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Some New Results Related in Fuzzy Soft Normed Space

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ABSTRACT

This study examines the concept of fuzzy soft normed space from several perspectives. Additionally, we analyze the concept of fuzzy soft metric space and its associated theorem. The ideas of Cauchy and convergence are also defined. Several theorems pertaining to these ideas have been proven.

1. Introduction

Most difficulties in reality are characterized by uncertainty or lack of precision. This is a result of insufficiently provided information. Zadeh created fuzzy sets in 1965 as a means to address such issues. Molodostov [5] introduced soft set theory in 1999 as a solution to the absence of parameterization tools in fuzzy set theory. Maji et al. introduced the new concept of fuzzy soft set in 2001 after studying how fuzzy set theory and soft set theory may be integrated. The original idea behind soft set theory was to broaden its use beyond cases with clear-cut answers to include cases involving uncertainty. Reference [6] is where Tanay and Kandemir started looking at fuzzy soft set theory from a topological standpoint. Zadeh [9] offered the concept of fuzzy soft norm for a set for the first time in 2013, and he also demonstrated the relationship between the two. Multiple perspectives on fuzzy soft normed space are explored in this research. We also examine several theorems and the notion of fuzzy soft metric space. Additionally defined are the concepts of convergence and Cauchy. Many theorems related to these concepts have been demonstrated to be true.

2.1 Fuzzy Soft Set

The section began with a definition of fuzzy soft set and continued such that a discussion of its properties.

Definition (2.1.1) [4]: A pair $\Gamma_{\mathcal{A}}$ can be defined as a fuzzy soft set over X if $\Gamma: \mathcal{A} \to I^X$ is a transition between $\mathcal{A}$ to I^X. For every ρ belonging to set $\mathcal{A}$, $\Gamma(\rho)$ denotes a fuzzy part of X, distinguished by its members function.

$$\Gamma_\rho : X \to [0,1]$$

Definition(2.1.2) [4]: Given two fuzzy soft sets $\Gamma_{\mathcal{A}}$ and $\mu_{\mathfrak{B}}$ defined over the same universe X, we may state the following:

i. $\Gamma_{\mathcal{A}}$ is a fuzzy soft part of $\mu_{\mathfrak{B}}$ denoted as $\Gamma_{\mathcal{A}} \leq \mu_{\mathfrak{B}}$ given:

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1. $\mathcal{A} \subseteq \mathfrak{B}$.

2. For any ρ within $\mathcal{A}$, $\Gamma_{\mathcal{A}}(x) \leq \mu_{\mathfrak{B}}(x)$ for any x within X.

ii. $\Gamma_{\mathcal{A}} = \mu_{\mathfrak{B}}$ if $\Gamma_{\mathcal{A}} \leq \mu_{\mathfrak{B}}$ with $\mu_{\mathfrak{B}} \leq \Gamma_{\mathcal{A}}$.

iii. The reciprocal of the fuzzy soft set $\Gamma_{\mathcal{A}}$ can be represented as ${\Gamma_{\mathcal{A}}}^{c}$, when $\Gamma^{c}: \mathcal{A} \to I^{X}$ and $\Gamma^{c}(\rho)$ indicates the inverse of the fuzzy set $\Gamma(\rho)$ given the member function ${\Gamma_{\mathcal{A}}}^{c} = 1 - \Gamma_{\rho}$ for any ρ in $\mathcal{A}$.

iv. A fuzzy set $\Gamma_{\mathcal{A}} = \Phi_{\mathcal{A}}$ is considered null fuzzy soft concerning $\mathcal{A}$ if, for every $\rho \in \mathcal{A}$, $\Gamma_{\rho}(x) = 0$ for any x in X.

v. Absolute fuzzy soft set $\Gamma_{\mathcal{A}}$ concerning set $\mathcal{A}$ is defined as $\Gamma_{\mathcal{A}} = \mathrm{X}_{\mathcal{A}}$, where for every element x in set X, $\Gamma_{\rho}(x) = 1$ for each ρ in set $\mathcal{A}$.

