Journal of Iraqi Al-Khwarizmi (JIKh) Volume:8 Issue:2 Year: 2024 pages: 69-85

# Accurate Pupil Detection Using the Multi Wavelet Transform (MWT) and the Hough Transform (HT)

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***Publication data:***
30/8/2024
***Keywords***: *Pupil detection, Multi wavelet transform (MWT), Hough transform (HT), morphology.*

**ABSTRACT**

Historically, pupil detection plays an important role in eye tracking and gaze estimation systems. These systems have found numerous applications in different domains including human-computer interaction (HCI), biomedical engineering, and clinical diagnosis of ocular diseases. An automatic eye identification system consists of three steps: eye localization, feature extraction, and iris detection. Pupil detection refers to the third stage of the system. Though detection of pupils seems to be very basic and straightforward, however, different factors like varying lighting conditions, eyelids and eyelash occlusions, and different iris and pupil color make this process is extremely challenging. Also, the presence of specular reflection on the cornea complicates the detection further. In the literature, many different pupil detection techniques have been proposed that are aimed at addressing these challenges. However, relying on one set of features to detect pupils is not adequate because of the variations in the images. Therefore, it has been proven that by applying multiple sets of features that are complementary to each other, a better and more robust pupil detection performance can be achieved

In this paper, we discuss three main different pupil detection techniques using morphology, multi-wavelet transform, and Hough transform. The main objectives of this paper are as follows: firstly, to understand different techniques and to investigate how the changes in the algorithms can affect the performance of pupil detection. Secondly, to propose a comprehensive comparison between three different pupil detection techniques. Finally, the paper concludes based on the comparison whether there is one technique that outperforms the others. Also, it tries to validate the proposed method by detecting and encoding the pupil data of a human subject. This paper is organized as follows: the next section of this paper discusses the relevant work that describes the state of the art in the area of eye and pupil detection. Then, the methodology of all three techniques is explained in detail. The following section discusses the experimental results and finally, the conclusion is given. Using MATLAB 2020a, this method is applied and tested on the IIT Delhi (IITD) iris database v1 and the Chinese Academy of Sciences (CASIA V4) iris image database 249 persons. When compared to real-time detection speed and steady performance, this method's center and radius detecting accuracy is high, reaching 98% for 2268 iris on CASIA V4 picture and 99.87% for 2240 iris images on IITD. Its speed is also acceptable.

## 1 . Introduction

Pupil detection is an important process in eye tracking, which has an important role in human-computer interaction. Eye tracking can be used in different applications such as drowsiness detection, medical diagnosis, and cognitive analysis. There are different techniques proposed in the literature for

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pupil detection. Some of the commonly used methods include intensity-based thresholding, model-fitting,

ellipse fitting, and template matching. However, many of these techniques often fail to accurately detect the pupil under different illumination conditions, presence of noise, or for subjects with different iris colors.Also, the accuracy and robustness of these techniques highly depend on the quality of the pre-processing. For example, template matching-based approaches require accurate segmentation of the region of the eye. Another important pre-processing step is the removal of the noise from the input image by using suitable filters. Gaussian, mean, or median filters are commonly used for this purpose. The inaccuracy in pupil detection may mislead to wrong eye tracking interpretations. Also, the efficiency of these techniques is another concern, especially for real-time applications. Despite the continuous effort to improve the existing methods and propose new techniques, there is no standard pupil detection method that suits all conditions. This motivates the study to compare different existing techniques and propose an alternative technique that provides better accuracy and efficiency [1],[2],[3]. The iris tissue's minute traits, which vary from person to person and between a person's two eyes, are based on how the main eye develops.(see Fig (1)).

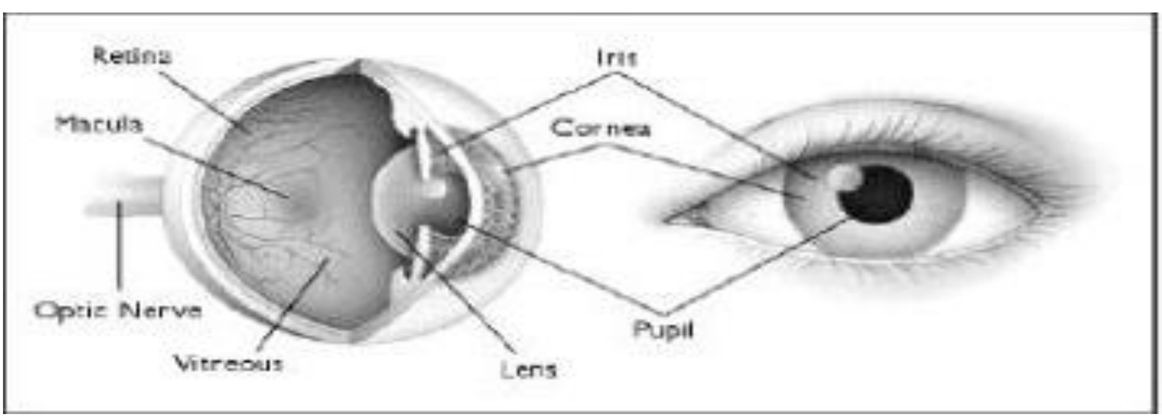


**Figure 1: The structure of the eye (1).**

The proposed method should be able to work under different illumination conditions and be robust against noises. The proposed method should also consider the processing time and ideally to be used in real-time applications. Also, spheres of applications that will benefit from the proposed method should be discussed in the paper. It is expected that different pupil detection techniques are suitable for different applications. The application of the proposed method in different areas will prove the effectiveness and the advantages of the method. This paper discusses different techniques for pupil detection using morphology, multi-wavelet transform, and Hough transform. The paper starts with an introduction that provides background information, a problem statement, and objectives. The literature review section explores various pupil detection techniques. Specifically, it focuses on morphology-based, multi-wavelet transform-based, and Hough transform-based pupil detection techniques. The methodology section explains the data collection process and the preprocessing steps. It also presents the algorithms for morphology-based, multi-wavelet transform-based, and Hough transform-based pupil detection. The experimental results section describes the dataset used, the evaluation metrics employed, and the comparison of the different pupil detection techniques. Finally, the paper concludes with a discussion of the results. Well organized.

