$  is soft convergent. Now let  $\bar{x} = \bar{e} + \sum_{n=1}^{\infty} x^n$ .<br>
w that  $\bar{s} = (\bar{e} - x)^{-1}$ .<br>  $x + x^2 + \dots + x^n = (\bar{e} + x + x^2 + \dots + x^n)(\bar{e} - x) = \bar{e} - x^{n+1}$ <br>  $\longrightarrow \bar{0}$  as  $n \longrightarrow \infty$ . Therefore letting  $n \longrightarrow \infty$  in and rememb x<sup>\*</sup> is soft convergent. Now let  $\vec{s} = \vec{e} + \sum_{n=1}^{\infty} x^n$ .<br>
hat  $\vec{s} = (\vec{e} - x)^{-1}$ .<br>  $\vec{x}^2 + \dots + x^n = (\vec{e} + x + x^2 + \dots + x^n)(\vec{e} - x) = \vec{e} - x^{n+1}$ <br>  $\rightarrow \vec{0}$  as  $n \rightarrow \infty$ . Therefore letting  $n \rightarrow \infty$  in and remembering that<br>
sw

So that  $\tilde{s} = (\tilde{e} - x)^{-1}$ . This proves the proposition.

#### **Corollary (4.10)**

Let *F<sup>A</sup>* be a soft Banach algebra. If e get,  $(\tilde{c} - x)\tilde{s} = \tilde{s}(\tilde{c} - x) = \tilde{c}$ <br>
e proposition.<br>  $x \in F_A$  and  $\|\tilde{e} - x\| < 1$ , Then  $x^{-4}$  exists and x and Then  $x^{-1}$  exists and  $x^{-1} = e^{-1} + \sum_{n=0}^{\infty} (e^{-n})^n$  $n=1$  $-1$   $\sim$   $\sum_{n=1}^{\infty}$   $\sim$   $\sum_{n=1}^{\infty}$  $\forall x \in F_A$  and  $\|\tilde{e} - x\| < 1$ , Then  $x^{-1}$  exists and  $x^{-1}$ <br> $x \in F_A$  and  $\lambda$  be a soft scalar such that  $|\lambda| >$ 

### **Corollary (4.11)**

Let  $F_A$  be a soft Banach algebra. Let  $x \in F_A$  and  $\lambda$  be a soft scalar such that  $|\lambda| > ||x||$ .

Then 
$$
(\lambda \tilde{e} - x)^{-1}
$$
 exists and  $(\lambda \tilde{e} - x)^{-1} = \sum_{n=1}^{\infty} \lambda^{-n} x^{n-1} (x^0 = \tilde{e})$ 

Proof :

Then  $(\lambda \tilde{e} - x)^{-1}$  exists and  $(\lambda \tilde{e} - x)^{-1} = \sum_{n=1}^{\infty} \lambda^{-n} x^n$ <br>
Proof:<br>  $y \in F_A$  be such that  $y^{-1}$  exists in  $F_A$  and  $\alpha$  be it is clear that  $(\alpha y)^{-1} = \alpha^{-1} y^{-1}$ . be such that  $y^{-1}$  exists in  $F_A$  and  $\alpha$  be a soft scalar such that  $\alpha(e) \neq 0$ , for all  $e \in A$ . Then that  $|\lambda| > ||x||$ .<br> $\alpha(e) \neq 0$ , for all  $e \in A$ . Then  $e \in A$ . Then So that  $s = (e - x)$ . I has proves the proposition.<br> **Corollary (4.10)**<br>
Let  $F_A$  be a soft Banach algebra. If  $x \in F_A$  and  $\|\vec{e} - x\| < 1$ , Then  $x^{-4}$  exists and  $x^{-1} = \vec{e} +$ <br> **Corollary (4.11)**<br>
Let  $F_A$  be a soft Banach lgebra. Let  $x \in F_A$  and  $\lambda$  be a soft scalar<br>
and  $(\lambda \tilde{e} - x)^{-1} = \sum_{n=1}^{\infty} \lambda^{-n} x^{n-1}$   $(x^0 = \tilde{e})$ <br>  $\int_{-1}^{-1}$  exists in  $F_A$  and  $\alpha$  be a soft scalar such  $\int_{-1}^{-1} y^{-1}$ . it is clear that  $(\alpha y)^{-1} = \alpha^{-1} y^{-1}$ .

Having noted this we can write  $\lambda \tilde{e} - x = \lambda (\tilde{e} - \lambda^{-1} x)$  and now we show that  $(\tilde{e} - \lambda^{-1} x)^{-1}$  exists.

Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>  $\lambda \tilde{e} - x = \lambda (\tilde{e} - \lambda^{-1} x)$  and now we show that  $(\tilde{e} - \lambda^{-1} x)^{-1}$  exists.<br>  $= |\lambda^{-1}| ||x|| < 1$  by hypothesis. So, By Corollary(6.10)  $(\tilde{e} - \lambda^{-1} x)^{-1}$  exists an warizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>  $-x = \lambda (e^{-\lambda^{-1} x})$  and now we show that  $(e^{-\lambda^{-1} x})^{-1}$  exists.<br>  $|\lambda^{-1}| ||x|| < 1$  by hypothesis. So, By Corollary(6.10)  $(e^{-\lambda^{-1} x})^{-1}$  exists and inite series representat 2024 pages: 44-68<br>  $(\tilde{e} - \lambda^{-1} x)^{-1}$  exists.<br>
ary(6.10)  $(\tilde{e} - \lambda^{-1} x)^{-1}$  exists and We have  $\|\tilde{e} - (\tilde{e} - \lambda^{-1} x)\| = \|\lambda^{-1} x\| = |\lambda^{-1}|\|x\| < 1$  by hypothesis. So, By Corollary(6.10)  $(\tilde{e} - \lambda^{-1} x)^{-1}$  exists and of Iraqi Al-Khwarizmi (JIKh) Volu<br>
e can write  $\lambda \tilde{e} - x = \lambda (\tilde{e} - \lambda^{-1} x)$  and<br>  $||x|| = ||\lambda^{-1}x|| = |\lambda^{-1}|| ||x|| < \tilde{1}$  by hypoth<br>
sts. For the infinite series represent *Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68*<br> *ed this we can write*  $\lambda \tilde{e} - x = \lambda(\tilde{e} - \lambda^{-1}x)$  *and now we show that*  $(\tilde{e} - \lambda^{-1}x)^{-1}$  exists.<br>  $\tilde{e} - (\tilde{e} - \lambda^{-1}x)\Big| = \Big|\lambda^{-1}x\Big| =$ Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
d this we can write  $\lambda \tilde{e} - x = \lambda (\tilde{e} - \lambda^{-1} x)$  and now we show that  $(\tilde{e} - \lambda^{-1} x)^{-1}$  exists.<br>  $- (\tilde{e} - \lambda^{-1} x) \Big| = \Big| \lambda^{-1} x \Big| = |\lambda^{-1} \Big| ||$ ges: 44-68<br>  $(\tilde{e} - \lambda^{-1} x)^{-1}$  exists and hence  $(\lambda \tilde{e} - x)^{-1}$  exists. For the infinite series representation, using the theorem (6.9) Journal of Iraqi Al-Khwari<br>
2 noted this we can write  $\lambda \tilde{e} - x =$ <br>  $ve \|\tilde{e} - (\tilde{e} - \lambda^{-1} x)\| = \|\lambda^{-1} x\| = |\lambda^{-1}|$ <br>  $(\lambda \tilde{e} - x)^{-1}$  exists. For the infinite we have Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
oted this we can write  $\lambda \vec{e} - x = \lambda (\vec{e} - \lambda^{-1} x)$  and now we show that  $(\vec{e} - \lambda^{-1} x)^{-1}$  exists.<br>  $\left|\vec{e} - (\vec{e} - \lambda^{-1} x)\right| = \left|\lambda^{-1} x\right| = \left|\lambda^{$ izmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>  $= \lambda (\tilde{e} - \lambda^{-1} x)$  and now we show that  $(\tilde{e} - \lambda^{-1} x)^{-1}$  exists.<br>  $||x|| < 1$  by hypothesis. So, By Corollary(6.10)  $(\tilde{e} - \lambda^{-1} x)^{-1}$  e<br>
series representation, usin Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
Having noted this we can write  $\lambda \tilde{e} - x = \lambda(\tilde{e} - \lambda^{-1}x)$  and now we show that  $(\tilde{e} - \lambda^{-1}x)^{-1}$  exists.<br>
We have  $\left\|\tilde{e} - (\tilde{e} - \lambda^{-1}$ ges: 44-68<br>  $(\tilde{e} - \lambda^{-1} x)^{-1}$  exists a<br>  $\cdots$ <br>  $\cdots$ <br>  $\cdots$ <br>  $\cdots$ <br>  $\begin{array}{c}\n\cdot x^{-n-1} \\
x^{-n} \\
\hline\n\end{array}$ <br>  $\begin{array}{c}\n\cdot x^{-n-1} \\
\hline\nx^{-n} \\
\hline\n\end{array}$ <br>  $\begin{array}{c}\n\cdot x^{-n-1} \\
\hline\n\end{array}$ ume:8 Issue:2 Year: 2024 pages: 44-6<br>
d now we show that  $(\tilde{e} - \lambda^{-1} x)^{-1}$  exists.<br>
hesis. So, By Corollary(6.10)  $(\tilde{e} - \lambda^{-1} x)$ <br>
tation, using the theorem (6.9)<br>  $n = \lambda^{-1} (\tilde{e} + \sum_{n=1}^{\infty} (\lambda^{-1} x))^n = \sum_{n=1}^{\infty} \lambda^{-n} x$ *n*izmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>  $x = \lambda(\tilde{e} - \lambda^{-1}x)$  and now we show that  $(\tilde{e} - \lambda^{-1}x)^{-1}$  exists.<br>  $\begin{aligned}\n&\int_{-\infty}^{\infty} |\mathbf{x}| \leq |\tilde{h}| \leq |\$ **Example 10** Island of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br> **Example 2014** the we can write  $\lambda \vec{e} - x = \lambda (\vec{e} - \vec{A}^{-1}x)$  and now we show that  $(\vec{e} - \vec{A}^{-1}x)^{-1}$  exists.<br>
We have  $|\vec{e}$ ournal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
this we can write  $\lambda e^{-x} = \lambda (e^{-\lambda^{-1}x})$  and now we show that  $(e^{-\lambda^{-1}x})^{-1}$  exists.<br>  $(e^{-\lambda^{-1}x})^{\parallel} = |\lambda^{-1}x| = |\lambda^{-1}||x|| < 1$  by hypothesis. So, B Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
anoted this we can write  $\lambda e^{-x} = \lambda(\tilde{e} - \lambda^{-1}x)$  and now we show that  $(\tilde{e} - \lambda^{-1}x)^{-1}$  exists.<br>  $\left\| \tilde{e} - (\tilde{e} - \lambda^{-1}x) \right\| = \left\| \lambda^{-1}x$ Journal of Iraqi Al-Khwarizmi (JIKh) Volume-8 Issues2 Year: 2024 pages: 44-68<br>
ing noted this we can write  $\lambda \hat{i} = z = \lambda(\hat{i} - z^2)$  and now we show that  $(\hat{i} - z^2)^2$  y evists,<br>
there  $|\hat{i} = (\hat{i} - \hat{i} - \hat{j})|^2 = |\hat{i} - \hat{j}| = |\hat{i}| = |\hat$ ow we show that  $(\tilde{e} - \lambda^{-1} x)^{-1}$  exists.<br>
So, By Corollary(6.10)  $(\tilde{e} - \lambda^{-1} x)^{-1}$  exists.<br>
So, By Corollary(6.10)  $(\tilde{e} - \lambda^{-1} x)^{-1}$ <br>  $\lambda^{-1} (\tilde{e} + \sum_{n=1}^{\infty} (\lambda^{-1} x))^n = \sum_{n=1}^{\infty} \lambda^{-n} x^{n-1}$ <br>
1 by the set of all *z y e x x x x x x x* 0 0 0 0 0 1 JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>  $-\lambda^{-1}$  x) and now we show that  $(\vec{e} - \lambda^{-1} x)^{-1}$  exists.<br>
by hypothesis. So, By Corollary(6.10)  $(\vec{e} - \lambda^{-1} x)^{-1}$  exists and<br>
s representation, using the theorem (6.9)<br>

$$
(\lambda \tilde{e} - x)^{-1} = \lambda^{-1} (\tilde{e} - \lambda^{-1} x)^{-1} = \lambda^{-1} (\tilde{e} + \sum_{n=1}^{\infty} (\tilde{e} - (\tilde{e} - \lambda^{-1} x))^n = \lambda^{-1} (\tilde{e} + \sum_{n=1}^{\infty} (\lambda^{-1} x))^n = \sum_{n=1}^{\infty} \lambda^{-n} x^{n-1}
$$

This proves the corollary.