If $\mathcal{A}$ is equal to E, then the null fuzzy soft set is indicated by Φ and the absolute fuzzy soft set is denoted by X.

vi. The combination of both fuzzy soft sets, represented by $\Gamma_{\mathcal{A}} \cup \mu_{\mathfrak{B}}$, is the fuzzy soft set $(\Gamma \cup \mu)_{c}$. Here, C represents the union of sets $\mathcal{A}$ and $\mathfrak{B}$. For any element ρ in C, the value of $(\Gamma \cup \mu)(\rho)$ is obtained by taking the union of $\Gamma(\rho)$ and $\mu(\rho)$

$$(\Gamma \cup \mu)_{\rho}(x) = \begin{cases} \Gamma_{\rho}(x) & if\ \rho \in \mathcal{A} - \mathfrak{B} \\ \mu_{\rho}(x) & if\ \rho \in \mathfrak{B} - \mathcal{A} \\ \max\{\Gamma_{\rho}(x), \mu_{\rho}(x)\} & if\ \rho \in \mathcal{A} \cap \mathfrak{B} \end{cases}$$

for any $x \in X$.

vii. The combination between the fuzzy soft sets $\Gamma_{\mathcal{A}}$ and $\mu_{\mathfrak{B}}$ is represented by $\Gamma_{\mathcal{A}} \cap \mu_{\mathfrak{B}}$. It is defined as the fuzzy soft set $(\Gamma \wedge \mu)_{c}$, where C is the non-empty intersection of sets $\mathcal{A}$ and $\mathfrak{B}$. For every element ρ in C, the value of $(\Gamma \wedge \mu)_{\rho}(x)$ is given by the minimum of $\Gamma_{\rho}(x)$ and $\mu_{\rho}(x)$, indicated as $(\Gamma \wedge \mu)_{\rho}(x) = \min\{\Gamma_{\rho}(x), \mu_{\rho}(x)\}$ for every $x \in X$.

2.2 Fuzzy Soft Normed space

Definition (2.2.1) [3]: Suppose that F represent either the field of real numbers or the field of complex numbers. Consider X as a linear space over F. The function $\|.\|$ is a norm on X. $X \to \mathbb{R}$ exhibits the following features and characteristics:

(1) $\|\hat{x}\|$ is greater than or equal to zero, for any $\hat{x}$ in X.

(2) The norm of $\hat{x}$ is equal to zero $\leftrightarrow$ $\hat{x}$ is equal to 0, for any $\hat{x}$ in X.

(3) $\|\lambda\hat{x}\| = |\lambda|\|\hat{x}\|$, for every $\hat{x} \in X$ and $\lambda \in F$.

(4) $\|\hat{x} + \hat{y}\| \leq \|\hat{x}\| + \|\hat{y}\|$, for every $\hat{x}, \hat{y} \in X$.

Definition(2.2.2) [2]: Consider * to be a binary operation on a set I, represented as $*: I \times I \rightarrow I$, where * is a function. A t-norm, also referred to as a triangular-norm, is regarded as * on the set I if it fulfills each of the following rules:

(1) $a * 1 = a$, for every $a \in I$, $a * 0 = 0 \; \forall \; a$.

(2) $*$ is commutative (i.e. $a * \mathfrak{b} = \mathfrak{b} * a$, for every $a, \mathfrak{b} \in I$).

(3) $*$ is monotone (i.e. if $\mathfrak{b}, c \in I$ which means $\mathfrak{b} \leq c$, then $a * \mathfrak{b} \leq a * c$, for every a in I).

(4) The operation * is associative, meaning that for all elements $a, \mathfrak{b}$ and c in the set I, the equation $a * (\mathfrak{b} * \mathrm{c}) = (a * \mathfrak{b}) * \mathrm{c}$ holds true. If, furthermore, * is continuous, it is referred to as a continuous t-norm.

Definition(2.2.3)[8]: Suppose that X be a linear space over F, where F can be either real or complex. Let * be a continuous t-norm on I. A function $\mathrm{N}: \mathrm{X} \times \mathrm{R}$, where R is the set of all real numbers, is considered a fuzzy norm if and only if for all x, y in X with α in F,

(1) $N(x, \tau) = 0$, $\forall \tau \in R$ and $\mathrm{t} \leq 0$.

