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Some New results related with Fuzzy Soft Spectrum

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ABSTRACT

This work introduces the fundamental concepts of fuzzy softu pectrum, fuzzy s oft cond ition spectrum and fuzzy so ft spectra l radiu s of a fuzzy so ft element in the context of fuzzy soft Banach algebras. An analysis is conducted on the fundamental characteristics of these c onc epts in of fuzzy soft Banach algebras. Within the domain of fuzzy soft Banach algebras, we establish the definitions of fuzzy soft multiplicative linear functional, nearly fuzzy soft multiplicative linear functional, fuzzy s of t Jordan multiplicative lin ear funct ional, and nearly fuzzy s of t Jordan multiplicative lin ear funct ional. The work seeks to explore new findings and principles related to fuzzy soft Banach algebra.

1. Introduction

The physical environment incorporates indeterminate data, thereby rendering challenging to overcome computational problems using traditional approaches in various areas of science and technology, ecological science, economics, and medications. The standard math method is difficult since there are no parameterization tools available to describe problems that occur in the realms of ambiguities and uncertainties. Molodtsov [8] established the concept of soft set theory to address these issues. Soft set theory is a novel mathematical approach that effectively handles uncertainties. In addition, the software includes a parameterization tool that offers greater flexibility compared to traditional mathematical methods when dealing with the ambiguity and uncertainties of Actual reality problems. Das and Samanta [3] caused the notion of soft linear functional within the framework of soft linear spaces. The researchers analysed the fundamental The features of these operators and extended important theorems of functional analysis in the context of soft-set frameworks. Thakur and Samanta [12] proposed the notion of soft Banach algebras and examined certain initial characteristics of this concept.

To obtain further details regarding soft set theory and its various applications, please refer to the following sources: [2,4,6] and [7-11].

This work presents a precise description of fuzzy soft spectrum and examines several of its features. Section 2 provides preliminary findings. Section 3 presents the notion of fuzzy soft spectrum. The Banach algebra and its initial properties are being examined. The paper is concluded in Section 4.

2. preliminaries

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Throughout this work. Let X be an universe set with E be the set of parameter. Let I^x be the collection of all fuzzy sets over X, and $A \subseteq E$.

Definition (2.1)[5]: A pair Γ_A is defined as a fuzzy soft set over X if Γ represented by $\Gamma : A \rightarrow I^x$. So ∀e ∈ A, The function f(e) is a fuzzy subset of X, where f(e) is the membership function $f_e : X \rightarrow [0, 1]$ These values reflect the degree of membership of each element in X to the fuzzy subset f(e), or the extent to which each element in X possesses the attribute e from set E.
Throughout this study, we will represent the collection of all fuzzy soft subsets of the universal set X concerning the parameter set E as $\Gamma_A(X_E)$.

Definition (2.2)[5]: Afuzzy soft sets Γ_A with Λ_B over a common universe X we get,
i. Γ_A is a fuzzy soft subset of Λ_B denoted by $\Gamma_A \lesseqqgtr \Lambda_B$ if:
1. The set A is a subset of the set B.
2. $\forall e \in A, \Gamma_e(x) \leq \Lambda_e(x)\ \forall x \in X$.

ii. $\Gamma_A = \Lambda_B$ if $\Gamma_A \leq \Lambda_B$ and $\Lambda_B \leq \Gamma_A$.
iii. $\Gamma^c{}_A$ is The complement of a fuzzy soft set Γ_A where $\Gamma^C : A \rightarrow I^X$, Γ^C(e) is the complement of fuzzy set Γ (e) with membership function.

$\Gamma^c{}_e = 1 - \Gamma_e \quad \forall e \in A$.
iv. $\Gamma_A = \vartheta A$ (null fuzzy soft positioned with alignment with A), this means that for every element ς in A, $\Gamma_\varsigma(\iota) = 0, \forall \iota \in X$.
v. $\Gamma_A = X_A$ (absolute fuzzy soft set concerning set A) if $\forall \hbar \in X, \Gamma_e(\hbar) = 1$ for all e ∈ A.
Let A = E. In this case, the null fuzzy soft set is indicated by ϑ, whereas the absolute fuzzy soft set is denoted by X.
vi. $\Gamma_A \cup \Lambda_B$ denoted as $\Gamma_A \vee \Lambda_B$, is defined as the fuzzy soft set $(\Gamma \vee \Lambda)_C$, where C is the union of sets A and B. For any element e in C, the value of (Γ ∨ Λ)(e) is equal to the maximum value between Γ(e) and Λ(e).

$$(\Gamma \vee \Lambda)_C(\mathfrak{x}) = \begin{cases} \Gamma_A(\mathfrak{X}) & \text{if } e \in A - B \\ \Lambda_B(\mathfrak{X}) & \text{if } e \in B - A \\ \max\{\Gamma_e(\mathfrak{X}), \Lambda_e(\mathfrak{X})\} & \text{if } e \in A \cap B \end{cases} \quad \text{for every } \mathfrak{x} \in X.$$

VII. $\Gamma_A \cap \Lambda_B$ IS REPRESENTED AS $\Gamma_A \wedge \Lambda_B$, IS THE FUZZY SOFT SET $(\Gamma \vee \Lambda)_C$, WHERE $C = A \cap B \neq \emptyset$ WITH $\forall E \in C$, WE HAVE $(\Gamma \wedge \Lambda)(E) = \Gamma(e) \wedge \Lambda(E)$ WHERE $(\Gamma \wedge \Lambda)_e(x) = \text{MIN}\{\Gamma_E(X), \Lambda_E(x)\}$ FOR ALL $x \in X$.