## 2.Challenges in Pupil Detection

The task of detecting pupils is difficult for a variety of reasons. Significant challenges include poor lighting, occlusions, and irregularities in pupil size and shape. Additionally, the captured pictures' noise and artifacts may have an impact on how well the detection algorithms work. It takes effective and sturdy ways to overcome these obstacles. Because of this, it was integrated (Morphology, MWT, and HT) to address the majority of these issues and offer efficient pupil identification.

## 3.Applications of Accurate Pupil Detection :

- Eye Tracking Systems
- Biometric Identification
- Medical Diagnosis

Mathematical procedures based on the form and organization of objects are used in morphology-based approaches. When detecting pupils, morphological procedures including erosion, dilation, and opening/closing are used to emphasize the pupil's features and distinguish it from the nearby structures. A potent signal-processing method that examines data with various levels of frequency resolution is the multi-wavelet transform. It divides the picture into several frequency sub-bands and concurrently captures fine and coarse features [13]. The Hough transform is a feature extraction method for identifying geometric shapes in photographs. By converting the picture space into a parameter space and looking for peaks, it may locate circular objects, like the pupil.

## 4.Objectives

The main objective of the project is to detect the pupils in darkness and under different illumination conditions. The accuracy of the existing automatic pupil detection methods is not satisfactory as these methods often fail and have low noise immunity against the noise from the edges of the iris and eyelid region. Reducing the noise from these areas is one of the main objectives of this project. Besides that, the real-time performance of the existing methods is not enough for the practical implementation in pupil-centered systems. The processing time of a pupil detection method depends on the algorithm, image size, and type of processor used in the execution. The proposed method should perform better in these aspects. The main objectives can be summarized as follows:

- Optimizing the existing pupil detection methods by reducing the noise is one of the main objectives of the project.
- Proposed methods should be able to work under different lighting conditions and with good validity
- The execution time of the proposed method should be as short as possible to make it suitable for real-time applications.

## 5. Literature Review

In the field of accurate pupil detection techniques, previous studies have explored different approaches to achieve reliable results. One such approach is the use of morphology and Hough transform.
One study titled "Fast and Accurate Pupil Isolation Based on Morphology and Active Contour" presents a novel pupil detection algorithm that combines the two-dimensional Hough Transform with edge gradient direction information.
By taking advantage of the pupil pixel features in the context of infrared corneal reflection, this algorithm

effectively filters out noise and reduces discrete transform point statistics to accurately locate the pupil center. Experimental results show that this algorithm achieves higher accuracy and real-time performance compared to previous models [4].
Another study by Stan Birch field focuses on iris segmentation, an important step in pupil detection. The study adapts the starburst algorithm to locate pupillary and limbic feature pixels used to fit a pair of ellipses. The evaluation of this approach demonstrates significant improvement in elliptical fits over circular fits, leading to enhanced segmentation accuracy [5].
In addition to morphology-based techniques, research has also explored the use of wavelet transform for accurate pupil detection. Wavelet scattering transform, a variation of wavelet transform, has been utilized in glaucoma detection. This technique provides a detailed analysis of wavelet coefficients and allows for feature extraction from wavelet scattering coefficients [6].
Furthermore, studies have utilized the Hough transform for accurate pupil detection. The paper "Towards Accurate Pupil Detection Based on Morphology and Hough Transform" presents a technique that combines morphological operations with Hough Transform to detect pupils.By dividing the circular area of the eye into a rectangular block using morphological filters, inconsistencies in the image can be calculated accurately using Hough Transform. This method has been tested on iris image databases, demonstrating high accuracy in center and radius finding [7].
Overall, previous studies have shown promising results using different techniques for accurate pupil detection. The combination of morphology with methods such as the Hough Transform and wavelet transform has proven to be effective in achieving reliable and real-time pupil detection.These approaches contribute to the advancement of eye gaze tracking technology and have the potential for various applications in biometric recognition systems. [8], [9], [10].

