Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 102-109

New Results On Fuzzy Soft Spectral Banach Space

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Article History

Publication data: 30 / 6 /2024

Keywords: Almost fuzzy Soft Jordan Functional, fuzzy soft spectrum, fuzzy soft Condition Spectrum, fuzzy Soft Spectral Radius, Almost fuzzy Soft Multiplicative Linear Functional.

ABSTRACT

This work introduces the concepts of fuzzy soft spectrum, fuzzy soft condition spectrum, and fuzzy soft condition spectral radius of a fuzzy soft element over a fuzzy soft Banach space. Next, we give our definitions of fuzzy soft multiplicative linear functioning, almost fuzzy soft multiplicative linear functional, fuzzy soft Jordan multiplicative linear functional, and very fuzzy soft Jordan multiplicative linear functional within the context of a fuzzy soft Banach space. The study focuses on exploring novel findings and theorems related to fuzzy soft Banach spaces.

1.Introduction

Uncertain data in the real world poses challenges in solving issues in mathematics, biological sciences, economics, engineering, and medical sciences using conventional mathematical approaches. The main obstacle to the normal mathematical approach is the absence of parameterization tools to describe problems that arise in the domains of ambiguities as well as uncertainties.

In order to address such issues The notion of soft set theory was proposed by Molodtsov [5]. Soft set theory is a novel mathematical approach that effectively handles uncertainty. Moreover, it possesses a parameterization tool that offers greater flexibility compared to conventional mathematical approaches for dealing with the ambiguity and uncertainties inherent in real-world issues.

Das with Samanta [1] proposed the concept of soft linear functionals in soft linear spaces. The researchers examined the fundamental characteristics of these operators and expanded upon important theorems in the field of functional analysis within the context of soft-set settings.

This work introduces the notions of fuzzy soft spectrum, fuzzy soft condition spectrum, as well as the fuzzy soft spectral radius of a fuzzy soft element in a fuzzy soft Banach space. Within the framework of fuzzy soft space, we present the precise definitions of four types of linear functionals: the text contains four types of linear functionals: fuzzy soft multiplicative linear functional, almost fuzzy soft multiplicative linear functional, almost fuzzy soft Jordan multiplicative linear functional, as well as the fuzzy soft Jordan multiplicative linear functional. The study investigates recent findings and fundamental ideas in the field of fuzzy soft Banach space.

2.1 Preliminaries

Throughout this piece of writing. Suppose that X denotes a universal set and E denotes a collection of parameters.. I^X is the set of every fuzzy sets over X, and A is a subset of E.

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Definition(2.1.1) [4]: A pair Γ_A is considered a fuzzy soft set over X if $\Gamma: A \to I^X$ is a mapping from A to elements of I^X. For every element e in the set A, the function $\Gamma(e)$ represents a fuzzy subset of X, characterized by its membership function.

$$\Gamma_e: X \to [0,1]$$

Definition(2.1.2) [4]: Given two fuzzy soft sets Γ_A as well as μ_B defined over a shared universe X, we may state the following:

i. Γ_A is a fuzzy soft subset of μ_B shown by $\Gamma_A \leq \mu_B$ if:

1. $A \subseteq B$

2. For all e in A, $\Gamma_A(x) \leq \mu_B(x)\ \forall x \in X$.

ii. $\Gamma_A = \mu_B\ if\ \Gamma_A \leq \mu_B\ and\ \mu_B \leq \Gamma_A$.

iii. The complement of the fuzzy soft set Γ_A is represented as ${\Gamma_A}^c$ where $\Gamma^c: A \to I^X$ as well as $\Gamma^c(e)$ is the complement of fuzzy set $\Gamma(e)$ with membership function ${\Gamma_A}^c = 1 - \Gamma_e\ \forall\ e \in A$.

iv. $\Gamma_A = \Phi_A$ (null fuzzy soft with regard to A), if for each e $\in A$, $\Gamma_e\ (x) = 0, \forall\ x \in X$.

v. $\Gamma_A = X_A$ (absoiute fuzzy soft set with regard to A) if $\forall\ x \in X, \Gamma_e\ (x) = 1\ for\ each\ e \in A$.