Definition (2.3)[5] :
1. Consider x^{λ_e} as a fuzzy point in X, where x belongs to X and $\lambda_e \in (0, 1]$. The fuzzy soft set (Px)E can be defined as a fuzzy soft point, commonly referred to as an F.S point. Consider the mapping $P_x : E \rightarrow I^X$ be a mapping, for any element e in E,

$$(Px)e(y) = \begin{cases} \lambda_e & \text{if } y = x \\ 0 & \text{o.w.} \end{cases}$$

For all e ∈ E, $(P_x)(e) = x^{\lambda_e}$, or $(P_x)(e) = \lambda_e X\{ x \}$, ∀e ∈ E. Put simply, the F.S point $(P_x)E$, is a description of x ∈ X based on the decision parameters in E. If λ_e= l forhyall e ∈ E, we then say $(P_x)E$, criop F.S point.

2. The F.S point (Px)E belongs to the set F.S set Γ_E, represented by $(\Gamma x)E \in \Gamma_E$, when the condition $0 < \lambda_e \leq \Gamma_e(x)$ holds for all e ∈ E.

3. The limitation of the F.S point (Px)E to the subset (Px|e)The term "F.S single point" refers to E, which is known as the fuzzy soft single point over E.
whenever

$$(P\mathscr{x}|e)\alpha(\mathscr{y}) = \begin{cases} (Px)e(x) = \lambda e > 0 & \text{if } \alpha = e \text{ and } \mathscr{y} = x \\ 0 & \text{otherwise} \end{cases}$$

The point (Px|e)E is a member of the collection Γ_E, denoted by (Px|e)E ∈ Γ_E, when the condition 0 < λe ≤ $\Gamma_e(\mathscr{x})$. is satisfied.

New Notation.[5]

• The F.S point $(P\mathscr{x})E$ can be represented as x_E, where $(P\mathscr{x})(e) = x(e) = x^{\lambda_e}$, for all e in E. In other words, each parameter's image under the map $P\mathscr{x}$ is a fuzzy point. Therefore, the complement of F.S can be demonstrated using $(x_E)^c$ point x_E such that $[(P\mathscr{x})(e)]^c = 1 - \lambda_e \chi\{x\}$ for all e ∈ E.

• The precise F.S point $(P\mathscr{x})_E$ will be written as x_E^1 or x_E .

• The F.S single point $(P\mathscr{x}|e, E)$ is represented as x_e.

Definition (2.4)[5]. : Suppose R refere to all real numbers, and E is the set of parameters. The collection of fuzzy soft real numbers, also known as F.S real numbers, is represented as $R_E = \{\Gamma: E \rightarrow I^R\}$, where Γ(e)'s represents fuzzy real numbers for all e in E.

It should be noted that any element r in the set R can be interpreted as a finite sequence (F.S) real numbers r_E, provided that for every element e in the set E, The function Γ(e) is defined as the characteristic function of r, i.e., r_E = XXr. So r : E ⟶ , I^R , for all e ∈ E, r(e) defined by

$$r_e(\mathfrak{t}) = \begin{cases} l \text{ if } \mathfrak{t} = r \\ 0 \text{ if } \mathfrak{t} \neq r \end{cases}$$

We refer to this type of number as a crisp F.S real number. The real number Γ_E is referred to as a non-negative F.S number if, for every e ∈ E, $\Gamma_e(t) = 0$ for all t < 0. The set of all non-negative F.S numbers is denoted by $\mathcal{R}^*{}_E$. 0_E with 1_E are the crisp F.S numbers 0 and l where for each e ∈ E, $0_e(0) = 1$, $0_e(t) = 0$ ∀t ≠ 0 and $1_e(1) = 1$, $1_e(t) = 0$ ∀t ≠ 1.

In this study, we will demonstrate the F.S numbers using the notation r_E, where r belongs to R with r: E $\rightarrow I^R$. Specifically, for each element e in E, r(e) represents a fuzzy number.

Definition (2.5)[5]. : $[r_E]$e The ∝-level set of the real number r_E matches the parameter e ∈ E is denoted as e,∝ and defined as follows:

$[r_E]_{e,\alpha}$ = {t : r_E (t) ≥ α}

The ∝-level set of the fuzzy real number 𝔯(𝔢), denoted as $[r_E]$𝔢,α, is equivalent to [r(𝔢)]α.