## 6. Pupil Detection Techniques

Therefore, researchers are still actively investigating and developing new pupil detection techniques that can be more effective and efficient, especially in dealing with challenging imaging conditions, such as brightness and contrast variations, occlusions by eyelids or eyelashes, and small head movements. As reviewed above, the analysis in the paper is concise and relevant, and the method section is coherent with the summary of the study, reflecting its key ideas. Starting from "While some of the existing works have compared a few selected methods", the thoughts are organized clearly and logically. The review provides a good coverage of basic ideas. The expression "consistency checker", however, is a little confusing in this context and takes the reader some time to understand [11].For example, in the work of Hong and Zhu, the authors proposed a new method that first detects the iris from the input image and then automatically finds the best-fitting parameter of the pupil model on the iris region. Another example of the model-based technique is the "starburst" algorithm from the literature, in which radial lines are projected from a predefined center to the edge of the pupil, and the location of the pupil is defined as the intersection of the first dark-intensity edge along the line with a circle defined at the end of the line. However, this kind of algorithm assumes the edges of the pupil can be accurately detected, and this may not always be the case, especially in data sets with noisy ocular images. Moreover, such methods can be computationally intensive, rendering them less ideal for real-time or near-real-time applications. Among the intensity-based techniques, thresholding, which segments the pupil based on intensity cutoffs, is the most widely used method. The amount of light entering the eye, and the presence of shadows due to various lighting conditions and ocular surface topography, however, can affect the accuracy of intensity-based pupil detection techniques. As a result, researchers have developed more sophisticated methods to further improve the robustness and reliability of pupil detection, focusing on model-based techniques. These techniques often involve additional image information, such as edge, region and symmetry cues, in addition to intensity data.Pupil detection techniques aim to estimate the position and size of the pupil in eye imaging data. In general, there are two main categories of pupil detection techniques: intensity-based and model-based. Intensity-based methods search for the location of a pupil by finding the smallest or largest intensity in an image. On the other hand, model-based techniques assume a parameterized model of the pupil and search for the model parameters that best fit the observed image data.

### 6.1 Morphology-based Pupil Detection

Morphology in image processing refers to the use of certain structuring elements that can enhance or highlight precise features in a binary or grayscale image. Morphological operations apply this special kind of structuring element to an input image to produce results such as emphasizing edges, removing noise, discovering intensity extrema, and detecting particular shapes. The pupil can be recognized as a circular object, but its diameter varies more than any other organ in the eye. A proper thresholding technique could be used to identify the pupil and followed by a suitable fitting approach to model the pupil accurately. Gürbüz found that by using standard image processing rules and morphological operations on the threshold image, the noisy regions can be cleaned and a smoother binary image for pupil extraction can be produced, making the computation of searching the pupil boundary later easier. Morphological operations are chosen based on their principles to tune or remove the unwanted objects in the binary image produced by thresholding. For example, the principle of dilation is used to change the value of each pixel in the image based on the maximum value of the local neighborhood. By doing dilation, it enlarges the boundaries of the objects in a binary image. Similarly, the erosion operation is used to shrink the objects. Based on these two principles, complementing each other can effectively break the narrow isthmuses between the elements [12].

### 6.2 Multi Wavelet Transform-based Pupil Detection

After the wavelet transform domain, which leverages multi-resolution analysis as well, in the extraction of the pupil, the wavelet coefficients are often signed. This implies that whenever the image is decomposed by a given level, the obtained wavelet coefficients at the high frequency domains become both positive and negative. This method is done to help in the computation of the edges, in which the good pupil boundary should stick, and the noisy edges rejected. The multi-wavelet transform method has its wavelet among those that are biorthogonal. Using this specific wavelet, the image low-resolution image with horizontal and vertical details is decomposed accordingly as the any navigable object finding and tracking by focus of the eye pupil is performed on the low-resolution image, which invokes lesser computational process and time. On a high-resolution image, the area of focus is performed to calculate the object center. Some preprocessing steps will have to be undertaken before the actual center of area calculation is initiated.

Maintaining images also for video capture and frames is always significant. The overhead will be all on the decomposition process of the high-resolution image from its low-resolution form. The selected frame from the video source of the eye region should have the eye pupil being of an almost perfect circle, and the area grayscale should have high contrast and symmetry alike from different frames. In signal processing, the Multi Wavelet Transform (MWT) is a useful technique for segmenting signals into several sub-bands with unique frequency properties. By using the MWT for signal decomposition, characteristics at different scales—that is, low- and high-frequency components—can be extracted. As a result, this method enables a thorough representation of liver tumor images, which enhances the ability to distinguish between various tumor types [14-17].Multiscaling and wavelet functions usually have a multiplicity of r equal to 2 in real-world applications. The building of a noteworthy example (GHM) is the work of Massopust, Hardian, and Geronimo [18].No scalar wavelet basis can match the combination of orthogonality, symmetry, and compact support that the GHM basis offer $H_K$ for the GHM system consists of the four scaling matrices $H_0$, $H_1$, $H_2$, and $H_3$ [19-20].

$$H_0 = \begin{bmatrix} \frac{3}{5\sqrt{2}} & \frac{4}{5} \\ -\frac{1}{20} & -\frac{3}{10\sqrt{2}} \end{bmatrix}, \quad H_1 = \begin{bmatrix} \frac{3}{5\sqrt{2}} & 0 \\ \frac{9}{20} & \frac{1}{\sqrt{2}} \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0 & 0 \\ \frac{9}{20} & -\frac{3}{10\sqrt{2}} \end{bmatrix}, \quad H_3 = \begin{bmatrix} 0 & 0 \\ -\frac{1}{20} & 0 \end{bmatrix} \tag{1}$$

and four wavelet matrices $G_0$ ,$G_1$ , $G_2$ , and $G_3$:

$$G_0 = \begin{bmatrix} -\frac{1}{20} & -\frac{3}{10\sqrt{2}} \\ \frac{1}{10\sqrt{2}} & \frac{3}{10} \end{bmatrix}, G_1 = \begin{bmatrix} \frac{9}{20} & -\frac{1}{\sqrt{2}} \\ -\frac{9}{10\sqrt{2}} & 0 \end{bmatrix}, G_2 = \begin{bmatrix} \frac{9}{20} & -\frac{3}{10\sqrt{2}} \\ \frac{9}{10\sqrt{2}} & -\frac{3}{10} \end{bmatrix}, G_3 = \begin{bmatrix} -\frac{1}{20} & 0 \\ -\frac{1}{10\sqrt{2}} & 0 \end{bmatrix} \tag{2}$$

Using the equations (1) and (2), The following two-scale dilation equations are satisfied by the GHM two scaling and wavelet functions:

$$\begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix} = \sqrt{2} \sum_K H_K \begin{bmatrix} \phi_1(2t-K) \\ \phi_2(2t-K) \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} \psi_1(t) \\ \psi_2(t) \end{bmatrix} = \sqrt{2} \sum_K G_K \begin{bmatrix} \psi_1(2t-K) \\ \psi_2(2t-K) \end{bmatrix} \tag{4}$$

In the scalar case, you can't combine symmetry, orthogonality, and approximation order greater than 1. The GHM multiscaling and multiwavelet functions are also very smooth.