When $A = E$, the null fuzzy soft set is represented by Φ, and the absolute fuzzy soft set is given as X.

vi. The union of both fuzzy soft set Γ_A as well as μ_B, is denoted by $\Gamma_A \cup \mu_B$, is the fuzzy soft set $(\Gamma \cup \mu)_c$, where $C = A \cup B$ and $\forall\ e \in C$, we have $(\Gamma \cup \mu)(\mathrm{e}) = \Gamma(\mathrm{e}) \cup \mu(e)$ where

$$(\Gamma \cup \mu)_e(x) = \begin{cases} \Gamma_e(\mathrm{x}) & if\ e \in A - B \\ \mu_e(x) & if\ e \in B - A \\ \max\{\Gamma_e(x), \mu_e(x)\} & if\ e \in A \cap B \end{cases}$$

for all x$\in X$.

vii. The intersection of two fuzzy soft sets Γ_A as well as μ_B is represented by $\Gamma_A \cap \mu_B$, and is given by the fuzzy soft the set $(\Gamma \wedge \mu)_c$. Here, $\mathrm{C} = \mathrm{A} \cap B \neq \emptyset$ with $\forall\ e \in C$, we have $(\Gamma \wedge)(\mathrm{e}) = \mu\Gamma(\mathrm{e}) \wedge \mu(e)$ where

$$(\Gamma \wedge \mu)_e(x) = \min\{\Gamma_e(x), \mu_e(x)\}\ for\ all\ x \in X.$$

3.1 fuzzy soft spectrum

Definition (3.1.1): Assume that $\mathbb{C}$ get the set of complex numbers with $\mathcal{g}(\mathbb{C})$ representing the set of every nonempty bounded subsets of $\mathbb{C}$. Consider a collection of parameters denoted by A. A mapping $\Gamma: A \longrightarrow \mathcal{g}(\mathbb{C})$ is defined as a fuzzy soft complex set. Γ_A denotes it. If the set Γ_A has only one element, it may be seen as being equivalent to the associated fuzzy soft element, which will be referred to as a fuzzy soft complex number. The set of every complex numbers that exhibit the characteristics of fuzziness and softness is denoted by the sign $\mathbb{C}(f)$.

Definition (3.1.2): Consider $\hat{\Gamma}_B$ as a fuzzy soft Banach space such that a fuzzy soft unit element I, and let $T: X \to X$ be a linear operator. The fuzzy soft spectrum of an element x belonging to set A is represented as $\hat{\Gamma}_S$ and can be defined as follows:

$$\hat{\Gamma}_S = \{\lambda \in \mathbb{C}(f): \lambda I - T \notin \mathcal{G}(x)\}.$$

$\mathcal{G}(\text{x})$ represents the collection of any fuzzy soft invertible elements in the fuzzy soft Banach space $\hat{\Gamma}_B$.

The fuzzy soft spectral radius of x, written as $\hat{r}(\text{x})$, is defined as $\hat{r}(\text{x}) = \text{Sup}\{|\lambda|: \lambda \in \hat{\Gamma}_S\}$.

Definition (3.1.3): Let $0 < \varepsilon < 1$. The fuzzy soft condition spectrum of x in G is denoted by $\hat{\Gamma}_C(\text{x})$ and is defined as follows:

$$\hat{\Gamma}_C(x) = \left\{\lambda \in \mathbb{C}(f): \|(\lambda I - T)\|\|(\lambda I - T)^{-1}\| > \frac{1}{\varepsilon}\right\}.$$

The fuzzy soft condition spectral radius of x is denoted by $\hat{r}_C(x)$ and define it by:

$$\hat{r}_C(x) = Sup\{|\lambda|: \lambda \in \hat{\Gamma}_C(x)\}.$$

Proposition(3.1.4) : Consider $\hat{\Gamma}_B$ as a fuzzy soft Banach space. If $\hat{T} \in \Gamma$ satisfies $\|\hat{T}\| < 1$, thus $(I - \hat{T})$ is invertible as well as its inverse is provided as $(\text{I} - \hat{T})^{-1} = \text{I} + \sum_{n=1}^{\infty} \hat{T}^n$.

Proof. Since $\hat{\Gamma}_B$ is fuzzy soft Banach space , so we have $\|\hat{T}^j\| \leq \|\hat{T}\|^j$ for any positive integer j, so that the infinite series $\sum_{n=1}^{\infty}\|\hat{T}\|^n$ is fuzzy soft convergent because $\|\hat{T}\|<1$. So the sequence of partial sum $\hat{s}_k = \sum_{n=1}^{k} \hat{T}_n$ is a fuzzy soft Cauchy sequence since $\left\|\sum_{n=1}^{k+p} \hat{T}^n\right\| < \sum_{n=k}^{k=p}\|\hat{T}\|^n$.