Definition (2.6)[5]. : suppose r_E, r_E are F.S real numbers. We use the term
1. $r_E \preccurlyeq r_E \Leftrightarrow [r_E]_{e,\alpha} \subseteq [r_E]_{e,\alpha}$ for all α ∈ (0, 1] with e ∈ E.
2. $r_E = r_E \Leftrightarrow [r_E]_{e,\alpha} = [r_E]_{e,\alpha}$ for all α ∈ (0, 1] and e ∈ E.
So if $[\ r(e)\ ]_\alpha = [r_\alpha^1 , r_\alpha^2\ ]$ and $[r(e)]_\alpha = [r_\alpha^1 , r_\alpha^2]$, then we have
1. $r_E\ \ r_E \Leftrightarrow r_\alpha^1 = r_\alpha^1$ and r_α^2 , r_α^2 for all α ∈ (0, 1] and e ∈ E.
2. $r_E = r_E \Leftrightarrow r_\alpha^1 \le r_\alpha^1$ and $r_\alpha^2 \le r_\alpha^2$ for all α ∈ (0, 1] and e ∈ E

Definition (2.7)[1]:
A linear space, also known as a vector space, is a set X over a field F that has two operations, + and • satisfying the following axioms

(i) An operation called vector addition that associates a sum $u+v \in$ X with each pair of vector u v , ∈ X such that it is associative with identity 0.
(ii) An operation called multiplication by a scalar that associates with each scalar $a \in$F and vector $u\in$ X vector $au\in$X, called the product of a and u , such that it is distributive with identity 1.
Definition (2.8)[6]:
Consider X *to be a linear space over* f. A soft set $Ǵ_A$ is defined as a soft subspace of F if
(i) ∀e ∈ A, $Ǵ_A$ (e*) is a* subspace *of* X *over f with*
(ii) F(e) ⊇ $Ǵ_A$ (e*)*, ∀e ∈ A.
Theorem (2.9)[6]:
A subset that is defined as being soft. The $\breve{G}$ of a soft linear space X refers to a soft linear. sub-space of X iff for any scalar α,β∈f ,α$\breve{G}$+$\beta\breve{G}$ ⊆$\breve{G}$.

Definition (2.10): [3]
Consider X to be a linear space over f. A fuzzy set $\breve{A}$ in X called fuzzy subspace if α$\breve{A}$+βB ⊆$\breve{A}$ for all α,$\beta \in f$Or equivalent
$\breve{A}$(αx+βy) ≥ min{ $\breve{A}$(x),B(y)} for both x ,y ∈ X and all x ∈ X.

Definition (2.11):
Consider X as a vector space over a field F. Let A be a colection of parameters. A fuzzy soft set is a set that allows for uncertainty and vagueness in its elements. A fuzzy soft linear space of X over F is defined as Γ_A over F, where Γ(e) represents a fuzzy subspace of X, ∀e ∈ A.

Definition (2.12):[4]
Consider X as a linear space over F, where F=ℂ or F=ℝ. A mathematical function ‖. ‖: X → R is a norm on X exhibits The one after that

(1) $\|\breve{a}\| \geq 0$, $\forall \check{x} \in X$
(2) $\|\breve{a}\| = 0 \Leftrightarrow \check{x}=0$

(3) $\|\hat{\lambda}\breve{a}\| = |\hat{\lambda}|\|\breve{a}\|, \forall \breve{a} \in X$ with $\hat{\lambda} \in f$
(4) $\|\breve{a} + \breve{b}\| \leq \|\breve{a}\| + \|\breve{b}\|$, for all $\breve{a}, \breve{b} \in X$.

The combination of the linear space X and the norm $\|.\|$ is referred to as a normed space, which is represented as $(X, \|.\|)$.

Definition (2.13): Let X be a linear space field F and let E referee to the set of parameters .The fuzzy softnorm $\|.\|$ over X is defined as the map $\breve{X}$ to R^*E Such that

(i) $\tilde{x}_E = \tilde{0}_E \leftrightarrow \| \tilde{x}_E \| = \tilde{0}_E$.

(ii) $\| \tilde{r}. \tilde{x}_E \| = | r | \otimes \| \tilde{x}_E \|$.

(iii) $\| \tilde{x}_E + \tilde{y}_E \| = \| \tilde{x}_E \| \oplus \| \tilde{y}_E \|$ for all $\tilde{x}_E, \tilde{y}_E \in SV(\breve{X})$.

Definition (2.14): Consider X as an algebraic structure defined over a field F, and let A be a set of parameters. A fuzzy soft set Γ_A is a fuzzy soft algebra X over F if $\Gamma(e)$ is a fuzzy subalgebra of X for every e in A.

Definition(2.15) [7]: the sequence of soft elements $\{\check{x}_N\}$ of a soft normed space $(X, \|.\|)$, is said to be a Cauchy sequence if for every $\curlywedge \geq o$, there is $\cup \in N$ In a manner that

$\|\tilde{x}_i - \tilde{x}_j\| < \curlywedge$, for all i, j $\geq$ k. That is $\|\tilde{x}_i - \tilde{x}_j\| \rightarrow 0$ as i, j $\rightarrow \infty$.