(2) $N(x, \tau) = 1 \leftrightarrow x = 0, \forall \tau \in R$ such that $\tau > 0$.

(3) $N(\alpha x, \tau) = N\left(x, \frac{\tau}{|\alpha|}\right)$, for all $\alpha \neq 0$, , $\forall \tau \in R$ such that $\tau > 0$.

(4) $N(x, \tau) * N(y, s) \leq N(x + y, \tau + s)$, , $\forall \tau, s \in R, x, y$ in X .

(5) $N(x, .)$ is continuous nondecreasing such that $lim_{\tau \rightarrow \infty} N(x, \tau) = 1$.

The triplet $(X, N, *)$ shall be denoted as a fuzzy normed space.

Definition (2.2.4) [1]: Consider $S_l(X)$ as a soft linear space. A mapping $\|.\| : S_l(X) \rightarrow R^+(E)$ is defined as a soft norm on $S_l(X)$ if it fulfills what follows issues such as

1) $\|\hat{x}\| \geq 0$ for any $S_l(X)$ with $\|\hat{x}\| = 0 \leftrightarrow \hat{x} = 0$

2) $\|v \, \hat{x}\| = |v| \|\hat{x}\|$ for any $\hat{x} \in S_l(X)$ with for any soft scalar v

3) $\|\hat{x} + \hat{y}\| \leq \|\hat{x}\| + \|\hat{y}\|$ for any $\hat{x}, \hat{y} \in S_l(X)$

The soft linear space $S_l(X)$ is a space equipped with a soft norm $\|.\|$ on X. This type of space is referred to as a soft normed space as well as is indicated by $(X, \|.\|)$.

Definition(2.2.3): Consider X as a linear space over the scalar field F, where X is an absolute soft linear space. Assume that $*$ is a continuous t-norm. The symbol $R(\mathcal{A}^*)$ denotes the set of all non-negative soft real numbers, whereas S_{SP} symbolises the set of all soft points on X. A fuzzy subset $\widetilde{F_S}$ on $S_{SP}(X) \times R(\mathcal{A}^*)$ is considered a fuzzy soft norm on X if it is the case if these conditions are met: $\hat{x}, \hat{y}$belong to $S_{SP}(X)$, as well as k belongs to F (in which k is a soft integer).

1) $\widetilde{F_S}(\hat{x}, \tau) = 0 \; \forall \tau \in R(\mathcal{A}^*)$.

2) $\widetilde{F_S}(\hat{x}, \tau) = 1 \; \forall \tau \in R(\mathcal{A}^*)$ with $\tau > 0 \leftrightarrow \hat{x} = \theta_0$.

3) $\widetilde{F_S}(k\,\hat{x}\,,\tau\,) = \widetilde{F_S}\left(\hat{x}\,,\frac{\tau}{|k|}\right)$ if $k\ \neq 0\ \forall\ \tau\ \in R(\mathcal{A}^*)$ with $\tau\ > 0$.
4) $\widetilde{F_S}(\hat{x}\oplus\hat{y}\,,\tau\,\oplus s) \geq \widetilde{F_S}(\hat{x}\,,\tau\,) * \widetilde{F_S}(\hat{y}\,,s\,)\ \forall\,\tau\,,s \in R(\mathcal{A}^*)$ with $\hat{x},\hat{y} \in S_{SP}(X)$
5) $\widetilde{F_S}(\hat{x}\,,.)$ is continuous concerning $R(\mathcal{A}^*)$ such that $\lim_{\tau\to\infty} \widetilde{F_S}(\hat{x}\,,t\,) = 1$.

The triangle $(X,\widetilde{F_S}\,,*)$ is denoted as a fuzzy soft normed space.