Since $\hat{\Gamma}_B$ is fuzzy soft complete so $\sum_{n=1}^{\infty} \hat{T}^n$ is fuzzy soft convergent.

Now let $s = \text{I} + \sum_{n=1}^{\infty} \hat{T}^n$.

Now it is only we have to show that $s = (I - \hat{T})^{-1}$.

We have

(1) $(\text{I} - \hat{T})(\text{e} + \hat{T} + \hat{T}^2 + \ldots \hat{T}^n) = (\text{I} + \hat{T} + \hat{T}^2 + \ldots \hat{T}^n)(\text{I} - \hat{T}) = \text{I} - \hat{T}^{n+1}$

Now again since $\|\hat{T}\| < 1$ so $\hat{T}^{n+1} \to \theta$ as $n \to \infty$. Therefore letting $n \to \infty$ in and remembering that multiplication in Γ is continuous we get, $(\text{I}-\hat{T})s = s(\text{I}-\hat{T}) = \text{e}$

So that $s = (\text{I} - \hat{T})^{-1}$. This proves the proposition.

Lemma (3.1.5): Consider $\hat{\Gamma}_B$ as a fuzzy soft Banach space equipped with an identity element I.

Suppose T is an element of G in which $\|\hat{T}\| \leq 1$, thus $(\mathrm{I} - \hat{\mathrm{T}})$ is invertible as well as its inverse is provided as $(\mathrm{I} - \hat{\mathrm{T}})^{-1} = \mathrm{I} + \sum_{n=1}^{\infty} \hat{\mathrm{T}}^{n}$ Furthermore we have $\left\|(\mathrm{I} - \hat{\mathrm{T}})^{-1}\right\| \leq \frac{\|\mathrm{I}\|}{\|\mathrm{I}\| - \|\hat{\mathrm{T}}\|}$.

Proof. For the proposition (4). Now
let $S_n = I + \hat{T} + \hat{T}^2 + \cdots + \hat{T}^n$ and $y = I + \sum_{n=1}^{\infty} \hat{T}^n$ Then by
the first part we know that $(\mathrm{I} - \hat{\mathrm{T}})^{-1} = \mathrm{y}$. Also we have

$$\|\mathrm{y}\| = \lim_{n \to \infty} \|\mathrm{S_n}\| = \lim_{n \to \infty} \left\|\mathrm{I} + \sum_{k=1}^{n} \hat{\mathrm{T}}^{k}\right\| \leq \|\mathrm{I}\| + \sum_{k=1}^{\infty} \|\hat{\mathrm{T}}\|^{k} = \frac{\|\mathrm{I}\|}{\|\mathrm{I}\| - \|\hat{\mathrm{T}}\|}.$$

If $\hat{\Gamma}_B$ is fuzzy soft with identity (i.e.,$\|\mathrm{I}\| = 1$) then we get $\left\|(I - \hat{T})^{-1}\right\| \leq \frac{1}{1 - \|\hat{T}\|}$

Corollary (3.1.6): Consider $\hat{\Gamma}_{\mathrm{B}}$ as a fuzzy soft Banach space. Consider $\hat{x} \in \hat{\Gamma}$ and $\hat{\mu}$ be a fuzzy soft scalar in which $|\hat{\mu}| > \|\hat{T}\|$. Then $(\hat{\mu}I - T)^{-1}$ exists and $(\hat{\mu}I - T)^{-1} = \sum_{n=1}^{\infty} \hat{\mu}^{-n} T^{n-1} (x^0 = I)$.