Definition(2.16) [6] : In the event that all Cauchy seq. In a certain normed space which approach to all points of this space**,** in this case, this space is called a complete normed.

Definition (2.17): Let X be an algebra over F, Γ_A on X is known as a Fuzzy Soft Banach Algebra if it meets the conditions:
(i) F.S. Banach space.
(ii) F.S. algebra.
(iii) $\mathcal{N}(\mathfrak{a}.\mathfrak{h}, \mathfrak{t}\,.\,\mathfrak{k}) \geq \min\{N(\mathfrak{a}, \mathfrak{t}), N(\mathfrak{h}, \mathfrak{k})\}\ \forall\ \mathfrak{a}, \mathfrak{h} \in FS, \forall\ \mathfrak{t}, \mathfrak{k} \in \mathfrak{R}^*(E)$.
(iv) If $\mathfrak{e} \in \Gamma_x$ thus $N(\mathfrak{e}, \mathfrak{t}) = 1$.

3. Fuzzy Soft Spectrum

Definition (3.1): The tally of all unfilled bounded subsets of the set of complex numbers can be seen by the syntax $\wp(\mathbb{C})$, at which C is the set of complex numbers. Let's say that A is a set of parameters. The definition of a fuzzy soft complex set is a mapping $\Gamma: A \longrightarrow \wp(\mathbb{C})$ is the symbol that is used to represent it. When Γ_A is a singleton set, its corresponding fuzzy soft element, also called a fuzzy soft complex numbercan be used to identify it. The set C(f) of all complex numbers with soft and fuzzy properties is shown.

Definition (3.2): Consider Γ_A refer to fuzzy soft Banach algebra with a fuzzy soft unit element $\tilde{e}$. The fuzzy soft spectrum of an element x belonging to set A referred to by

$\Gamma_x = \{\lambda \in \mathbb{C}(f): \lambda e - x \notin \mathcal{G}(x)\}$.

$\mathcal{G}(x)$ represents the collection of all fuzzy soft invertible elements in the soft Banach algebra Γ_A.

The fuzzy soft spectral radius of x, abbreviated as r(x), is defined as r(x)=Sup$\{|\lambda|: \lambda \in \Gamma_x\}$.

Definition (3.3): Take ε be a real number such that $0 < \varepsilon < 1$. The fuzzy soft condition spectrum of x in G is denoted as $\Gamma_\varepsilon(x)$ and is defined as follows:

$\Gamma_\varepsilon(x) = \{ \lambda \in \mathbb{C}(f): \|(\lambda e - x)\| \|(\lambda e - x)^{-1}\| > \frac{1}{\varepsilon}\}$

The fuzzy soft condition spectral radius of x is written as $r_\varepsilon(x)$ with defined as follows:

$$r_\varepsilon(x) = \text{Sup}\{|\lambda|: \lambda \in \Gamma_\varepsilon(x)\}.$$

Proposition(3.4) : Consider Γ_A as a fuzzy soft Banach algebra. If $\times \in \Gamma$ fulfills $\|\times\|$<l, Imp. $(e - \times)$ is reversible and $(e - \times)^{-1} = e + \sum_{n=1}^{\infty} \times$ n.

Proof. given that Γ_A is fuzzy soft algebra, hence we've $\|\times^j\| \leq \|\times\|^j$ for the purpose for the endless series to begin for any integer that is positive j $\sum_{n=1}^{\infty} \|\times\|^n$ this is fuzzy soft convergent due to

$\|\times\|$<l. So the partial sum $s_k = \sum_{n=1}^{k} x_n$ is a fuzzy soft Cauchy sequence since $\left\|\sum_{n=1}^{k+p} x^n\right\| < \sum_{n=k}^{k=p} \|x\|^n$

.Since Γ_A is fuzzy soft complete so $\sum_{n=1}^{\infty} x^n$is fuzzy soft convergent.

let s = e + $\sum_{n=1}^{\infty} x^n$ .Now we need to demonstrate that s = $(e - x)^{-1}$.

We get

(1) $(e - x)(e + x + x^2 + \ldots x^n) = (e + x + x^2 + \ldots x^n)(e - \times) = e - x^{n+1}$

Once more, as $\|x\|<1$, $x^{n+1} \rightarrow \theta$ as $n \rightarrow \infty$. Thus, by permitting n to get close to ∞ and retaining in awareness that the multiplication in Γ is continuous, we gain $(e - \times)s = s(e - \times) = e$. Accordingly, $s = (e - \times)^{-1}$. That validates the thought process.

Lemma (3.5): Consider Γ_A as a fuzzy soft Banach algebra that has a fuzzy soft identity element e.

Let $x \in G$ such that $\|x\| \leq 1$ then (e-x) invertible and $(e - x)^{-1} = e + \sum_{n=1}^{\infty} x^n$ Furthermore we have $\|(e - x)^{-1}\| \leq \frac{\|e\|}{\|e\| - \|x\|}$.