Definition(2.2.4): Suppose that $\tilde{\tau}\ > 0$ be a soft real number and suppose that $(X,\widetilde{F_S}\,,*)$ be a fuzzy soft normed space. The following are the definitions of an open ball, a closed ball, with a sphere such that a center at $\hat{x}$ and a radius v:

$\mathfrak{B}(\hat{x},v,\tilde{\tau}\,) = \{\,\hat{y} \in X\colon \widetilde{F_S}\,(\hat{x}-\hat{y}\,,\tilde{\tau}+\,) > 1-v\}$

$\overline{\mathfrak{B}}(\hat{x},v\,,\tilde{\tau}) = \{\,\hat{y} \in$ X: $\widetilde{F_S}(\hat{x}-\hat{y},\tilde{\tau}\,) \geq 1-v\,\}$

$S(\hat{x}\,,v\,,\tilde{\tau}\,) = \{\,\hat{y} \in X\colon \widetilde{F_S}(\hat{x}-\hat{y}\,,\tilde{\tau}\,) = 1-v\,\}$

The terms "fuzzy soft closed ball," "fuzzy soft sphere" and "fuzzy soft open ball" refer to $\mathfrak{B}(\hat{x},v,\tilde{\tau}\,),\overline{\mathfrak{B}}(\hat{x},v\,,\tilde{\tau})$, and $S(\hat{x}\,,v\,,\tilde{\tau}\,)$ respectively. These terms are used to describe a geometric shape having a centre $\hat{x}$ and a radius v.

Definition(2.2.5) [7]: A mapping $\Delta\colon\ S_{SP}(X) \times S_{SP}(X) \times R(\mathcal{A}^*) \to [0,1]$ fuzzy soft metric on X is defined as [0,1] if Δ fulfills a particular circumstance

1. $\Delta(\hat{x},\hat{y},\tilde{\tau}) > 0$, for any $\tilde{\tau} > \tilde{0}$
2. $\Delta(\hat{x},\hat{y},\tilde{\tau}) = 1$ for every $\tilde{\tau}\ > \tilde{0} \leftrightarrow \hat{x} = \hat{y}$
3. $\Delta(\hat{x},\hat{y},\tilde{\tau}) = \Delta(\hat{y},\hat{x},\tilde{\tau})$.
4. $\Delta(\hat{x},\hat{z},\tilde{\tau}\oplus\tilde{s}\,) \geq \Delta(\hat{x},\hat{y}\,,\tilde{s}) * \Delta(\hat{y},\hat{z},\tilde{\tau})\ \ \forall\,\tilde{\tau}\,,\tilde{s} > \tilde{0}$
5. $\Delta(\hat{x},\hat{y},.) : (0,\infty) \to [0,1]$ is continuous .

A fuzzy soft metric space, denoted as $(X,\Delta\,,*\,)$, is defined as a soft set X is equipped using the fuzzy soft metric Δ.

Theorem(2.2.6): A normed space that exhibits fuzzy soft features can alternatively be regarded as a fuzzy soft metric space.

Proof : The fuzzy soft metric space is described as follows:

$\Delta(\hat{x},\hat{y},\tilde{\tau}) = \widetilde{F_S}(\hat{x}-\hat{y},\tilde{\tau})$ ………………….*

for every $\hat{x},\hat{y}\ \in S_{SP}(X)$.

It can be readily demonstrated that the axioms of the fuzzy soft metric space are fulfilled.

$\Delta(\hat{x},\hat{y},\tilde{\tau}) = \widetilde{F_S}(\hat{x}-\hat{y},\tilde{\tau}) = 0$ if $\tilde{\tau} \leq 0$

$\Delta(\hat{x},\hat{y},\tilde{\tau}) = \widetilde{F_S}(\hat{x}-\hat{y},\tilde{\tau}) = 1\ \ if\ \tilde{\tau}\ > 0$

$\Delta(\hat{x}\,,\hat{y}\,,\tilde{\tau}) = \widetilde{F_S}(\hat{x}-\hat{y},\tilde{\tau})$

$=\widetilde{F_S}(\hat{y}-\hat{x},\tilde{\tau})$

$=\Delta(\hat{y},\hat{x},\tilde{\tau})$

$\Delta(\hat{x},\hat{y},\text{t}) = \Delta(\hat{y},\hat{x},\tilde{\tau})$

$\Delta(\hat{x},\hat{z},\tilde{s}\oplus\tilde{\tau}) = \widetilde{F_S}(\hat{x}-\hat{z},\tilde{\tau}\oplus\tilde{s})$