Proof. $y \in \hat{\Gamma}$ be such that y^{-1}exists in $\hat{\Gamma}$ and $\hat{\mu}$ be afuzzy soft scalar such that

$\hat{\mu}(\lambda) \neq 0, \forall \lambda \in A$. Then it is clear that

$$(\hat{\mu}y)^{-1} = \hat{\mu}^{-1} y^{-1}.$$

Having noted this we can write

$$\hat{\mu}I - T = \hat{\mu}(I - \hat{\mu}^{-1}T)$$

and now we show that $(I - \hat{\mu}^{-1}T)^{-1}$ exists. We have $\|I - (I - \hat{\mu}^{-1}T)\| = \|\hat{\mu}^{-1}T\| = |\hat{\mu}|^{-1}\|T\| < 1$ by hypothesis. So, by Corollary 10 $(I - \hat{\mu}^{-1}T)^{-1}$ exists and hence $(\hat{\mu}I - T)^{-1}$ exists. For the infinite series representation, using the Proposition 9 we obtain

$$(\hat{\mu}I - T)^{-1} = \hat{\mu}^{-1}(I - \hat{\mu}^{-1}T)^{-1}$$

$$= \hat{\mu}^{-1}\left(I + \sum_{n=1}^{\infty} [I - (I - \hat{\mu}^{-1} T)]^n\right) = \hat{\mu}^{-1}\left(I + \sum_{n=1}^{\infty} (\hat{\mu}^{-1}T)^n\right) = \sum_{n=1}^{\infty} \hat{\mu}^{-n} T^{n-1}.$$

This proves the corollary.

Corollary (3.1.7): Consider a fuzzy soft Banach space denoted by $\hat{\Gamma}_B$, which has a fuzzy soft unit element I such that $\|\mathrm{I}\| = \check{1}$ and $\check{T} \in \mathcal{G}(\mathrm{x})$. Suppose $\lambda \in \widetilde{\mathbb{C}(f)} - \{0\}$ such that $\|\hat{T}\| \ddot{<} \check{\lambda}$. Then $(\lambda I - T)$ is invertible and $(\hat{\mu}I - T)^{-1} = \sum_{n=1}^{\infty} \check{\mu}^{-n} {T_n}^{n-1} : (\hat{T}^0 = \check{I})$ .Furthermore, we have

$$\left\|(\check{\lambda}I - 1)^{-1}\right\| \lesssim \frac{\bar{1}}{|\bar{\lambda}| - \|\hat{T}\|}$$

Proof. To demonstrate the last conclusion, we just need to replace $\frac{\hat{T}}{\bar{\lambda}}$ by $\check{x}$ in Lemma (5) and obtain the outcome for corollary (6).

Theorem (3.1.8): Consider a fuzzy soft Banach space denoted by $\hat{\Gamma}_B$, which possesses a fuzzy soft identity element I. It is required that the norm of I, denoted by $\|I\|$, be equal to 1. Additionally, let x

belong to the set A. Next, we can observe that the upper bound of $\hat{r}(\mathrm{x}) \leq r_C(x) \leq \frac{1+\varepsilon}{1-\varepsilon}\|\hat{T}\|$, whereas the lower bound is $\hat{r}(\mathrm{x})$.

Proof. Since $\hat{\Gamma}_S \subseteq \Gamma_C(x)$, so we have r(x)≤ $\hat{r}_C(x)$. Suppose that $\lambda \in \Gamma_C(x)$. If $|\lambda| \leq \|\hat{T}\|$, then we can easily prove that $|\lambda| \leq \frac{1+\varepsilon}{1-\varepsilon}\|\hat{T}\|$ Thus we have $\hat{r}_C(x) \leq \frac{1+\varepsilon}{1-\varepsilon}\|\hat{T}\|$.

Now suppose that $|\lambda| > \|\hat{T}\|$. Then $(\lambda I - \hat{T})$ is invertible and by corollary (3.5) we have $\|(\lambda I - \hat{T})^{-1}\| \leq \frac{1}{|\lambda| - \|\hat{T}\|}$. Consequently by some computations we get $|\lambda| \leq \frac{1+\varepsilon}{1-\varepsilon}\|\hat{T}\|$. Thus we conclude that $\hat{r}_C(x) \leq \frac{1+\varepsilon}{1-\varepsilon}\|\hat{T}\|$

Definition (3.1.9): Consider $\hat{\Gamma}_B$ as a fuzzy soft Banach space, as well as consider $T: \Gamma_G \to \mathbb{C}(A)$ to be a fuzzy soft linear functional. T is considered to be almost fuzzy soft multiplicative if there exists a positive value δ such that for any x and y belonging to G:

$$|T(xy) - T(x)T(y)| \leq \delta\|x\|\|y\|.$$

Proposition (3.1.10): Consider $\hat{\eta}$ as a fuzzy soft linear functional on a fuzzy soft Banach space $\hat{\Gamma}_B$, where $\hat{\Gamma}_B$ has an identity element I and $\hat{\eta}(I) = 1$. The subsequent requirements are of equal significance.