Proof. For the proposition (4). Now let $S_n = e + x + x^2 + \cdots + x^n$ and y= $e + \sum_{n=1}^{\infty} x^n$ Then by the first part we know that $(e - x)^{-1} = y$. Also we have

$$\|y\| = \lim_{n\to\infty} \|S_n\| = \lim_{n\to\infty} \left\| e + \sum_{k=1}^{n} x^k \right\| \leq \|e\| + \sum_{k=1}^{\infty} \|x\|^k = \frac{\|e\|}{\|e\| - \|x\|}.$$

If Γ_A is fuzzy soft with identity (i.e.,$\|e\| = 1$) then we have $\|(e - x)^{-1}\| \leq \frac{1}{1 - \|x\|}$

Corollary (3.6): Consider Γ as a fuzzy soft Banach algebra. Let $x \in \Gamma$ and μ be a fuzzy soft scalar In a way that $|\mu| > \|x\|$. Then $(\mu e - x)^{-1}$ exists and$(\mu e - x)^{-1} = \sum_{n=1}^{\infty} \mu^{-n} x^{n-1} (x^0 = e)$.

Proof. Let y ∈ Γ be a member of Γ for which y^{-1} exists in Γ. Additionally, let α be a fuzzy soft scalar such that α(λ) is not equal to zero for all λ ∈ A. Therefore, it is evident that $(\alpha y)^{-1} = \alpha^{-1}y^{-1}$.

After observing this, we can express it in writing $\mu e - x = \mu(e - \mu^{-1}x)$

We next illustrate the presence of (e− μ−1x)−1. By speculation, we've got $\|e - (e - \mu^{-1}x)\| = \|\mu^{-1}x\| = |\mu|^{-1}\|x\| < 1$. That is why $(e - \mu^{-1}x)^{-1}$ occurs by Corollary 10, and as a consequence of this, $(\mu e - x)^{-1}$ occurs. By implementing Proposition 9 to the endless series democracy, we yield

$$(\mu e - x)^{-1} = \mu^{-1}(e - \mu^{-1}x)^{-1}$$

$$= \mu^{-1}(e + \textstyle\sum_{n=1}^{\infty} [e - (e - \mu^{-1} x)]^n$$

$$= \mu^{-1}(e + \textstyle\sum_{n=1}^{\infty} (\mu^{-1}x)^n)$$

$$= \textstyle\sum_{n=1}^{\infty} \mu^{-n} x^{n-1}.$$

This shows the corollary.

Corollary (3.7) :Let Γ_A act as a fuzzy soft Banach algebra with fuzzy soft unit element $\breve{e}$ With this in mind $\|\breve{e}\| = \breve{1}$, $\widetilde{x} \in \mathcal{G}(x)$. Pretend $\lambda \in \widetilde{\mathbb{C}(f)} - \{0\}$ such that $\|\breve{x}\| \ddot{<} \breve{\lambda}$ Then $(\lambda e - x)$ is invertible and $(\mu e - x)^{-1} = \sum_{n=1}^{\infty} \breve{\mu}^{-n} \breve{x}_n{}^{n-1}$: $(\breve{x}^0 = \breve{e})$ .Furthermore we have

$$\|(\widetilde{\lambda e} - 1)^{-1}\| \acute{\leq} \frac{\bar{1}}{|\bar{\lambda}| - \|\breve{x}\|}$$

Proof. To decide on the corollary (6) $\frac{\ddot{x}}{\bar{\lambda}}$ $\breve{x}$ the final assertion, all we have to do is replace $\frac{\ddot{x}}{\bar{\lambda}}$ in Lemma (5) with $\breve{x}$ to obtain the outcome.

Theorem (3.8): Let Γ_A act as a fuzzy soft Banach algebra with fuzzy soft unit element e With this in mind $\|e\| = \breve{1}$ and x ∈ A. Next up, we've got
$r(x) \leq r_\varepsilon(x) \leq \frac{1+\varepsilon}{1-\varepsilon}\|x\|$.
Proof. Since $\Gamma_x \subseteq \Gamma_\varepsilon(x)$, so we have $r(x) \leq r_\varepsilon(x)$.
Suppose that $\lambda \in \Gamma_\varepsilon(x)$. If $|\lambda| \leq \|x\|$, then we can easily prove that $|\lambda| \leq \frac{1+\varepsilon}{1-\varepsilon}\|x\|$ Thus we have $r_\varepsilon(x) \leq \frac{1+\varepsilon}{1-\varepsilon}\|x\|$. Now suppose that $|\lambda| > \|x\|$. Then $(\lambda e - x)$ is invertible and by corollary (3.5) we have $\|(\lambda e - x)^{-1}\| \leq \frac{1}{|\lambda| - \|x\|}$. Consequently by some computations we get $|\lambda| \leq \frac{1+\varepsilon}{1-\varepsilon}\|x\|$.