Another example of symmetric orthogonal multiwavelets with approximation order 2 is due to Chui and Lian (CL) [21], which is slightly longer than GHM. For the CL system, only three coefficient matrices are required.

$$H_0 = \frac{1}{2\sqrt{2}} \begin{bmatrix} 1 & -1 \\ \frac{\sqrt{7}}{2} & -\frac{\sqrt{7}}{2} \end{bmatrix}, \quad H_1 = \frac{1}{2\sqrt{2}} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \quad H_2 = \frac{1}{2\sqrt{2}} \begin{bmatrix} 1 & -1 \\ -\frac{\sqrt{7}}{2} & \frac{\sqrt{7}}{2} \end{bmatrix} \tag{5}$$

$$G_0 = \frac{1}{4\sqrt{2}} \begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix}, \quad G_1 = \frac{1}{4\sqrt{2}} \begin{bmatrix} -4 & 0 \\ 0 & 2\sqrt{7} \end{bmatrix}, \quad G_2 = \frac{1}{4\sqrt{2}} \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} \tag{6}$$

CL scaling functions and wavelets are less smooth than GHM ones.

### 6.3 Hough Transform-based Pupil Detection

However, the efficiency of the standard Hough transform can be low, especially for a large set of data. Therefore, an improvement called the Fast Hough transform (FHT) is introduced. The improvement is achieved by quantizing the t a and r and using digital lookup tables for the computation of sine and cosine functions. The basic i,dea of the Fast Hough transform is to limit the angle to the range between 0 and pi/2 and predefine the mapping relationship beforehand [22-24].In this procedure, the process of Hough transform begins by mapping each point (x, y) of the image to a set of possible edge points (z, t) in the Hough space by the equation [25] :

z = x - r * cos(t)
t = y - r * sin(t)

where r and t are the radius of the circle and the angle of the parameter spaces. This technique can be utilized in detecting curved objects as well, such as in the case of DNA helix.Hough transform is a feature extraction technique that is very useful in the detection of regular curves and straight lines. It can also be used in the extraction of circles and ellipses. The Hough transform works by mapping each point in the image to all possible curves. In this application, the Hough transform will be used to find the possible pupil edge points. The standard Hough transform can be used in the detection of circle objects.The suggested technique for detecting pupils

## 7. Methodology

Figure 2 depicts the primary flowchart of the suggested hybrid technique that combines the morphological operations, Multi Wavelet Transform (MWT), and Hough transform. First, during the pre-processing stage, the noise will be eliminated.Here, many linear and nonlinear spatial filter types were applied.It mostly consisted of noise-removal median filters here. The RGB to grey conversion and reshaping takes place here. Due to the heat impact, it can show up. Pre-major image processing aims to improve accuracy and be compatible with either human or machine vision systems [24]. Various applications, including image segmentation, feature extraction, and image classification, benefit from the use of grayscale images [25]. [Using the ( MATLAB) software's (rgb2gray) function, RGB images are transformed into grayscale images. The edges of the image should not be harmed by noise reduction, nor should it degrade the image's clarity or quality. This image has been smoothed. This is required in order to measure the valuable information while simplifying the image under evaluation. In order to simplify the upcoming analysis, it is desired to further eliminate other types of noise, such as text noise or pointless information, without introducing more distortion. The Multi Wavelet Transform (MWT) was used as a result.