- i) $\hat{\eta}(\mathrm{x}) = 0$ implies $\hat{\eta}(x^2) = 0\ for\ all\ x \in G,$
- ii) $\hat{\eta}(x^2) = (\hat{\eta}(x))^2, x \in G,$
- iii) $\hat{\eta}(\mathrm{x}) = 0$ implies $\hat{\eta}(xy) = 0\ for\ all\ x, y \in G$
- iv) $\hat{\eta}(\mathrm{xy}) = \hat{\eta}(\mathrm{x})\hat{\eta}(\mathrm{y})\ for\ all\ x, y \in G$

proof: (i)→(ii)

$\hat{\eta}(I) = 1$ implies

$\hat{\eta}(x - \hat{\eta}(x))(\lambda) = \hat{\eta}(Ix - \hat{\eta}(x))(\lambda\hat{\eta}(I)(\lambda)\hat{\eta}(x)(\lambda) - \hat{\eta}(x)(\lambda) = 0\,; \forall\lambda \in A.$

So we have 0=$\hat{\eta}(x - \hat{\eta}(x)) = 0.$ By (i) we have

$\hat{\eta}\left(\left(x - \hat{\eta}(x)\right)^2\right) = \hat{\eta}(x^2 - 2x\hat{\eta}(x) + \left(\hat{\eta}(x)\right)^2 = \hat{\eta}(x^2) - \left(\hat{\eta}(x)\right)^2 = 0\,.$

Thus we deduce that $\hat{\eta}(x)^2 = \left(\hat{\eta}(x)\right)^2$.

(ii)⟹(iii)

By substituting the expression $u + v$ with the variable (x) in (ii) we obtain the result $\hat{\eta}(uv + vu) = 2\hat{\eta}(u)\hat{\eta}(v); u, v \in G, (1).$

Let x, y be in G with $\hat{\eta}(x) = 0$. According to (1) we have $\hat{\eta}(xy + yx) = 0. (2)$

Hence by (ii) we obtain $\hat{\eta}(xy + yx)^2) = 0$. Since $(xy + yx)^2$=2$\hat{\eta}(x(yxy) + (yxy)x) = 4\hat{\eta}(x)\hat{\eta}(yxy) = 0.$

By combining equations (2) and (3), we can deduce that $\hat{\eta}(xy) = 0$.

(iii)⟹(iv)

Consider $x, y \in G. we\ have\ \hat{\eta}\big(x - \hat{\eta}(x)\big) = 0$. Hence for each $\lambda \in A$ we have

$\hat{\eta}\big(x - \hat{\eta}(x)\big)(\lambda) = 0$. Thus by (iii) we have $\hat{\eta}\left(\big(x - \hat{\eta}(x)\big)y\right)(\lambda) = 0$.

Then we get

$\hat{\eta}\left(\big(x - \hat{\eta}(x)\big)y\right)(\lambda) = \hat{\eta}(xy - \hat{\eta}(x)y)(\lambda) = \hat{\eta}(xy)(\lambda) - \hat{\eta}(x)(\lambda)\hat{\eta}(y)(\lambda) = 0\ ;\ \lambda\forall \in A$.

Thus we have $\hat{\eta}(xy) - \hat{\eta}(x)\hat{\eta}(y) = 0$. Consequently we get $\hat{\eta}(xy) = \hat{\eta}(x)\hat{\eta}(y)$.

(iv)⟹(i)

From (iv) we have$\hat{\eta}(xy)(\lambda) = \hat{\eta}(x)(\lambda)\hat{\eta}(y)(\lambda)$. If $\hat{\eta}(x) = 0, then\ \hat{\eta}(x)(\lambda) = 0; \forall \in A$ so we have $\hat{\eta}(xy)(\lambda) = \hat{\eta}(x)(\lambda)(y)(\lambda) = 0.\hat{\eta}(y)(\lambda) = 0$

Therefore $\hat{\eta}(xy) = 0$.

If $\hat{\eta}$ is a fuzzy soft multiplicative linear functional, it is also a fuzzy soft Jordan multiplicative functional. Currently, we possess the subsequent corollary.