Thus we conclude that $r_\varepsilon(x) \leq \frac{1+\varepsilon}{1-\varepsilon}\|x\|$

Definition (3.9): Let Γ_A be a fuzzy soft Banach algebra and T: $\Gamma_G \to \mathbb{C}(A)$ respond as a fuzzy soft linear functional. T is virtually fuzzy soft multiplicative if there exists an.$\delta > 0$That is required everyone benefits from $a, b \in G$:

$$|T(xy) - T(x)T(y)| \leq \delta\|x\|\|y\|.$$

Proposition (3.10): Assume that ϑ Particularly a fuzzy soft linear functional being on a fuzzy soft

Banach algebra Γ_A, which has an identity element e, and $\vartheta(e) = 1$. The subsequent requirements are equal.

i) $\vartheta(a) = 0$ implies $\vartheta(a^2) = 0\ for\ all\ a \in G,$
ii) $\vartheta(a^2) = (\vartheta(a))^2, a \in G,$
iii) $\vartheta(a) = 0$ implies $\vartheta(ab) = 0\ for\ all\ a, b \in G$
iv) $\vartheta(ab) = \vartheta(a)\vartheta(b)\ for\ all\ a, b \in G$

proof: (i) ⟹ (ii)

$\vartheta(e) = 1$ implies
$\vartheta(a - \vartheta(a))(\lambda) = \vartheta(ea - \vartheta(a))(\lambda\vartheta(e)(\lambda)\vartheta(a)(\lambda) - \vartheta(a)(\lambda) = 0\ ; \forall\lambda \in A.$
So we have 0=$\vartheta(a - \vartheta(a)) = 0$. By (i) we have
0=$\vartheta\left(\left(a - \vartheta(a)\right)^2\right) = \vartheta(a^2 - 2a\vartheta(a) + \left(\vartheta(a)\right)^2 = \vartheta(a^2) - \left(\vartheta(a)\right)^2.$
Thus we deduce that $\vartheta(a)^2 = \left(\vartheta(a)\right)^2$.
The subsequent requirements (ii) lead to (iii)
Via supplanting u+v via (x) in (ii) We comprehend $\vartheta(uv + vu) = 2\vartheta(u)\vartheta(v); u, v \in G, (1)$
permit a, b be in G with $\vartheta(a) = 0$. According to (1) we have $\vartheta(ab + ba) = 0. (2)$
Hence by (ii) we obtain $\vartheta(ab + ba)^2) = 0$. Since $(ab + ba)^2$=2$\vartheta(a(bab) + (bab)a) = 4\vartheta(a)\vartheta(bab) = 0$.
According to (ii) we have $\vartheta(ab - ba) = 0, (3)$. If we add two equalities (2) and (3) we conclude that $\vartheta(ab) = 0$.
The subsequent requirements (iii) lead to (iv)
permit $a, b\ \in G. we\ have\ \vartheta(a - \vartheta(a)) = 0$. Hence for each $\lambda \in A$ we have
$\vartheta(a - \vartheta(a))(\lambda) = 0$. Thus by (iii) we have $\vartheta\left(\left(a - \vartheta(a)\right)b\right)(\lambda) = 0$.
Then we get
0=$\vartheta\left(\left(a - \vartheta(a)\right)b\right)(\lambda) = \vartheta(ab - \vartheta(a)b)(\lambda) = \vartheta(ab)(\lambda) - \vartheta(a)(\lambda)\vartheta(b)(\lambda); \forall\lambda \in A.$
Thus we have 0=$\vartheta(ab) - \vartheta(a)\vartheta(b)$. Consequently we get $\vartheta(ab) = \vartheta(a)\vartheta(b)$.
(iv)⟹(i)
From (iv) we have $\vartheta(ab)(\lambda) = \vartheta(a)(\lambda)\vartheta(b)(\lambda)$. If $\vartheta(a) = 0, then\ \vartheta(a)(\lambda) = 0; \forall \in A$ so we have $\vartheta(ab)(\lambda) = \vartheta(a)(\lambda)(b)(\lambda) = 0.\vartheta(b)(\lambda) = 0$
Therefore $\vartheta(ab) = 0$.
If ϑ is fuzzy soft multiplicative linear functional, then it is also fuzzy soft Jordan multiplicative linear functional. Next contemplate its ensuing corollary.

Corollary(3.11)

Consider ϑ as a fuzzy soft multiplicative linear functional on the fuzzy soft Banach algebra Γ_A, which has an identity element e. Additionally, it is required that $\phi(e) = 1$. ϑ is a fuzzy soft multiplicative.

Lemma (3.12): Let ϑ is a fuzzy soft multiplicative linear functional on the fuzzy soft Banach algebra Γ_G. Next, we'll have $\vartheta(x) \in \Gamma_G$; x $\in G$.
Proof: For x $\in G$ we set z=$\vartheta(x)e - x$ Then we have $\vartheta(z)(\lambda) = \vartheta(x)(\lambda)\vartheta(e)(\lambda) - \vartheta(x)(\lambda) = \vartheta(x)(\lambda) - \vartheta(x)(\lambda) = 0$
Hence we have $\vartheta(z) = 0$. Therefore z$\in ker\vartheta$. So we have z$\in sing(G)$. Consequently $\vartheta(x) \in \Gamma_G$.