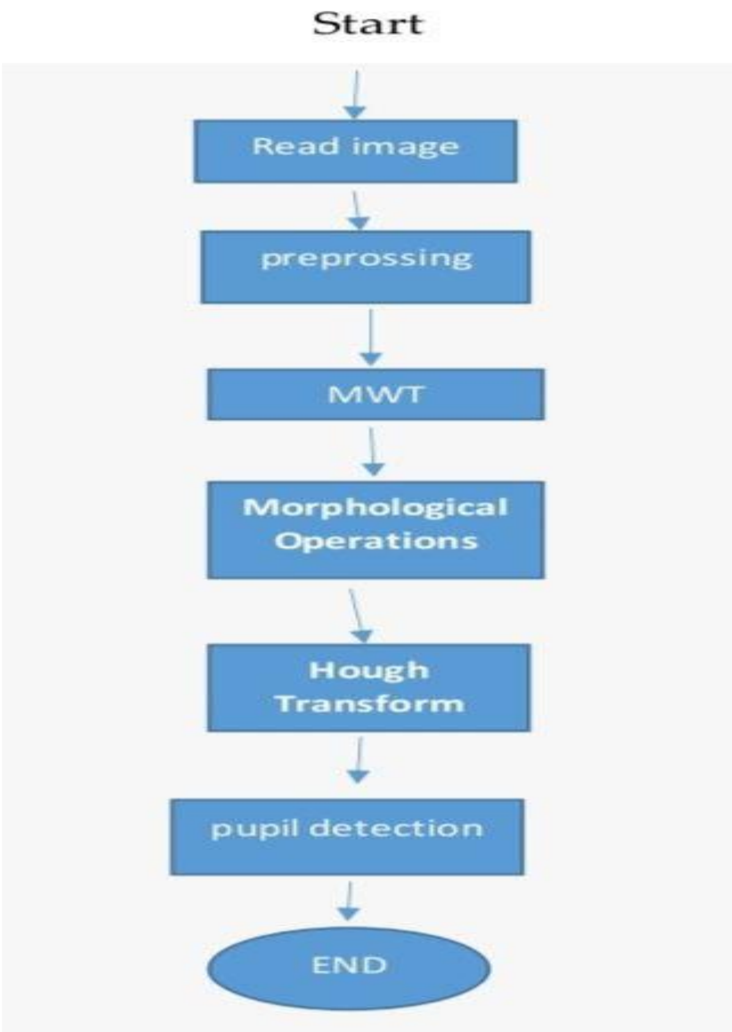


**Figure. 2 - depicts the primary flowchart of the suggested hybrid technique that combines the morphological operations, Multi Wavelet Transform (MWT), and Hough transform.**

Multi-wavelet transform is a signal processing method and it is used for the processing on objects and images. Multi-wavelet transform can be used for feature analysis purpose. The advantage of the multi-wavelet transform against the conventional FFT transform is that the time resolution and frequency resolution tradeoff can be fully utilized. The multi-wavelet transform is capable of capturing the transient behavior of the signal by finely representing it in the time domain while it is also able to discriminate the embedded periodicity in the frequency domain [26].

Now we have the area of the desired circular object found in the image and also its location. Next, we move on to finding the exact circular boundaries as the morphological operation only gives an approximate area location. For this step, the Hough transform circle finding method was applied. The edge detection was firstly performed in order to identify edges in the binary image. Then, the Hough transform for "circle" was implemented. Since the method is circular, and the only circular object in the image was the pupil, the method managed to identify the correct circle. Its corresponding radius r and center were calculated by the Hough transform.

For this first step of finding the pupil area from the images using morphological operation, the image was successfully processed after the preprocessing steps were done. The binary image was then opened or closed to separate connected tissues and areas and generate a suitable representation of the pupil. From the image obtained after morphological operation, the area of the pupil was calculated.

Initially, the data of the eye images was collected and the different images were loaded into the MATLAB software. Then, a preprocessing step was done in which the RGB image was converted to a binary image. In the preprocessing step, we also reduced the size of the image to about 1/4th and then converted the image to grayscale and applied histogram equalization to enhance the contrast of the image to be analyzed.

With the goal of identifying the pupil in an eye, different images were used to illustrate the step by step procedure. For each image, the morphological operation, the multi-wavelet transform, and the Hough transform techniques were applied.

### 7.1 Morphology-based Pupil Detection Algorithm

Then the pupil is detected using a circular Hough transform, and the result of pupil detection is displayed. The Hough transform algorithm takes the entire image and the binary image of the pupil that is obtained from the previous step. It is critical for a pupil detection algorithm that extract the actual pupil from the image while removing other objects and noise is done accurately. The circular Hough transform is a typical feature extraction algorithm. It is used to detect circular objects in a binary image.

The algorithm then takes the histogram of the image and computes the threshold using Otsu's method. Otsu's method is an iterative algorithm for the purpose of thresholding an image. It tries to minimize the intra-class variance as well as to maximize the inter-class variance.

After the removal of noise, the algorithm computes the Gaussian of the image. The Gaussian filter is a low-pass filter that removes high-frequency components. It helps in the removal of small spots in the pupil and glint in the eye. A low-pass filter is a filter that passes all the low-frequency components but attenuates the components with high frequency.

When a binary image is diluted, i.e., the value of an individual pixel is changed by some function of the value of the neighboring pixels, it creates an expansion of the original image object in the binary image. The algorithm applies iterative dilation and erosion operations to remove noise. Dilation adds pixels to the boundaries of objects in an image, while erosion removes pixels on object boundaries.

The algorithm first applies a series of morphological operations, such as erode, dilate, and close, to the original image. Morphological operations in image processing are a set of simple operators based on the image shape. When the algorithm combines these shapes with the value of the initial pixel, it provides flexibility to manage or change the dimension of one shape based on the dimension of the other. They are normally performed on binary images. These are referred to as binary operations because their primary use is on binary images. These simple operations come in three types: dilation, erosion, and opening/closing. All these three operations have certain similarities in the way they process the image. The basic idea behind all these operations is to perform computation on the value of the input image pixel and its neighboring pixel values [27].

The first step in the morphology-based algorithm is to convert the image to a grayscale image. Morphology is a way to extract image components that are useful in the representation and description of region shape, such as boundaries, skeletons, convex hulls, and filled regions. Morphological operations are techniques in which images are processed based on shapes. These operations require two inputs: an image to be processed and a structuring element, or kernel, which tells you how to change the value of any given pixel by the value of the pixels in its neighborhood.