Corollary(3.1.11): Consider $\hat{\eta}$ as a fuzzy soft Jordan multiplicative linear functional on the fuzzy soft Banach space $\hat{\Gamma}_B$, which has an identity element I such that $\phi(I) = 1$. $\hat{\eta}$ is a fuzzy soft multiplicative.

Lemma (3.1.12): Assume that $\hat{\eta}$ is a fuzzy soft multiplicative linear functional on the soft fuzzy Banach space $\hat{\Gamma}_B$. Next, we consider the estimated value of $\hat{\eta}$, denoted as $\hat{\eta}(x)$, which belongs to the set $\hat{\Gamma}_B$. Here, x is an element of the set G.

Proof: For x ∈ G we set z= $\hat{\eta}(x)I - T$ Then we have

$$\hat{\eta}(z)(\lambda) = \hat{\eta}(x)(\lambda)\hat{\eta}(I)(\lambda) - \hat{\eta}(x)(\lambda) = \hat{\eta}(x)(\lambda) - \hat{\eta}(x)(\lambda) = 0.$$

Hence we have $\hat{\eta}(z) = 0$. Therefore z ∈ $ker\hat{\eta}$. So we have z ∈ $sing(G)$. Consequently $\hat{\eta}(x) \in \hat{\Gamma}_B$.

Remark (3.1.13): Let's assume that $\hat{\eta}$ is a fuzzy soft multiplicative linear functional. Consider z to be an element of $\hat{\Gamma}_B$, where T is an element of G. Therefore, if ze-xis is not invertible, we may $\hat{\eta}(zI - T) = 0$.

It is understood that when an invertible element y is present, the value of $\hat{\eta}(y) = 0$.

Therefore z$\hat{\eta}(I) - \hat{\eta}(x) = 0$. So we have z=$\hat{\eta}(x)$.

Thus, we present the following theorem.

Theorem (3.1.14): Consider $\hat{\Gamma}_B$ as a commutative fuzzy soft Banach space. Let x be an element of G, then

$$\hat{\Gamma}_B = \{\hat{\eta}(x): \hat{\eta} \text{ is a fuzzy soft multiplicative linear functional}\}.$$

Proof. The conclusion may be derived from lemma (3.12) and the last sentence.

Lemma(3.1.15): Consider T as a fuzzy soft bounded linear functional mapping from $\Gamma_E(A)$ to $\mathbb{C}(f)$. Then T is nearly indistinctly malleable.

Proof: For any $\hat{x}, \hat{y} \in A$, we obtain
$|T(\hat{x}\check{y}) - T(\hat{x})T(\check{y})| \leq |T(\hat{x}\check{y})| + |T(\hat{x})T(\check{y})| \leq \|T\|\|\hat{x}\check{y}\| + \|T\|^2\|\hat{x}\|\|\check{y}\| = (\|T\| + \|T\|^2)\|\hat{x}\|\|\check{y}\|$
Thus T is almost soft multiplicative where $\bar{\bar{\delta}} = (\|T\| + \|T\|^2)$.

Proposition (3.1.16): Let $\hat{\Gamma}_B$ is a fuzzy soft Banach space and $T_1: \Gamma_E(A) \longrightarrow \mathbb{C}(f)$ is a fuzzy soft multiplicative linear functional and $T_2: \Gamma_E(A) \longrightarrow \mathbb{C}(f)$ is a soft bounded linear functional.
The sum of T_1 and T_2 is a nearly fuzzy soft multiplicative functional, although it is not strictly multiplicative.

Proof: For each $\hat{x}, \check{y} \in A$ we have
$|(T_1 + T_2)(\hat{x}\check{y}) - (T_1 + T_2)(\hat{x})(T_1 + T_2)(\check{y})| =$

$|T_1(\hat{x}\check{y}) + T_2(\hat{x}\check{y}) - (T_1(\hat{x}) + T_2(\check{y}))(T_1(\check{y}) + T_2(\check{y})| =$

$|T_1(\hat{x}\check{y}) + T_2(\hat{x}\check{y}) - T_1(\hat{x})T_1(\check{y}) - T_2(\hat{x})T_2(\check{y}) - T_1(\hat{x})T_2(\check{y}) - T_2(\hat{x})T_1(\check{y})| \leq$