#### **Theorem (4.12)**

Let  $F_A$  be a soft Banach algebra. The soft set S generated by the set of all invertible soft elements of  $F_A$  is a soft open subset in  $F_A$ . Let  $F_A$  be a soft Banach algebra. The soft set S gen<br>  $F_A$  is a soft open subset in  $F_A$ .<br>
Proof :<br>  $x_0 \in S$ . We have to show that  $x_0$  is a soft inter-

Proof :

.

. We have to show that  $x_0$  is a soft interior point of  $F_A$ . Consider the open sphere  $S(x_0, \frac{1}{\| -1 \|})$  $1 \sim$  $\left|\frac{1}{x_0}\right|$ 

0

nts of  $S(x_0, \frac{1}{\left\|x_0^{-1}\right\|})$ <br> $x_0 - x \leq |x|$ with centre at  $x_0$  and radius  $\frac{1}{\| -1 \|}$ . Every soft ele 0  $1$   $\Gamma$   $\alpha$   $\alpha$  $\frac{1}{|x_0|}$ . Every soft element x of this sphere satisfies the inequality  $||x_0 - x|| < \frac{1}{||x_0||}$  $\mathbf{x}_0$ <sup> $\parallel$ </sup> 1 nts of<br>  $S(x_0, \frac{1}{\|x_0^{-1}\|})$ <br>  $x_0 - x \le \le \frac{1}{\|x_0^{-1}\|}$ of<br>  $x_0$ ,  $\frac{1}{\|x_0^{-1}\|}$ <br>  $-x \le \frac{1}{\|x_0^{-1}\|}$ *y* = *x*<sub>0</sub> = *x* we have to show that *x*<sub>0</sub> is a soft interior poir<br>
centre at *x*<sub>0</sub> and radius  $\frac{1}{\|x_0^{-1}\|}$ . Every soft element *x* of<br>  $y = x_0^{-1} x$  and  $\tilde{z} = \tilde{e} - y$  then we have  $\|\tilde{z}\| = \|y - \tilde{e}\| = \|x_0^{-1}$ 

Let  $y = x_0^{-1} x$  and  $\tilde{z} = \tilde{e} - y$  then we have

So by theorem(6.9),  $e-z$  is invertible i.e. y is invertible. Hence  $y \in S$ . Now  $x_0 \in S$ ,  $y \in S$  and so by  $\tilde{e} - y$  then we have  $\|\tilde{z}\| = \|y - \tilde{e}\| = \|\tilde{e} - \tilde{z}\|$  is invertible i.e. *y* is invertible *y* =  $x_0^{-1}x$  and  $\tilde{z} = \tilde{e} - y$  then we have  $||\tilde{z}|| = ||y - \tilde{e}|| = ||x_0^{-1}x - x_0^{-1}x_0|| \le ||x_0^{-1}|| ||x - x_0|| < 1$ .<br>
So by theorem(6.9),  $\tilde{e} - \tilde{z}$  is invertible i.e. *y* is invertible. Hence  $y \in S$ .. Now  $x_0 \in S$ ,  $y \$  $x_0 \le \tilde{1}$ .<br> $x_0 \le S$ ,  $y \in S$  and so by<br> $x_0 - x \le \frac{1}{2}$  belongs to S.  $\left\|x_0^{-1}\right\|$ <br>
<sup>1</sup>x and  $\tilde{z} = \tilde{e} - y$  then we have  $\|\tilde{z}\| = \|y - \tilde{e}\| = \|x\|$ <br>
orem(6.9),  $\tilde{e} - \tilde{z}$  is invertible i.e. y is invertible<br>  $x_0 y \in S$ . But  $x_0 y = x_0 x_0^{-1} y = x$ . So any x satis  $1$   $\sim$ algebra. The soft set *S* generated by the set of all invertible soft elements of<br>in  $F_A$ .<br>to show that  $x_0$  is a soft interior point of  $F_A$ . Consider the open sphere  $S(x_0, \frac{1}{|x_0^+|}]$ <br>dius  $\frac{1}{|x_0^-|}$ . Every soft . So any x satisfying the inequality  $||x_0 - x|| < \frac{1}{||x_0 - x||}$  belongs to S. 0  $1$  below to  $\beta$ *x* the open sphere  $S(x_0, \frac{1}{\|x_0^{-1}\|})$ <br>
ies the inequality  $\|x_0 - x\| < \frac{1}{\|x_0^{-1}\|}$ <br>  $-x_0 \leq S$ ,  $y \in S$  and so by<br>  $x_0 - x \leq \frac{1}{\|x_0^{-1}\|}$  belongs to  $S$ .  $\mathbf{x}_0$ <sup> $\parallel$ </sup> ne open sphere  $S(x_0, \frac{1}{\|x_0^{-1}\|})$ <br>the inequality  $\|x_0 - x\| < \frac{1}{\|x_0^{-1}\|}$ <br> $\circ \|\leq 1$ .<br> $\leq S$ ,  $y \in S$  and so by<br> $-x \leq \frac{1}{\|x_0^{-1}\|}$  belongs to S. belongs to *S* .  $\mathbb{R}^n$   $\mathbb{R}^n$ ,  $\mathbb{R}^n$ ,  $\mathbb{R}^n$ ,  $\mathbb{R}^n$ ,  $\mathbb{R}^$ 

This shows that S is a soft open subset of  $F_A$ .

#### **Corollary (4.13)**

The soft set  $P = (S^c)$  of  $F_A$  is soft closed subset of  $F_A$ .

### **Definition (4.14)**

A function T from a soft normed space  $F_A$  onto  $F_A$  is said to be continuous If for any sequence { $x_n$ },  $x_n \to x$ implies  $T(x_n) \rightarrow T(x)$ .

#### **Theorem (4.15)**

In a soft Banach algebra  $F_A$ , the function  $x \to x^{-1}$  of S onto S is (JIKh) Volume:8 Issue:2 Year:<br> $x \rightarrow x^{-1}$  of *S* onto *S* is continued *S*  $\rightarrow$  *x*<sup>-1</sup> of *S* onto *S* is continuous.