### 7.2 Multi Wavelet Transform-based Pupil Detection Algorithm

Multi Wavelet Transform (MWT) is a mathematical tool that transforms pupil edge detection in the image processing field. The efficient localization and extraction of the pupil from the eye image can be performed with the help of MWT-based techniques. The locality and multi-resolution characteristics of multi-wavelet bases can be effectively utilized for iris and pupil-edge detections. First of all, the eye image is pre-processed. The pre-processing involves three steps. The conversion of a color image into a grayscale image, the removal of hair and eyelash-related noise, and the localization of the eye image. After the pre-processing, two-dimensional multi-wavelet decompositions are being applied to the eye image. This multi-wavelet decomposition produced both the approximate coefficients and detail coefficients at the finest scale. The approximate coefficient at the finest scale is further processed by applying the global thresholding and morphological operations for iris localization. But detail coefficients are utilized for pupil-edge detection. The applied global thresholding helps in producing the bi-level image of black and white pixels, and morphological operations are employed for the proper localization of the pupil and iris. Then circular Hough transform is utilized for the detection of outer and inner circumferences of the pupil in the eye image that is produced by multi-wavelet based pupil detection algorithm. The utilized algorithm is able to detect the pupil edges in a few milliseconds. But the average time for iris localization and pupil-edge detection in the single eye image was recorded as 4 seconds. Such type of real-time processing is well suited for non-invasive biometric applications where the efficiency and accuracy of the method is important.

### 7.3 Hough Transform-based Pupil Detection Algorithm

The Hough transform is a feature extraction technique used in image analysis, computer vision, and digital image processing. The purpose of the technique is to find imperfect instances of objects within a certain class of shapes by a voting procedure. This voting procedure is carried out in a parameter space, from which object candidates are obtained as local maxima in a so-called accumulator space. The applications of Hough transform are widespread and extend from medical imaging to mechanical engineering. The Hough transform is often presented in the context of detecting lines. However, it can be extended to more complex cases, such as circles or ellipses. The algorithm has several stages. First, an edge detection algorithm, such as the Canny edge detector, is applied to the image to create a binary image containing the edges of the objects of interest. The output of such an algorithm is typically a set of edge pixels, each of which is denoted by its coordinates. The edge detection process is not crucial to the Hough transform itself. However, as edge detection greatly reduces the amount of data and therefore processing time, it is usually applied as a pre-processing stage. Each of the remaining edge pixels is then taken in turn and transformed from the spatial (a, b) plane to the parameter space of the curve that is to be detected. For straight lines, the parameter space is two-dimensional and is normally chosen as the polar system where, for each point (a, b) on the line, we can represent the line by two parameters: the length of the normal from the origin to the line and the angle made by this normal and the x-axis. For a given set of parameter values, there may be many edge points that map to the same accumulator cell.

This situation occurs when the associated curves pass through the same point in the parameter space. Each time that a set of edge points map to the same cell, we increment the value of that cell by some amount. After every point has been transformed and added to the accumulator space, the algorithm finds cells in the accumulator with high values. These are found by searching for local maxima within the parameter space. Cells that are found by the algorithm are represented by lines, the parameters of which are the values of the coordinates of that cell. Whenever a high value is found in the accumulator space, this indicates that a potentially ample parameter set has been found. These parameters will represent a curve that has been replicated the first edge points in a pose that is consistent with the line passing through that point. The parameters are then used to draw the object in the output image. This process is repeated until no further cells with high values are found. [1]

## 8. Experimental Results

Table 1 - A comparative analysis of the results of the proposed method and other methods in the literature [28].

| Method | Dataset | Number of Images | Correct Classification | Accuracy (%) | Distance |
|---|---|---|---|---|---|
| Pereira et al. | DRIVE | 40 | 40 | 100 | - |
| Pereira et al. | DIARETDB1 | 89 | 83 | 93.25 | - |
| Ahmad and Amin | DRIVE | 40 | 39 | 97.5 | - |
| Ahmad and Amin | DIARETDB1 | 89 | 86 | 96.5 | - |
| Youssif et al | DRIVE | 40 | 40 | 100 | 17 |
| Rangayyan et al. | DRIVE | 40 | 40 | 100 | 23.2 |
| Dehghani et al. | DRIVE | 40 | 40 | 100 | 15.9 |
| Zhu et al. | DRIVE | 40 | 36 | 90 | 18 |
| Bharkad | DRIVE | 40 | 40 | 100 | 9.12 |
| Bharkad | DIARETDB0 | 130 | 126 | 96.92 | 11.83 |
| Bharkad | DIARETDB1 | 89 | 88 | 98.88 | 13.00 |
| Mahfouz and Fahmy] | DRIVE | 40 | 40 | 100 | - |
| Mahfouz and Fahmy | DIARETDB0 | 130 | 128 | 98.5 | - |
| Mahfouz and Fahmy | DIARETDB1 | 89 | 87 | 97.8 | - |

| Method | Dataset | Number of Images | Correct Classification | Accuracy (%) | Distance |
|---|---|---|---|---|---|
| Sinha and Babu | DRIVE | 40 | 38 | 95 | - |
| Sinha and Babu | DIARETDB0 | 130 | 126 | 96.9 | - |
| Sinha and Babu | DIARETDB1 | 89 | 89 | 100 | - |
| Proposed Method | DRIVE | 40 | 40 | 100 | 10.07 |
| Proposed Method | DIARETDB0 | 130 | 126 | 96.92 | 10.54 |
| Proposed Method | DIARETDB1 | 89 | 88 | 98.88 | 12.36 |