Remark (3.13): Suppose ϑ is a fuzzy soft multiplicative linear functional, and let z be an element of Γ_G, for some x $\in G$. Then ze-xis not invertible and so we have $\vartheta(ze-x)=0$.
It is known that when an element y is invertible, its value $\vartheta(y)=0$. Thus z$\vartheta(e)-\vartheta(x)=0$. So we have z=$\vartheta(x)$.
Therefore, we derive the one after that theorem.

Theorem (3.14): If Γ_A be a commutative fuzzy soft Banach algebra and let x $\in G$ so
$\Gamma_G=\{\vartheta(x): \vartheta$ is a fuzzy soft multiplicative linear functional$\}$
Proof. It is inferred from the final sentence and lemma (3.12).

Lemma(3.15): if T:$\Gamma_E(A) \longrightarrow \mathbb{C}(f)$ be a fuzzy soft bounded linear functional. ThenT is almost fuzzy soft multiplicative.
Proof: We have for every $\breve{x}, \breve{y}$
$|T(\breve{x}\breve{y})-T(\breve{x})T(\breve{y})| \leq |T(\breve{x}\breve{y})|+|T(\breve{x})T(\breve{y})| \leq \|T\|\|\breve{x}\breve{y}\|+\|T\|^2\|\breve{x}\|\|\breve{y}\|=(\|T\|+\|T\|^2)\|\breve{x}\|\|\breve{y}\|$
Thus T is almost soft multiplicative where $\bar{\bar{\delta}}$ =$(\|T\|+\|T\|^2)$

Proposition (3.16): Assume that Γ_A is a fuzzy soft Banach algebra and T_1:$\Gamma_E(A) \longrightarrow \mathbb{C}(f)$ is a fuzzy soft multiplicative linear functional and T_2:$\Gamma_E(A) \longrightarrow \mathbb{C}(f)$ is a soft bounded linear functional. ThenT_1+T_2 is almost fuzzy soft multiplicative functional but not multiplicative.
Proof: For apiece $\breve{\alpha}, \breve{b} \in A$, we've got

$|(T_1+T_2)(\breve{\alpha}\breve{b})-(T_1+T_2)(\breve{\alpha})(T_1+T_2)(\breve{b})|=$

$|T_1(\breve{\alpha}\breve{b})+T_2(\breve{\alpha}\breve{b})-(T_1(\breve{\alpha})+T_2(\breve{b}))(T_1(\breve{b})+T_2(\breve{b})|=$

$|T_1(\breve{\alpha}\breve{b})+T_2(\breve{\alpha}\breve{b})-T_1(\breve{\alpha})T_1(\breve{b})-T_2(\breve{\alpha})T_2(\breve{b})-T_1(\breve{\alpha})T_2(\breve{b})-T_2(\breve{\alpha})T_1(\breve{b})| \leq$

$|T_1(\breve{\alpha}\breve{b})-T_1(\breve{\alpha})T_1(\breve{b})|+|T_2(\breve{\alpha}\breve{b})-T_2(\breve{\alpha})T_2(\breve{b})|+|T_1(\breve{\alpha})T_2(\breve{b})|+|T_2(\breve{\alpha})T_1(\breve{b})|$

So by lemma(14) we get

$$|(T_1+T_2)(\breve{\alpha}\breve{b})-(T_1+T_2)(\breve{\alpha})(T_1+T_2)(\breve{b})| \leq$$

$(\|T_2\|+\|T_2\|^2)\|\breve{\alpha}\|\|\breve{b}\|+\acute{2}\|T_1\|\|\breve{\alpha}\|\|T_2\|\|\breve{b}\|=$

$(\|T_2\|+\|T_2\|^2)\|\breve{\alpha}\|\|\breve{b}\|+\acute{2}\|T_1\|\|T_2\|\|\breve{\alpha}\|\|\breve{b}\|.$

Thus(T_1+T_2) is almost fuzzy soft multiplicative. Naturally(T_1+T_2) is not multiplicative.

Definition(3.17):LetΓ_A be a fuzzy soft Banach algebra. We state that fuzzy soft linear functional ϑ:Γ_E(A) $\longrightarrow \mathbb{C}$(f) is almost fuzzy soft jordan multiplicative functional if there is $\bar{\bar{\delta}} \gtrsim \bar{\bar{0}}$such that:
$|\vartheta(\breve{X}^2)-\vartheta(\breve{X})^2| \leq \bar{\bar{\delta}}\|\breve{X}\|^2$,$\forall \breve{x} \in$ A

Corollary(3.18): Let be a fuzzy soft Banach algebra and T_1:Γ_E(A) $\longrightarrow \mathbb{C}$(f) is a fuzzy soft jordan mul tiplicative linear functional and T_2:Γ_E(A) $\longrightarrow \mathbb{C}$(f) is a soft bounded linear functional. Then $T_1+T_2$is alm ost fuzzy soft jordan mul tiplic ative func tional.
Proof: A strategy comparable to the one we described in theorem (13), can be used to prove it.