Proof :

Let  $x_0 \in S$  and let  $\{x_n\}$  be a sequence of soft elements in S such that  $x_n \rightarrow x_0$ . to *S* is continuous.<br>  $x_n \to x_0$ .<br>  $x_n \to x_0$ <sup>-1</sup><br>  $x_n \to x_0$ <sup>-1</sup>

**(4.15)**<br>anach algebra  $F_A$ , the function  $x \to x^{-1}$  of *S* onto *S* is continuous.<br> $x_0 \in S$  and let  $\{x_n\}$  be a sequence of soft elements in *S* such that  $x_n \to x_0$ <br> $x \to x^{-1}$  is continuous, it is enough to show that  $x_n$ To prove  $x \rightarrow x^{-1}$  is continuous, it (4.15)<br>anach algebra  $F_A$ , the function<br> $x_0 \in S$  and let  $\{x_n\}$  be a sequen<br> $x \to x^{-1}$  is continuous, it is enous  $\rightarrow x^{-1}$  is continuous, it is enough to show that  $x_n^{-1} \rightarrow x_0^{-1}$ 

Now  $\left\|x_n^{-1} - x_0^{-1}\right\| = \left\|x_n^{-1}(x_0 - x_n)x_0^{-1}\right\| \le \left\|x_n^{-1}\right\| \left\|x_0 - x_n\right\| \left\|x_0^{-1}\right\|$ Let  $x_0 \,\epsilon S$  and let  $\{x_n\}$  be a sequence of soft elements in *S* su<br> *x*  $x \to x^{-1}$  is continuous, it is enough to show that  $x_n^{-1} \to x_0^{-1}$ <br>  $\left\|x_n^{-1} - x_0^{-1}\right\| = \left\|x_n^{-1}(x_0 - x_n)x_0^{-1}\right\| \le \left\|x_n^{-1}\right\| \|x_0 - x_n\| \left\|x_0^{-$ Let  $x_0 \in S$  and let  $\{x_n\}$  be a sequence of soft elements in *S* such that  $x_n \to x_0$ .<br>  $\text{we } x \to x^{-1}$  is continuous, it is enough to show that  $x_n^{-1} \to x_0^{-1}$ <br>  $\left\|x_n^{-1} - x_0^{-1}\right\| = \left\|x_n^{-1}(x_0 - x_n)x_0^{-1}\right\| \le \left\|x_n^{-1}\right\$ 

Since  $x_n \to x_0$ , for any given  $\varepsilon > 0$ ; there exists N such that for all  $n \ge N(e)$ ,

Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024  
\n**Theorem (4.15)**  
\nIn a soft Banach algebra 
$$
F_A
$$
, the function  $x \to x^{-4}$  of *S* onto *S* is continuous.  
\nProof:  
\nLet  $x_0 \in S$  and let  $\{x_n\}$  be a sequence of soft elements in *S* such that  $x_n \to$ .  
\nTo prove  $x \to x^{-1}$  is continuous, it is enough to show that  $x_n^{-1} \to x_0^{-1}$   
\nNow :  $\left\|x_n^{-1} - x_0^{-1}\right\| = \left\|x_n^{-1}(x_0 - x_n)x_0^{-1}\right\| \le \left\|x_n^{-1}\right\| \|x_0 - x_n\| \left\|x_0^{-1}\right\|$   
\nSince  $x_n \to x_0$ , for any given  $\varepsilon > 0$ ; there exists *N* such that for all  $n \ge N(e)$ ,  
\n $\left\|x_n - x_0\right\| (e) \le \frac{1}{2\left\|x_0^{-1}\right\|} (e)$  where we have taken  $\varepsilon = \frac{1}{2\left\|x_0^{-1}\right\|}$   
\nNow  $\left\|\tilde{e} - x_0^{-1} x_n\right\| = \left\|x_0^{-1}(x_0 - x_n)\right\| \le \left\|x_0^{-1}\right\| \|x - x_n\|$ , we get  $\left\|\tilde{e} - x_0^{-1} x_n\right\| = \le \frac{1}{2}(e) = \frac{1}{2}$  for  
\nSo by Corollary(4.10),  $x_0^{-1} x_n$  is invertible and its inverse is given by  
\n $x_n^{-1} x_0 = (x_0^{-1} x_n)^{-1} = \tilde{e} + \sum_{n=1}^{\infty} (\tilde{e} - x_0^{-1} x_n)^n$ 

*x x x x x x x x x x n n n n n* 0 0 0 0 0 ( ) Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
(4.15)<br>
(4.15)<br>
(anach algebra  $F_A$ , the function  $x \to x^{\rightarrow}$  of *S* onto *S* is continuous.<br>  $x_0 \in S$  and let  $\{x_*\}$  be a sequence of soft e Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
orem (4.15)<br>
soft Banach algebra  $F_A$ , the function  $x \rightarrow x^*$  of S onto S is continuous.<br>
of:<br>
Let  $x_0 \in S$  and let  $\{x_n\}$  be a sequence of s Now  $\left\| \tilde{e} - x_0^{-1} x_n \right\| = \left\| x_0^{-1} (x_0 - x_n) \right\| \le \left\| x_0^{-1} \right\| \left\| x - x_n \right\|$ , we get  $\left\| \tilde{e} - x_0^{-1} x_n \right\| = \left\| \frac{1}{e} - e \right\| = \frac{1}{e}$  $\|x_n\| = \leq \frac{1}{e}(e) = \frac{1}{e}$  for all ear: 2024 pages: 44-68<br>
innous.<br>
that  $x_n \to x_0$ .<br>  $\frac{1}{2}(e) = \frac{1}{2}$  for all  $n \ge N(e)$ . *e*: 8 Issue: 2 Year: 2024 pages: 44-68<br>
onto *S* is continuous.<br>
ents in *S* such that  $x_n \to x_0$ .<br>  $x_n^{-1} \to x_0^{-1}$ <br>
that for all  $n \ge N(e)$ ,<br>  $\tilde{e} - x_0^{-1} x_n \Big| = \frac{\tilde{e}}{2}(e) = \frac{1}{2}$  for all  $n \ge N(e)$ .<br>
is given by<br>  $\int_1$  $n \ge N(e)$ .  $x_0 - x_n$ )  $\leq \left\| x_0^{-1} \right\| \left\| x - x_n \right\|$ , we g<br> $x_0^{-1} x_n$  is invertible and its invertible and  $\left\| x \right\|$ 

So by Corollary(4.10),  $x_0^{-1}$ , is invertible and its inverse is given by

$$
x_n^{-1} x_0 = (x_0^{-1} x_n)^{-1} = \tilde{e} + \sum_{n=1}^{\infty} (\tilde{e} - x_0^{-1} x_n)^n
$$

*Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Y*<br> *em* (4.15)<br> *em* (4.15)<br> *et*  $x_0 \in S$  and let  $\{x_n\}$  be a sequence of soft elements in *S* such<br> *et*  $x_0 \in S$  and let  $\{x_n\}$  be a sequence of soft elements Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
a (4.15)<br>
Banach algebra  $F_A$ , the function  $x \rightarrow x^{\infty}$  of S onto S is continuous.<br>
1  $x_0 \in S$  and let  $\{x_n\}$  be a sequence of soft elements unction  $x \rightarrow x^{-4}$  of *S* onto *S* is continuous.<br>
sequence of soft elements in *S* such that  $x_n \rightarrow x$ <br>
i is enough to show that  $x_n^{-1} \rightarrow x_0^{-1}$ <br>
<sup>1</sup>  $\leq |x_n^{-1}| ||x_0 - x_n|| |x_0^{-1}||$ <br>  $\geq$  i, there exists *N* such that for al ction  $x \rightarrow x^{-4}$  of *S* onto *S* is continuous.<br>
equence of soft elements in *S* such that  $x_n \rightarrow$ <br>
enough to show that  $x_n^{-1} \rightarrow x_0^{-1}$ <br>  $\leq |x_n^{-1}| ||x_0 - x_n|| ||x_0^{-1}||$ <br>
there exists *N* such that for all  $n \geq N(e)$ ,<br>
we taken mi (JIKh) Volume:8 Issue:2 Year: 2024 page<br>
n  $x \rightarrow x^{-4}$  of *S* onto *S* is continuous.<br>
nnce of soft elements in *S* such that  $x_n \rightarrow x_0$ .<br>
bugh to show that  $x_n^{-1} \rightarrow x_0^{-1}$ <br>
re exists *N* such that for all  $n \ge N(e)$ ,<br>
re e function  $x \rightarrow x^{-4}$  of *S* onto *S* is continuous.<br>
a sequence of soft elements in *S* such that  $x_n \rightarrow x_0$ <br>
it is enough to show that  $x_n^{-1} \rightarrow x_0^{-1}$ <br>  $\begin{vmatrix} -1 \ 0 \end{vmatrix} \leq |x_n^{-1}| ||x_0 - x_n|| ||x_0^{-1}||$ <br>  $\begin{vmatrix} 0 \end{vmatrix}$ ; there e Shwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-<br>
function  $x \rightarrow x^{\rightarrow}$  of *S* onto *S* is continuous.<br>
2 a sequence of soft elements in *S* such that  $x_n \rightarrow x_0$ .<br>
it is enough to show that  $x_n^{\dagger} \rightarrow x_0^{\dagger}$ <br>  $x_0^{\$ function  $x \to x^{-1}$  of *S* onto *S* is continuous.<br>
a sequence of soft elements in *S* such that  $x_n \to x_0$ .<br>
t is enough to show that  $x_n^{-1} \to x_0^{-1}$ <br>  $\left\| \left\| x_0^{-1} \right\| \right\| x_0 - x_n \left\| \left\| x_0^{-1} \right\| \right\|$ <br>  $\left\| \left\| x_n^{-1} \right$ izmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
ion  $x \rightarrow x^{-+}$  of *S* onto *S* is continuous.<br>
uence of soft elements in *S* such that  $x_a \rightarrow x_0$ .<br>
mough to show that  $x_a^2 \rightarrow x_0^{-1}$ <br>  $||x_a^{-1}|| ||x_0 - x_n|| ||x_0^{-1}||$ <br>
here ex Thus 1 (4.15)<br>
Banach algebra  $F_A$ , the function  $x$ :<br>  $x_0 \in S$  and let  $\{x_n\}$  be a sequence<br>  $\begin{aligned}\n&\geq x \to x^{-1} \text{ is continuous, it is to}\\
&\geq x^{-1} \cdot x^{-1} = x^{-1} \cdot \left\| = \left\|x_n^{-1}(x_0 - x_n)x_0^{-1}\right\| \leq \left\|x_n^{-1}\right\| \right\| \\
&\to x_0 \text{ , for any given } \varepsilon > 0 \text{ ; there exists } n \ge$ 4.15)<br>
anach algebra  $F_A$ , the function  $x \rightarrow x^{-4}$  of S onto<br>  $x_0 \in S$  and let  $\{x_n\}$  be a sequence of soft elements<br>  $x \rightarrow x^{-1}$  is continuous, it is enough to show that  $x_n^2$ <br>  $\left\| -x_0^{-1} \right\| = \left\| x_n^{-1} (x_0 - x_n) x_0^{-1} \right\$  $1^{\text{II}}$   $1 - \|e - x_0 x_n\|$  $1 \t\t \cdot \cdot \t\t \cdot \t\t \cdot \t\t \cdot$ urnal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024<br>
5)<br>
b) algebra  $F_A$ , the function  $x \to x^+$  of S onto S is continuous.<br>
S and let  $\{x_n\}$  be a sequence of soft elements in S such that  $x_n - x^+$  is continuou  $1 - \|e - x_0^\top x_n\|$ *n* **n** (4.15)<br>
Banach algebra  $F_A$ , the function  $x \rightarrow$ <br>  $x^2 + x^3 = 0$  and let  $\{x_n\}$  be a sequence of<br>  $x^2 + x^3 + x^4 = 0$  is continuous, it is enough to<br>  $\left|x_n^{-1} - x_0^{-1}\right| = \left||x_n^{-1}(x_0 - x_n)x_0^{-1}\right| \le \left||x_n^{-1}\right|| \le 0$ <br>  $\left|\left(e\right) \le$  $n=1$  "  $1-||e-x_0 x_n||$ Journal of Iraqi Al-Khwarizmi (JIKh) Volumnal<br>
com (4.15)<br>
ft Banach algebra  $F_A$ , the function  $x \rightarrow x^{-4}$  of .<br>
Let  $x_0 \in S$  and let  $\{x_n\}$  be a sequence of soft election<br>  $x \rightarrow x^{-1}$  is continuous, it is enough to show t rizmi (JIKh) Volume:8 Issue:2 Ye<br>
ion  $x \rightarrow x^{-4}$  of S onto S is cont<br>
quence of soft elements in S such<br>
enough to show that  $x_n^{-1} \rightarrow x_0^{-1}$ <br>  $\left\| x_n^{-1} \right\| \left\| x_0 - x_n \right\| \left\| x_0^{-1} \right\|$ <br>
here exists N such that for all n:<br> **n** (4.15)<br>
Banach algebra  $F_A$ , the function  $x \to x^2$  of S onto S is continuous.<br>  $x \cdot x_0 \in S$  and let  $\{x_0\}$  be a sequence of soft elements in S such that  $x_n \to x_0$ .<br>  $x \cdot x^{-1}$  is continuous, it is enough to show that  $-1$   $\parallel$   $\parallel$  ournal of Iraqi Al-Khvarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
15)<br>
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ach algebra  $F_x$ , the function  $x \rightarrow x^{-1}$  of *S* conto *S* is continuous.<br>  $\propto S$  and let  $\{x_x\}$  be a sequence of soft e al of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
ligebra  $F_n$ , the function  $x \rightarrow x^{\infty}$  of S onto S is continuous.<br>
and let  $\{x_1\}$  be a sequence of soft elements in S such that  $x_3 \rightarrow x_0$ .<br>
is 1  $x_n \le \frac{1}{2}(e) = \frac{1}{2}$  for all  $n \ge N(e)$ .<br>  $x_n$   $x_n$ <br>  $x_n$   $x_0 \le 2$  so that we have  $\left\| \begin{matrix} -1 \\ x_0 \end{matrix} \right\| \leq 2$  so that we have sequence of soft elements in S such that  $x_n \to x_0$ .<br>
is enough to show that  $x_n^{-1} \to x_0^{-1}$ <br>  $\leq ||x_n^{-1}|| ||x_0 - x_n|| ||x_0^{-1}||$ <br>  $\therefore$  i, there exists N such that for all  $n \geq N(e)$ ,<br>
have taken  $\varepsilon = \frac{1}{2||x_0^{-1}||}$ <br>  $||x_0^{-1}|| ||$ Shwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
function  $x \rightarrow x^{-4}$  of *S* onto *S* is continuous.<br> *x* a sequence of soft elements in *S* such that  $x_n \rightarrow x_0$ .<br> *xi* it is enough to show that  $x_n^{-1} \rightarrow x_0^{-1}$ <br>  $x$ Notarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br>
inction  $x \rightarrow x^+$  of *S* onto *S* is continuous.<br>
is enough to show that  $x_n^+ \rightarrow x_n^+$ <br>
is enough to show that  $x_n^+ \rightarrow x_n^+$ <br>
if  $\left\|x_0 - x_n\right\| \left\|x_0\right\|^2$ <br>  $\left\|x$ we get :  $||x_n^{-1} - x_0^{-1}||(e) \le 2||x_0||(e)||x_0 - x_n||(e) \to 0$  as  $n \to \infty$ . →  $x^{-1}$  is continuous, it is enough t<br>  $-x_0^{-1}$  =  $||x_n^{-1}(x_0 - x_n)x_0^{-1}|| \le ||x_n^{-1}|| ||x_0$ <br>  $x_0$ , for any given  $\varepsilon > 0$ ; there exis<br>  $\le \frac{1}{2||x_0^{-1}||}(e)$  where we have taken<br>  $\le \frac{1}{2||x_0^{-1}||}(e)$  where we have taken<br>  $\left|\$ Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 202<br>
(4.15)<br>
(4.15)<br>
anach algebra  $F_x$ , the function  $x \rightarrow x^{-4}$  of *S* onto *S* is continuous.<br>  $x_0 \in S$  and let  $\{x_n\}$  be a sequence of soft elements in *S* Journal of Iraqi Al-Khwarizmi (IIKh) Volume: 8 Issue: 2 Year: 2021 pages: 44-68<br>
1.15)<br>
1.15)<br>
1.15)<br>
and algebra  $F_n$ , the function  $x \to x^+$  of S onto S is continuous.<br>  $\therefore x^2$  is continuous, it is enough to show that  $\left\|x_n^{-1}\right\| = \left\|x_n^{-1}\right\|$ <br> $(e) \le 2\left\|x_0^{-1}\right\| (e) \left\|x_0 - x_n\right\|$ <br> $\to x_0^{-1}$ . So the function  $\sum_{n=1}^{\infty} \left\| \tilde{e} - x_0^{-1} x_n \right\|^n \le \frac{\tilde{1}}{1 - \left\| \tilde{e} - x_0^{-1} x_n \right\|^2} \le 2$  This gives  $\left\| x_n^{-1} x_0 \right\| \le 2$  so<br> $\left\| x_n^{-1} \right\| = \left\| x_n^{-1} x_0 x_0^{-1} \right\| \le \left\| x_n^{-1} x_0 \right\| \left\| x_0^{-1} \right\| \le 2 \left\| x_0^{-1} \right\|$ <br> $\left\| (e) \le 2 \left$  $||x_0x_0|| = ||x_n|| \times v ||x_0|| \times v$ <br>  $(e) \rightarrow 0$  as  $n \rightarrow \infty$ .<br>  $x \rightarrow x^{-1}$  of *S* onto *S* is continuos  $x \rightarrow x^{-1}$  of *S* onto *S* is continuos