$|T_1(\hat{x}\check{y}) - T_1(\hat{x})T_1(\check{y})| + |T_2(\hat{x}\check{y}) - T_2(\hat{x})T_2(\check{y})| + |T_1(\hat{x})T_2(\check{y})| + |T_2(\hat{x})T_1(\check{y})|$

So by lemma(14) we get

$$|(T_1 + T_2)(\hat{x}\check{y}) - (T_1 + T_2)(\hat{x})(T_1 + T_2)(\check{y})| \leq$$

$(\|T_2\| + \|T_2\|^2)\|\hat{x}\|\|\check{y}\| + \acute{2}\|T_1\|\|\hat{x}\|\|T_2\|\|\check{y}\| =$

$(\|T_2\| + \|T_2\|^2)\|\hat{x}\|\|\check{y}\| + \acute{2}\|T_1\|\|T_2\|\|\hat{x}\|\|\check{y}\|.$

Thus $(T_1 + T_2)$ is almost fuzzy soft multiplicative. Clearly $(T_1 + T_2)$ is not multiplicative.

Definition(3.1.17): Consider a fuzzy soft Banach space denoted by $\hat{\Gamma}_B$. A fuzzy soft linear functional $\hat{\eta}: \Gamma_E(A) \longrightarrow \mathbb{C}(f)$ is considered an essentially fuzzy soft Jordan multiplicative functional if there exists a positive value $\bar{\bar{\delta}} \gtrsim \bar{\bar{0}}$ such that:

$\left|\hat{\eta}(\hat{X}^2) - \hat{\eta}(\hat{X})^2\right| \leq \bar{\bar{\delta}}\left\|\hat{X}\right\|^2, \forall \check{x} \in A.$

Corollary(3.1.18): Consider $\hat{\Gamma}_B$ as a fuzzy soft Banach space. Let $T_1: \Gamma_E(A) \longrightarrow \mathbb{C}(f)$ be a fuzzy soft Jordan multiplicative linear functional, and let $T_2: \Gamma_E(A) \longrightarrow \mathbb{C}(f)$ be a fuzzy soft bounded linear functional. The sum of T_1 and T_2 may be described as a nearly fuzzy, soft, Jordan multiplicative linear functional.

Proof: The proof may be demonstrated using the same approach outlined in theorem (13).

Definition(3.1.19): Consider a fuzzy soft Banach space with identity element I, denoted as $\widehat{\Gamma}_B$. Let $\bar{\bar{\varepsilon}} \acute{>} \bar{0}$ and $T: X \to X$ be a linear operator. The fuzzy soft ε condition spectrum of an element is denoted by $\Gamma_{\bar{\varepsilon}}(\breve{X})$ and is defined as follows:

$$\Gamma_{\bar{\varepsilon}} = \left\{\breve{\lambda} \,\breve{\in}\, \mathbb{C}(F): \left\|\breve{\lambda}\, I - T)^{-1}\right\| \,\bar{\bar{\geq}}\, \frac{\breve{1}}{\varepsilon}\right\}.$$

Theorem(3.1.20): Let $\hat{\Gamma}_B$ be a fuzzy soft Banach space with identity element $\breve{I}$ and let $\bar{\bar{\varepsilon}} \acute{>} \bar{0}$. Let $\hat{\eta}$:Γ_E(A) $\longrightarrow \mathbb{C}$(f) be a fuzzy soft linear functional such that $\hat{\eta}(\breve{I})$=$\breve{1}$ and $\hat{\eta}(e) \,\bar{\bar{\in}}\, \Gamma_{\breve{\varepsilon}}$ for$\hat{X} \in \breve{A}$ .Then is fuzzy soft multiplicative functional.