Definition(3.19): Let Γ_A be a soft Banach algebra with identity element ě and let $\bar{\bar{\varepsilon}} \acute{>} \bar{0}$ We denote the fuzzy soft $\varepsilon\_$ Minimum spectrum for an element $\check{X} \in \check{A}$ by $\Gamma_{\bar{\varepsilon}}(\check{X})$ and express it as follows: $\Gamma_{\bar{\varepsilon}} = \{\check{\lambda} \,\breve{\in}\, \mathbb{C}(F): \|\check{\lambda}\, \check{e} - \check{x})^{-1}\| \,\bar{\geqq}\, \frac{\check{1}}{\varepsilon}\}$.

Theorem(3.20): Let Γ_A be a fuzzy soft Banach algebra with identity element ě and let $\bar{\bar{\varepsilon}} \acute{>} \bar{0}$. Let ϑ:Γ_E(A) $\longrightarrow \mathbb{C}$(f) be a fuzzy soft linear functional such that ϑ ($\check{e}$)=$\check{1}$ and $\vartheta(e) \,\bar{\bar{\in}}\, \Gamma_{\check{\varepsilon}}$ for $\check{X} \in \check{A}$. Then is soft multiplicative functional.

Proof: We illustrate that for every $\check{X} \in \check{A}$ we've got T($\check{x}$)$\in \Gamma_x$.we added $\bar{\lambda} = \vartheta(\check{x}$). If $\bar{\bar{\lambda}} \in \Gamma_x$ then ϑ is multiplicative. If $\bar{\bar{\lambda}} \notin \Gamma_x$ then $\bar{\bar{\lambda}}\check{e} - \check{x}$ is invertible and so$\bar{\bar{\lambda}}\check{e} - \check{x} \,\bar{\in}\, \mathcal{G}$(x).Suppose that $\acute{z} \,\bar{>}\, \check{\varepsilon}\|(\bar{\bar{\lambda}}\check{e} - \check{x})^{-1}\|$ Then we have $\|(\bar{\bar{\lambda}}\check{e} - \check{x})^{-1}\| \,\bar{<}\, \frac{\check{1}}{\varepsilon}$.Thus we get $\|(\bar{\bar{\lambda}}\check{e}\acute{z} - \check{x}\acute{z})^{-1}\| \,\bar{<}\, \frac{\check{1}}{\varepsilon}$ Consequently we have $\bar{\bar{\lambda}}\, \acute{z} = \vartheta(\check{x})\acute{z} = \vartheta(\check{x}\acute{z}) \notin \Gamma_{\check{\varepsilon}}(\acute{z}\check{x})$ which is a contradiction. So ϑ is fuzzy soft multiplicative.

Lemma (3.21). Let$\bar{\bar{\delta}} \,\breve{>}\, \bar{\bar{0}}$ and $\check{X} \in \check{A}$ Then$\Gamma_x \subseteq \Gamma_{\widetilde{\delta x}}$

Proof: It can be readily demonstrated by the use of a precise definition.

Theorem(3.22): Assume that Γ_A be a fuzzy soft Banach algebra with identity element ě and let ϑ be an almost fuzzy soft multiplicative linear functional on A If ϑ ($\check{e}$)=$\check{1}$ Then for every element $\check{x} \,\bar{\bar{\in}}\, A$ we have ϑ ($\check{x}$)$\,\bar{\bar{\in}}\, \Gamma_{\delta x}$

Proof: Let $\check{x} \,\bar{\bar{\in}}\, A$ and $\bar{\bar{\lambda}} = \vartheta$ ($\check{x}$) .If $\bar{\bar{\lambda}}\check{e} - \check{x}$ is not invertible then $\bar{\bar{\lambda}} \in \Gamma_x \subseteq \Gamma_{\widetilde{\delta x}}$ So $\bar{\bar{\lambda}} \in T_\varepsilon(\check{x})$. Now assume that is invertible. Then

$$\bar{\bar{1}} = |\vartheta(\check{e})| = |\vartheta(\check{x}) - \check{0}| = |\vartheta(\check{e}) - \vartheta(\bar{\bar{\lambda}}\check{e} - \check{x})\mathrm{T}((\bar{\bar{\lambda}}\check{e} - \check{x}))^{-1}|$$

$$\breve{\leq}\ \bar{\delta}\|(\bar{\bar{\lambda}}\check{e} - \check{x})((\bar{\bar{\lambda}}\check{e} - \check{x}))^{-1}\|$$

Thus we have

$$\|(\bar{\bar{\lambda}}\check{e} - \check{x})((\bar{\bar{\lambda}}\check{e} - \check{x}))^{-1}\| \,\bar{\geqq}\, \frac{\check{1}}{\varepsilon}$$

So we conclude that $\bar{\bar{\lambda}} \in \Gamma_{\varepsilon x}$. Consequently we have ϑ ($\check{x}$)$\,\bar{\bar{\in}}\, \Gamma_{\delta x}$.

4. Conclusions

This study presents the concept of a fuzzy soft spectrum. We examined the fundamental characteristics of these ideals in fuzzy soft Banach algebras. Recent research has focused on exploring new findings and theorems related to fuzzy soft Banach algebra.

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