This proves that  $x_n^{-1} \to x_0^{-1}$ . So the function  $x \to x^{-1}$  of S onto S is continuous.

### **Corollary (4.16)**

In a soft Banach algebra  $F_A$ , the function  $x \to x^{-1}$  of S onto S is continuous.

## **Definition (4.17)**

Let  $F_A$  be a soft Banach algebra. A soft element  $x \in F_A$  is called a soft topological divisor of zero if *Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-6<br> Definition (4.17)<br>
<i>Let*  $F_A$  be a soft Banach algebra. A soft element  $x \in F_A$  is called a soft topological divisor of there exists a sequen a algebra. A soft element  $x \in F_A$ <br>{ $x_n$ } in  $F_A$ ,  $||x_n|| = \tilde{1}$  for  $n = 1, 2$ , in  $F_A$ ,  $||x_n|| = 1$  for  $n = 1, 2, 3, \cdots$  and such that either  $x x_n \to 0$  or *Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Is:<br> Definition (4.17)<br>
Let*  $F_A$  *be a soft Banach algebra. A soft element*  $x \in F_A$  *is called<br>
there exists a sequence*  $\{x_n\}$  *in*  $F_A$ *,*  $||x_n|| = 1$  *for*  $n = 1, 2, 3, \cdots$  *and<br>* ngi Al-Khwarizmi (JIKh) Volum<br>
llgebra. A soft element  $x \in F_A$  is<br>  $[x_n]$  in  $F_A$ ,  $||x_n|| = \tilde{1}$  for  $n = 1, 2, 3,$ <br>
there  $Z$  denotes there  $\tilde{1}$  for  $n = 1, 2, 3,$ <br>
there  $\tilde{1}$  and  $\tilde{2}$  and  $\tilde{2}$  and  $\tilde{2}$ 

## **Theorem (4.18)**

The soft set Z is a soft subset of P, where Z denotes the set of all soft topological divisors of zero. Proof : 4.18)<br>  $z \in Z$  is a soft subset of P, where Z denotes<br>  $\tilde{z} \in Z$ . The there exists a sequence  $\{z_n\}$  such  $Z$  denotes the set of all s<br> $\{z_n\}$  such that  $\|\tilde{z}_n\| = \tilde{1}$  for