$= \widetilde{F_S}(\hat{x}-\hat{y}+\hat{y}-\hat{z},\ \tilde{\tau}\oplus\tilde{s})$

$\geq \widetilde{F_S}(\hat{x}-\hat{y},\tilde{s}) * \widetilde{F_S}(\hat{x}-\hat{z},\tilde{\tau})$

$=\Delta(\hat{x},\hat{y},\tilde{s}) * \Delta(\hat{x},\hat{z},\tilde{\tau})$

$\Delta(\hat{x},\hat{z},\tilde{s}\oplus\tilde{\tau}) \geq \Delta(\hat{x},\hat{y},\tilde{s}) * \Delta(\hat{x},\hat{z},\tilde{\tau})$ Given to the concept of * of Δ it can be concluded that Δ is a constant function denoted as $\Delta(\hat{x},\hat{y},.) : (0,\infty) \to [0,1]$.

Definition(2.2.6): A fuzzy soft normed space $(X,\widetilde{F_S},*)$ is considered convex if, for each $\tilde{x}_1,\tilde{x}_2 \in S_{SP}(X)$, the line segment between $\tilde{x}_1$ and $\tilde{x}_2$ is included within $S_{SP}(X)$, meaning that $K\odot\tilde{x}_1 \oplus(1-K)\odot\tilde{x}_2 \in S_{SP}(X)$.

Theorem(2.2.7) : Every ball that is open and has a fuzzy soft structure in a normed space with a fuzzy soft norm is convex.

Proof: Let $\tilde{y}_1,\tilde{y}_2$ be two points in a fuzzy soft open ball $\mathfrak{B}(\hat{x},v,\tilde{\tau})$ with centre $\tilde{x}_1$ and radius v . By definition, $\widetilde{F_S}(\tilde{x}_1-\tilde{y}_2,\tilde{\tau}) \gtrsim 1-v$.

The fuzzy soft open ball $\mathfrak{B}(\hat{x},v,\tilde{\tau})$ is said to be convex if the line segment

$K\odot\tilde{y}_1 \oplus(1-K)\odot\tilde{y}_2$ joining points $\tilde{y}_1,\tilde{y}_2$ lies in $\mathfrak{B}(\hat{x},v,\tilde{\tau})$.

Consider

$\widetilde{F_S}(K\odot\tilde{y}_1 \oplus(1-K)\odot\tilde{y}_2 - \tilde{x}_{1_1},\tilde{\tau})$

$=\widetilde{F_S}(K\odot\tilde{y}_1 \oplus(1-K)\odot\tilde{y}_2 - \tilde{x}_1 \oplus K\tilde{x}_\rho - K\tilde{x},\tilde{\tau})$

$\geq \widetilde{F_S}\left(c_n \odot\left(\tilde{x}_{\rho_n}-\tilde{x}_\rho\right),\frac{\tilde{\tau}}{2}\right) * \widetilde{F_S}(c_n - c)\odot\tilde{x},\frac{\tilde{\tau}}{2})$

$= \widetilde{F_S}\left(\tilde{x}_{\rho_n} - \tilde{x}_\rho\right),\frac{\tilde{\tau}}{2c_n}) * \widetilde{F_S}(\tilde{x},\frac{\tilde{\tau}}{2(c_n-c)}$

$\longrightarrow 1$ as $n \longrightarrow \infty$

Definition(2.2.8) : The sequence $\{\breve{x}_n\}$ of soft linear elements with a fuzzy soft normed space $(X,\ \widetilde{F_S},*)$ is considered to converge to $(X,\ \widetilde{F_S},*)$ in terms of fuzzy soft norm $\widetilde{F_S}$ if

$\widetilde{F_S}(\breve{x}_n - \breve{x}_0,\tau) \geq 1-v$ for any $n \geq n_0$ with $v \in (0,1]$ where n_0 is a positive integer such that $\tau > 0$ or $\lim_{n\to\infty} \widetilde{F_S}(\breve{x}_n - \breve{x}_0,\tau) = 1$ as $\tau \to \infty$.