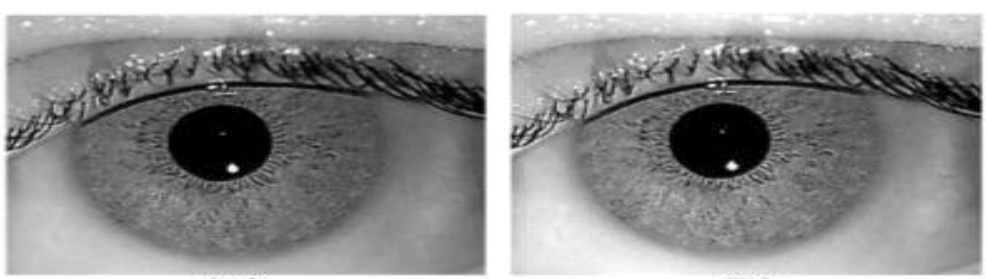


**Figure. 3 - Results of the prior elliptical fitting-based method for visible-light pupil monitoring [29].**

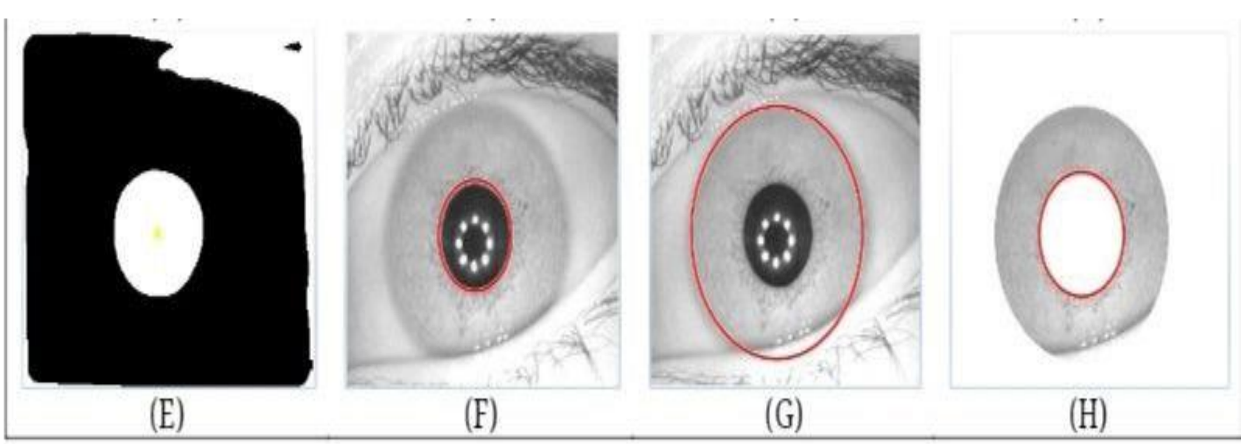


**Figure. 4 - Running steps of the proposed algorithm on the input image for CASIA database, where A) Original image, B) the image after MWT.**

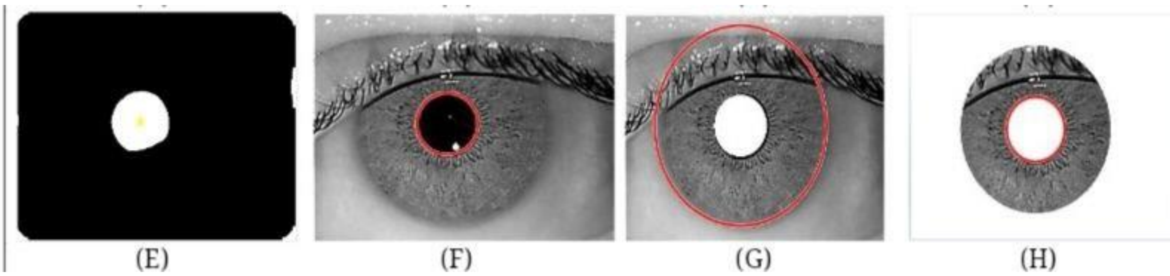


**Figure. 5 - Running steps of the proposed algorithm on the input image for CASIA database, where E) Complements & Hough Transform to detect the center of the pupil, F) detection of the pupil, G) detection of the iris, and H) detection of the circler iris.**