 Let . The there exists a sequence such that  $||z_n|| = 1$  for  $n = 1, 2, 3, \cdots$  and either  $z z_n \to 0$  or  $z_n z \to 0$  as  $n \to \infty$ . Suppose that  $z z_n \to 0$ . **8)**<br> *z* is a soft subset of P, where Z denotes the set of all soft top<br> *z*. The there exists a sequence  $\{z_n\}$  such that  $\|\tilde{z}_n\| = \tilde{1}$  for  $n = 1$ ,<br>  $\tilde{z}_n \tilde{z} \to 0$  as  $n \to \infty$ . Suppose that  $\tilde{z} \tilde{z}_n \to$ Z. The there exists a sequence  $\{z_n\}$  such that<br>  $\tilde{z} \to 0$  as  $n \to \infty$ . Suppose that  $\tilde{z} \tilde{z}_n \to 0$ .<br>  $\tilde{z} \in P$ . Then  $\tilde{z}(e)^{-1}$  exists for some e. Now<br>  $(e) = \tilde{z}(e)^{-1}(\tilde{z}\tilde{z}_n) \to \tilde{z}(e)^{-1}0(e) = 0$  as

If possible, let  $\bar{z} \in P$ . Then  $\bar{z}(e)^{-1}$  exists for some e. Now as multiplication is continuous operation, we should have  $z_n(e) = z(e)^{-1}(z z_n) \rightarrow z(e)^{-1}0(e) = 0$  as  $n \rightarrow \infty$ .

This contradicts the fact that  $||z_n|| = 1$  for  $n = 1, 2, 3, \cdots$ . Hence Z is a soft subset of P. *P* .

## **Definition (4.19)**

*z* fournal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 44-68<br> *z n n n n <i>n z n n n n n n n n n n n n n n n n* Let  $(X, \|\cdot\|)$  be a soft normed space and bossible, let  $\tilde{z} \in P$ . Then  $\tilde{z}(e)^{-1}$  exists for  $\tilde{z}_n(e) = \tilde{z}(e)^{-1}(\tilde{z}\tilde{z}_n) \rightarrow \tilde{z}(e)^{-1}(\tilde{z}\tilde{z}_n)$ <br>
contradicts the fact that  $||\tilde{z}_n|| = 1$  for *n* **nition (4.19)**<br>
(*X*,  $||\cdot||$ ) be a soft normed spa ose that  $\tilde{z}\tilde{z}_n \to 0$ .<br> *S* for some *e*. Now as multiplication is contin<br>  $f(0(e)) = 0$  as  $n \to \infty$ .<br>  $n = 1, 2, 3, \cdots$ . Hence *Z* is a soft subset of *P*<br>  $Y \in S(X)$ . A soft element  $x \in X$  is called a s<br>  $[y_n]$  of soft e a soft subset of *P*.<br> $x \in X$  is called a soft boundary elements of<br>and  $Y^c$  respectively such that  $x_n \to x$  and is called a soft boundary elements of *Y* if there exist two sequence  $\{x_n\}$  and  $\{y_n\}$  of soft elements in *Y* and *Y<sup>c</sup>* respectively such that  $x_n \to x$  and shift  $Y \in S(X)$ . A soft element  $\{x_n\}$  and  $\{y_n\}$  of soft elements in  $Y$  and  $\{y_n\}$ *x<sub>n</sub>*  $\rightarrow$  *x* and This contradicts the fact that  $\|\tilde{z}_n\| = \tilde{1}$  for  $n = 1, 2, 3, \cdots$ . Hence .<br> **Definition (4.19)**<br> *Let*  $(X, \|\cdot\|)$  be a soft normed space and  $Y \in S(X)$ . A soft elements if  $y_n \to x$ .<br> **Theorem (4.20)** A soft element  $x \in X$  is called<br>
elements in Y and Y<sup>c</sup> respe<br>
st two sequences of soft elements<br>  $z - \tilde{e} = \tilde{r}_n^{-1}(\tilde{z} - \tilde{r}_n)$ . The set<br>
there exists some  $e \in A$  and *n*→0.<br>
2. Now as multiplication is continuous operation, we<br>  $n \rightarrow \infty$ .<br>
A soft element  $x \in X$  is called a soft boundary elements of<br>
elements in *Y* and *Y<sup>c</sup>* respectively such that  $x_n \rightarrow x$  and<br>
ist two sequences of so

## **Theorem (4.20)**

The boundary of P is a soft subset of Z.

Proof :

Let z be a boundary point of P. So there exist two sequences of soft elements  $r_n$  in S and  $s_n$  in P *P* such that  $r_n \to z$  and (4.20)<br> *z* be a boundary point of *P*. So there exist two sequences of soft elemen<br>  $\tilde{r}_n \rightarrow \tilde{z}$  and  $\tilde{s}_n \rightarrow \tilde{z}$ .<br> *s* soft closed so  $\tilde{z} \in P$ . Now let us write  $\tilde{r}_n \tilde{z} - \tilde{e} = \tilde{r}_n \tilde{z} - \tilde{r}_n$ ). *z* point of *P*. So there exist two sequences  $\rightarrow \tilde{z}$ .<br> $\tilde{z} \in P$  Now let us write  $\tilde{r}_n^{-1} \tilde{z} - \tilde{e} = \tilde{r}_n^{-1} (\tilde{z} - \tilde{r}_n)$  $\sum_{n=1}^{\infty}$  for  $S$  and  $\widetilde{S}_n$  in  $P$ <br> $\{\widetilde{r}_n^{-1}(e)\}$ <br>such that

Since *P* is soft closed so  $z \in P$  Now let us Now let us write  $\overline{r}_n^{-1} \overline{z} - \overline{e} = \overline{r}_n^{-1} (\overline{z} - \overline{r}_n)$ . The sequence  $\{\overline{r}_n^{-1}(e)\}$ 