Proof: We prove that for all $\hat{X} \in \breve{A}$ we have T($\breve{x}$)$\in \Gamma_x$.We put $\bar{\lambda} = \hat{\eta}(\breve{x}\,)$. If $\bar{\bar{\lambda}} \in \Gamma_x$ then $\hat{\eta}$ is multiplicative. If $\bar{\bar{\lambda}} \notin \Gamma_x$ then $\bar{\bar{\lambda}}I - T$ is invertible and so$\bar{\bar{\lambda}}I - T \,\bar{\in}\, \mathcal{G}$(x).Suppose that $\acute{z} \,\bar{>}\, \breve{\varepsilon}\left\|(\bar{\bar{\lambda}}I - T)^{-1}\right\|$ Then we have $\left\|(\bar{\bar{\lambda}}I - T)^{-1}\right\| \,\bar{<}\, \frac{\breve{1}}{\varepsilon}$.Thus we get $\left\|(\bar{\bar{\lambda}}I\acute{z} - T\acute{z})^{-1}\right\| \,\bar{<}\, \frac{\breve{1}}{\varepsilon}$ Consequently we have $\bar{\bar{\lambda}}\,\acute{z} = \hat{\eta}(\breve{x})\acute{z} = \hat{\eta}(\breve{x}\acute{z}) \notin \Gamma_{\breve{\varepsilon}}(\acute{z}\breve{x})$ which is a contradiction. So $\hat{\eta}$ is fuzzy soft multiplicative.

Lemma(3.1.21). Let $\overline{\overline{\delta}} \,\bar{>}\, \bar{\bar{0}}$ and $\breve{X} \in \breve{A}$ Then$\Gamma_x \subseteq \Gamma_{\widetilde{\delta x}}$.

Proof: It may be readily demonstrated by the use of a precise definition.

Theorem(3.1.22): Let $\hat{\Gamma}_B$ be a fuzzy soft Banach space with identity element I and $\hat{\eta}$ be an almost fuzzy soft multiplicative linear functional on A If $\hat{\eta}\,(I)$=$\breve{1}$ Then for every element $\breve{x} \,\bar{\bar{\in}}\, A$ we have $\hat{\eta}\,(\breve{x}) \,\bar{\bar{\in}}\, \Gamma_{\delta x}$.

Proof: Let $\hat{x} \,\bar{\bar{\in}}\, A$ and $\bar{\bar{\lambda}} = \hat{\eta}\,(\hat{x})$ .If $\bar{\bar{\lambda}}I - T$ is not invertible then $\bar{\bar{\lambda}} \in \Gamma_x \subseteq \Gamma_{\widetilde{\delta x}}$ So $\bar{\bar{\lambda}} \in T_\varepsilon(\hat{x})$. Now assume that is invertible. Then

$$\bar{\bar{1}} = |\hat{\eta}(\mathrm{I})| = \left|\hat{\eta}(\hat{x}) - \breve{0}\right| = \left|\hat{\eta}(\mathrm{I}) - \hat{\eta}\big(\bar{\bar{\lambda}}\mathrm{I} - T\big)\mathrm{T}((\bar{\bar{\lambda}}\mathrm{I} - T))^{-1}\right|$$

$$\breve{\leq}\; \bar{\bar{\delta}}\left\|(\bar{\bar{\lambda}}\mathrm{I} - T)((\bar{\bar{\lambda}}\mathrm{I} - T))^{-1}\right\|.$$

Thus we have

$$\left\|(\bar{\bar{\lambda}}\mathrm{I} - T)((\bar{\bar{\lambda}}\mathrm{I} - T))^{-1}\right\| \,\bar{\bar{\geq}}\, \frac{\breve{1}}{\varepsilon}.$$

So we conclude that $\bar{\bar{\lambda}} \in \Gamma_{\varepsilon x}$. Consequently we have $\hat{\eta}\,(\hat{x}) \,\bar{\bar{\in}}\, \Gamma_{\delta x}$.

References

[1] S. Das and S. K. Samanta, Soft linear functionals in soft normed linear.

[2] Kamel, A. and Mirzavaziri, M., Closability of farthest point in fuzzy normed spaces, Bulletin of Mathematical Analysis and Applications, Vol.2,140-145,2010.

[3] Maji, P.K. , Biswas, R. and Roy, A.R., Fuzzy Soft Set ,Journal of Fuzzy Mathematics 9,589-602 , (3) (2001)-8843.

[4] P. K. Maji,R. Biswas, A.R. Roy, Fuzzy soft, Journal of Fuzzy Mathmatics 9(3)(2001) 589602.

[5] D. Molodtsov, Soft set theory first results, Comput. Math. Appl. 37(1999) 19-31.

[6] S. H. J. Petroudi, S. A. Sadati, A. Yaghobi, Almost Soft multiplicativelinear functional, The first national conference on Applied mathematicsin engineering and basic science. (2017), Shiraz, Iran.