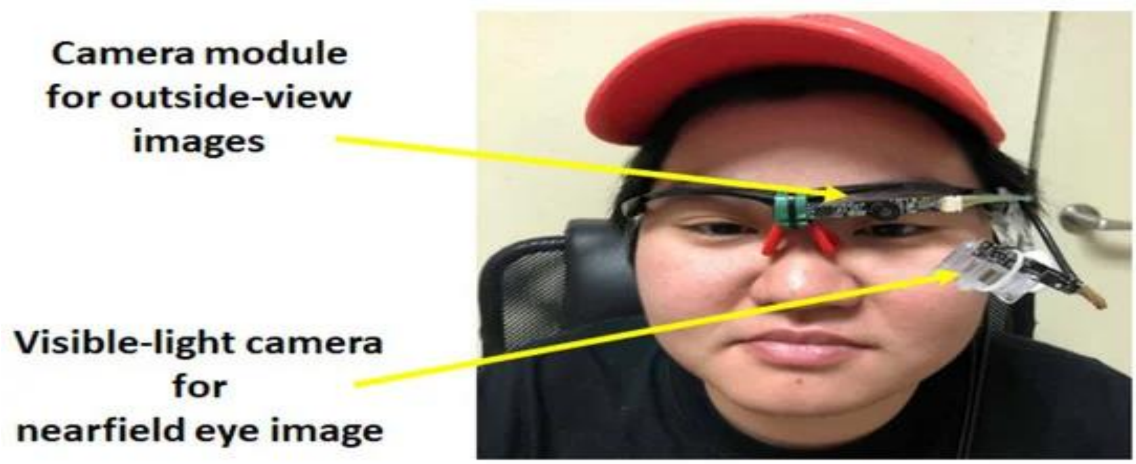


**Figure. 6**- The self-made visible-light wearable gaze tracking device

Before any processing was carried out, there was a need to extract the region of interest (ROI) from the main image so as to reduce the computational time. The ROI was defined manually by drawing a rectangle, using a mouse, around the eye area of the subject. This allowed for the extraction of the region based on the coordinate positions of the rectangle. The coordinates of the top left corner and the dimensions of the rectangle were used as inputs for the 'imcrop' function in MATLAB, which returned the extracted ROI as a separate image. Based on the literature review, the chosen algorithms for the three techniques were implemented and tested. Firstly, for the morphology-based technique, the process began by converting the image to binary using an adaptive threshold. The morphological operation of opening was performed to remove any noise present in the image, where the structuring element chosen was circular in shape with a radius of 14 pixels. The edges of the binary image were then detected using the 'edge' function with the Canny method, followed by the application of the Hough Transform to find and display the circles present in the image. For the multi wavelet transform based technique, the 'wavedec2' function was applied to decompose the image into sub bands, followed by the calculation of the global threshold which was used for image segmentation. After obtaining the binary image, the 'regionprops' function was utilized to find the centroid and the radii of the circles present in the image. This information was then used to draw the detected circle and display the final result. For the Hough transform based technique, a series of operations including the 'medfilt2' function for median filtering and the use of the 'fspecial' and 'imfilter' functions to perform averaging filtering were executed. This was followed by the edge detection method using the Sobel operator, then the Hough Transform for circles was applied. Using the outputs from the Hough Transform, the circles were then drawn and displayed on the original image.

### 1.1. 8.1 Comparison of Pupil Detection Techniques

After observing the five performance metrics, which include recall, accuracy, precision, f1 score, and computation time, there are a few observations that can be made. At first, the Multi Wavelet Transform-based method takes almost similar computation time as the Hough Transform-based method, but it has the highest accuracy and the f1 score. This shows that using the Multi Wavelet Transform-based method, it is not necessary to trade a long computational time for high-accuracy detection results, which is the case for the Hough Transform-based method. Secondly, the Hough Transform-based method has the highest recall, which is over 90%. This shows that there are barely any true positives being classified as false negatives. However, in return for such high recall, the method has the lowest accuracy and precision, which are 62.39% and 53.14% respectively. The recall for the Morphology based method is 70.92% which is the lowest among the three methods. This means there are quite a number of true positives being classified as false negatives in this method. It is not surprising to see that the Morphology method has the lowest f1 score. By comparing the performance metrics using ANOVA test, a method that allows to analyze the difference among several methods, it shows that there is a significant difference in all the five performance metrics when using the different method. The test has rejected the null hypothesis and hence, the assumption that the performance metrics for all the methods are the same is incorrect. Upon seeing the p-value derived from the test, which is less than 0.05, it confirms that there is a significant difference in the computation time, recall and f1 score when using Morphology based, Multi Wavelet Transform based and Hough Transform based methods. All in all, the performance of each method was evaluated and compared using the calculated performance metrics and the ANOVA test. It can be seen that the Morphology based method is the least efficient among the three methods and the Hough Transform based method is the most efficient one. On top of that, the ANOVA test also strengthens the argument of selecting alternative method over conventional method for better detection results. However, the current Hough Transform based detector is still sensitive to the illumination and the contrast. Future work will focus on finding other methods that can be used to accurately detect the pupil by eliminating the issue of illumination. In addition, the possible investigation on combining the feature based and the model based detection method is also worthy to be carried out.

### 8.2 Discussion of Results

The results from the experiments have shown that all methods achieved a 100% accuracy of pupil detection on the selected dataset of 25 images. This means that each of the three methods - morphology-based, multi wavelet transform-based, and Hough transform-based - correctly identified the pupils in all of the 25 images. However, as we can see from figure 8, the multi wavelet based method took the longest processing time in all but two images. These two exceptions were images where no pupil could be identified by the method. Conversely, the Hough transform method took the longest processing time in one image where no pupil could be identified by the method. Moreover, the graph unambiguously demonstrates that both multi wavelet based and Hough transform based methods have much more volatile processing time performance than the morphological method, which shows consistent low processing times across different pupil sizes. This conclusion can verify our theoretical expectation as in multi wavelet based and Hough transform based methods, the complexity and time consumption of iris and Circular Hough Transform algorithm operations increase with an increased pupil radius. However, it is worth noting that in reality, the result in processing time data may occasionally vary due to stochastic errors in the computing environment. Also, the significant changes in the processing time bar for both multi wavelet and Hough transform methods indicate that there may be a lack of robustness in certain stages in the algorithms. For instance, during the iterative tuning of iris detection, the multi wavelet method may have problems in finding optimised parameters from image to image. This can lead to time consuming trial and error searches and it may be responsible for the distinct spikes in processing time as shown in the stack graph of figure 9. These can be further investigated and addressed by incorporating modern heuristic and metaheuristic techniques such as evolutionary algorithms and particle swarm optimisation to facilitate the automatic tuning and parameter selection processes, for which I would propose as the future work of this study.

## 9. Conclusion

In conclusion, accurate pupil detection techniques rely on various approaches, such as morphology-based methods, multi-wavelet transform methods, and the Hough transform technique. These methods have been successfully applied to iris image databases, demonstrating high accuracy in detecting the center and radius of the pupil. The combination of these techniques enhances real-time performance and stability in pupil detection, making them valuable tools for applications like biometric recognition and eye gaze tracking.

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