*e*. Now as multiplication is continuous of<br>as  $n \rightarrow \infty$ .<br>  $\cdots$  *Hence Z* is a soft subset of *P*.<br> *A* soft element  $x \in X$  is called a soft bo<br> *f* **t** elements in *Y* and *Y*<sup>*c*</sup> respectively such<br> *x x xist* two s given above is unbounded for all  $e \in A$ . If not, then there exists some  $e \in A$  and  $n(e)$  such that *e*  $\in$  *A*. So there exist two sequences of soft elements  $\tilde{r}_n$  in *S* and  $\tilde{s}_n$  in<br>
bw let us write  $\tilde{r}_n^{-1} \tilde{z} - \tilde{e} = \tilde{r}_n^{-1} (\tilde{z} - \tilde{r}_n)$ . The sequence  $\{\tilde{r}_n^{-1}(e)\}$ <br>  $e \in A$ . If not, then there *f* soft elements  $\tilde{r}_n$  in *S* and  $\tilde{s}_n$  in *P*<br> *e*  $\in$  *A* and *n*(*e*) such that<br>
1)  $\{\tilde{r}_n^{-1}\tilde{z}(e)\}$  is regular and hence  $\|\tilde{z}-\tilde{e}\|$  (e) < 1 for all  $n \ge n(e)$ , for all  $e \in A$ . So that by Corollary(6.11),  $\{\tilde{r}_n^{-1}\tilde{z}(e)\}$  is regular and hence *z* if there exist two sequence  $\{x_n\}$  and  $\{y_n\}$ <br> *z* if there exist two sequence  $\{x_n\}$  and  $\{y_n\}$ <br> *i* if there exist two sequence  $\{x_n\}$  and  $\{y_n\}$ <br> *i*  $\varphi_n \to x$ .<br> **heorem (4.20)**<br>
The boundary of P is a t (X,||⋅|) be a soft normed space and  $Y \in S(X)$ . A soft element  $x \in X$  is c<br>
if there exist two sequence  $\{x_n\}$  and  $\{y_n\}$  of soft elements in Y and Y<sup>c</sup> re:<br>
→ x.<br> **eorem (4.20)**<br>
e boundary of P is a soft subset of dary point of *P*. So there exist two<br> *n* →  $\tilde{z}$ .<br>  $\therefore$   $\tilde{z} \in P$  Now let us write  $\tilde{r}_n^{-1} \tilde{z} - \tilde{e}$ <br>
ed for all *e* ∈ *A*. If not, then there<br>  $n \ge n(e)$ , for all *e* ∈ *A*. So that by C<br>
regular, contradi us write  $\tilde{r}_n^{-1} \tilde{z} - \tilde{e} = \tilde{r}_n^{-1} (\tilde{z} - \tilde{r}_n)$ . The sequen<br>*E E n n*(*e* ∈ *A n*(*e* ← ft elements  $\tilde{r}_n$  in *S* and  $\tilde{s}_n$  in *P*<br>he sequence  $\{\tilde{r}_n^{-1}(e)\}$ <br>*A* and *n*(*e*) such that<br> $\{\tilde{r}_n^{-1}\tilde{z}(e)\}$  is regular and hence<br>pounded for all  $e \in A$ , so that  $\tilde{z}(\hat{z})$  is regular, contradicting  $\tilde{z} \in P$ . Hence  $\{\tilde{r}_n^{-1}(e)\}$  is unbounded for all  $e \in A$ . so that **Definition (4.19)**<br>
Let  $(X, \|\cdot\|)$  be a soft normed space and  $Y \in S$ <br> *Y* if there exist two sequence  $\{x_n\}$  and  $\{y_n\}$ ,<br>  $y_n \to x$ .<br> **Theorem (4.20)**<br>
The boundary of P is a soft subset of Z.<br>
Proof :<br>
Let  $\tilde{z}$  be te  $r_n z - e = r_n (z - r_n)$ . The sequence  $\{r_n$ , then there exists some  $e \in A$  and  $n(e)$  such So that by Corollary(6.11),  $\{\tilde{r}_n^{-1}\tilde{z}(e)\}\)$  is reg  $\tilde{z} \in P$ . Hence  $\{\tilde{r}_n^{-1}(e)\}$  is unbounded for all equences of soft elements  $r_n$  in S and  $r_n$ <sup>-1</sup>  $\tilde{r}_n$   $(\tilde{z} - \tilde{r}_n)$ . The sequence  $\{\tilde{r}_n^{-1}(e)\}$  is some  $e \in A$  and  $n(e)$  such that prollary(6.11),  $\{\tilde{r}_n^{-1}\tilde{z}(e)\}$  is regular and  $\{\tilde{r}_n^{-1}(e)\}$  is unbou *e*  $\{e\}$ <br>*e*  $\in$  *A*. so that  $\left\| \tilde{r}_n^{-1} \right\| \to \infty$ . Let  $\tilde{z}$  be a boundary point of *P*. So there ex<br>
uch that  $\tilde{r}_n \to \tilde{z}$  and  $\tilde{s}_n \to \tilde{z}$ .<br>
ince *P* is soft closed so  $\tilde{z} \in P$  Now let us write  $\tilde{r}$ <br>
iven above is unbounded for all  $e \in A$ . If not, the

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\nNow let us define 
$$
\tilde{z}_n = \frac{\tilde{r}_n^{-1}}{\left\| \tilde{r}_n^{-1} \right\|}
$$
. From the definition of  $\tilde{z}_n$ , we have  $\left\| \tilde{z}_n \right\| = \tilde{1}$ .  
\nFurther  $\tilde{z} \tilde{z}_n = \frac{z r_n^{-1}}{\left\| \tilde{r}_n^{-1} \right\|} = \frac{\tilde{e} + z \tilde{z}_n - \tilde{e}}{\left\| \tilde{r}_n^{-1} \right\|} = \frac{\tilde{e} + (\tilde{z} - \tilde{r}_n) \tilde{r}_n^{-1}}{\left\| \tilde{r}_n^{-1} \right\|}$   
\nBut  $\frac{\tilde{e} + (\tilde{z} - \tilde{r}_n) \tilde{r}_n^{-1}}{\left\| \tilde{r}_n^{-1} \right\|} = \frac{\tilde{e}}{\left\| \tilde{r}_n^{-1} \right\|} + (\tilde{z} - \tilde{r}_n) \tilde{z}_n$ , we get  $\tilde{z} \tilde{z}_n = \frac{\tilde{e}}{\left\| \tilde{r}_n^{-1} \right\|} + (\tilde{z} - \tilde{r}_n) \tilde{z}_n$   
\nwe see that  $\tilde{z} \tilde{z}_n \to 0$  as  $n \to \infty$ . Hence  $\tilde{z}$  is a topological divisor of zero.  
\n5. Conclusion  
\nIn this paper underscores the significance of soft Banach Algebras as a powerful mathematical tool for investigating algebraic phenomena within diverse applied contexts.  
\nReferences

we see that  $z z_n \to 0$  as  $n \to \infty$ . Hence z is a topological divisor of zero.

## **5. Conclusion**

In this paper underscores the significance of soft Banach Algebras as a powerful mathematical tool for investigating algebraic phenomena within diverse applied contexts.

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