Definition(2.2.9) : A sequence $\{\breve{x}_n\}$ in a fuzzy soft normed space $(X,\ \widetilde{F_S},*)$ is considered a Cauchy sequence if the fuzzy soft norm F_S satisfies the condition $\widetilde{F_S}(\breve{x}_n - \breve{x}_m, \tau) \geq 1 - v$ for all $m, n \geq n_0$ such that $v \in (0,1]$ where n_0 is a positive integer with $\tau > 0$ or $\lim\limits_{n \to \infty} \widetilde{F_S}(\breve{x}_n - \breve{x}_m, \tau) = 1$ as $\tau \to \infty$.

Theorem(2.2.10) : In a fuzzy soft normed space $(X, \widetilde{F_S},*)$, any sequence that converges also satisfies the Cauchy criterion.

Proof : Assume that $\{\breve{x}_n\}$ be a sequence in a fuzzy soft normed space $(X, \widetilde{F_S},*)$. If the sequence $\{\breve{x}_n\}$ converges to $\breve{x}_0$, then there exists a natural number n_0 such that

$\widetilde{F_S}(\breve{x}_n - \breve{x}_0, \tau) \geq 1 - v$, for every $n \geq n_0$, $v \in (0,1]$ where $\tau > 0$.

Consider $\widetilde{F_S}(\breve{x}_n - \breve{x}_m, \tau) = \widetilde{F_S}(\breve{x}_n - \breve{x}_m \oplus \breve{x}_0 - \breve{x}_0, \tau)$

$= \widetilde{F_S}((\breve{x}_n - \breve{x}_0) \oplus (\breve{x}_m - \breve{x}_0), \tau)$

$\geq \widetilde{F_S}(\breve{x}_n - \breve{x}_0, \frac{1}{2}) * \widetilde{F_S}(\breve{x}_m - \breve{x}_0, \frac{1}{2})$

$\geq (1 - v) * (1 - v)$

$= min\{1 - v, 1 - v\}$

$= 1 - v$

Theorem(2.2.11): If a limit of a sequence exists in a fuzzy soft normed space, it is guaranteed to be unique.

Proof : Assume that a contrary that the sequence $\{\breve{x}_n\}$ in a fuzzy soft normed space $(X,\ \widetilde{F_S},*)$ converges to x_ρ and $x_{\grave{\rho}}$ respectively. That is $\lim\limits_{n \to \infty} \widetilde{F_S}(\breve{x}_n - x_\rho, \tau) = 1$ and $\lim\limits_{n \to \infty} \widetilde{F_S}(\breve{x}_n - x_{\grave{\rho}}, \tau) = 1$. By definition, there exist positive integers n_1, n_2

such that, $\widetilde{F_S}(\breve{x}_n - x_\rho, \tau) \geq 1 - v$ for any $n \geq n_1$ and $v \in (0,1]$ and

$\widetilde{F_S}(x_\rho^n - x_{\grave{\rho}}, \tau) \geq 1 - v$ for any $n \geq n_2$ and $v \in (0,1]$.

Choose $n \geq n_0$, where $n_0 = \min\{n_1, n_2\}$

$\widetilde{F_S}(x_\rho - x_{\grave{\rho}}, \tau) = \widetilde{F_S}(x_\rho - \breve{x}_n \oplus \breve{x}_n - x_{\grave{\rho}}, \tau)$

$= \widetilde{F_S}((\breve{x}_n - x_\rho) \oplus \widetilde{F_S}(\breve{x}_n - x_{\grave{\rho}}), \tau)$

$\geq \widetilde{F_S}(\breve{x}_n - x_\rho, \frac{1}{2}) * \widetilde{F_S}(\breve{x}_n - x_{\grave{\rho}}, \frac{1}{2})$

$\geq (1 - v) * (1 - v)$

$= min\{1 - v, 1 - v\} = 1 - v$

Or $\widetilde{F_S}(x_\rho - x_{\grave{\rho}}, \tau) = 1$ as $n \to \infty$.

The fuzzy soft normed space defines that $\widetilde{F_S}(x_\rho - x_{\grave{\rho}}, \tau) = 1$ if and only if $x_\rho - x_{\grave{\rho}} = 0$, where τ is greater than 0. Thus, $x_\rho = x_{\grave{\rho}}$

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