Anti Intuitionistic Fuzzy Soft HX Ring

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ABSTRACT

In this paper we prove some results about of anti intuitionistic fuzzy soft HX ring of HX ring, after that we study some types of anti intuitionistic fuzzy soft HX ring of HX ring and prove some results about them. Also we study the image and pre image of anti intuitionistic fuzzy soft HX ring of HX ring and prove some results about them.

1- Introduction

The concept of fuzzy sets was first introduced by L. A. Zadeh [15] in 1965 who extended the classical notion of a set and studied their properties. Atanassov [1] in 1986 proposed intuitionistic fuzzy set and detailed some more properties. In 1999, Molodtsov [8] defined the notion of soft set and proved some results. The fuzzy soft set and intuitionistic fuzzy soft set are introduced by P. K. Maji and R. Biswas in 2001 [9,10]. Ummahan Acar, Fatih Koyuncuand [2] in 2010 defined soft rings. In 2011, Jayanta Ghosh, and T.K. Samanta [14] defined fuzzy soft ring and studied some of its algebraic properties. In 2012, Zhiming Zhang [16] initiated the study of intuitionistic fuzzy soft rings. The concept of intuitionistic fuzzy soft rings introduced by B. A. Ersoy and S. Onar [5] in 2013. In 1988, Li Hong Xing [7], proposed the concept of HX ring and derived some of its properties. In 2014, R. Muthuraj and M. S. Muthuraman [11] defined the notion of intuitionistic fuzzy HX ring of a HX ring and some of their related properties. In 2016, R. Muthuraj and N. Ramila Gandhi [13] stated the concept of an anti fuzzy HX ring. In 2016 R. Muthuraj and s. Muthuraman [12] defined intuitionistic anti-fuzzy Hx ring. In 2018, Walaa Hasan Ashour [3] defined the notion of intuitionistic fuzzy soft HX subring of a HX ring and some of their its related properties. In this paper we prove some results about of anti intuitionistic fuzzy soft HX ring of HX ring, after that we study some types of anti intuitionistic fuzzy soft HX ring of HX ring and prove some results about them. Also we study the image and pre image of anti intuitionistic fuzzy soft HX ring of HX ring and prove some results about them.

2- Preliminaries

Definition 2.1[8]: Let X be an initial universe set and E be a set of parameters. A pair (F, E) is called a soft set over X if $F: E \to P(X)$ is a function, where $P(X)$ is the power set of X. The set of all soft set over X is denoted by $SS(X, E)$.

Definition 2.2 [9]: Let X be an initial universe set, E a set of parameters and I^X is the set of all fuzzy sets of X. A pair (F, E) is called a fuzzy soft set over X ,where $F: E \to I^X$ is mapping from E in to I^X.

Definition 2.3 [2]: Let R be a ring and (F, E) be a soft set over R. then $(F, E)$is said to be a soft ring over R if $F(e)$ is a subring of R for all $e \in E$.

Definition 2.4 [**14**]: The fuzzy soft set (F, E) over a ring R is called a fuzzy soft ring over R if, $\forall x, y \in R, e \in E$

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i- $F_e(x - y) \geq F_e(x) \wedge F_e(y)$
ii- $F_e(xy) \geq F_e(x) \wedge F_e(y)$

Definition 2.5 [4,5]: An intuitionistic fuzzy sets $A = \langle x, \mu_A(x), \upsilon_A(x) \rangle$ in X of a ring R is called an intuitionistic fuzzy ring of R if it satisfies the following condition $\forall x, y \in R$,

i- $\mu_A(x - y) \geq \mu_A(x) \wedge \mu_A(y)$
ii- $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$
iii- $\upsilon_A(x - y) \leq \upsilon_A(x) \wedge \upsilon_A(y)$
iv- $\upsilon_A(xy) \leq \upsilon_A(x) \wedge \upsilon_A(y)$

Definition 2.6 [10]: Let X be an initial universe and E be a set of parameters. A pair $(I\tilde{F}, E)$ is called an intuitionistic fuzzy soft set over X, where $\tilde{F}$ is a mapping given by $\tilde{F}: E \to I\tilde{F}(X)$. An intuitionistic fuzzy soft set is a parameterized family of intuitionistic fuzzy subsets of X. For any $e \in E, \tilde{F}(e)$ is referred to as the set of a-approximation elements of the intuitionistic fuzzy soft set $(I\tilde{F}, E)$, which is actually an intuitionistic fuzzy set on X and can be written as $\tilde{F}(e) = \{\langle x, \mu_{\tilde{F}(e)}(x), \upsilon_{\tilde{F}(e)}(x) \rangle | x \in X\}$. Here, $\mu_{\tilde{F}(e)}(x)$ *and* $v_{\tilde{F}(e)}(x)$ are respectively the membership degree and non-membership degree that object x holds on parameter e.

Definition 2.7 [16]: Let R be a ring and $(I\tilde{F}, E)$be intuitionistic fuzzy soft set over R. Then $(I\tilde{F}, E)$ is said to be intuitionistic fuzzy soft ring of R if for each $e \in E$ and $x, y \in R$,

i- $\mu_{\tilde{F}_e}(x - y) \geq \mu_{\tilde{F}_e}(x) \wedge \mu_{\tilde{F}_e}(y)$
ii- $\mu_{\tilde{F}_e}(xy) \geq \mu_{\tilde{F}_e}(x) \wedge \mu_{\tilde{F}_e}(y)$
iii- $\upsilon_{\tilde{F}_e}(x - y) \leq \upsilon_{\tilde{F}_e}(x) \wedge \upsilon_{\tilde{F}_e}(y)$
iv- $\upsilon_{\tilde{F}_e}(xy) \leq \upsilon_{\tilde{F}_e}(x) \wedge \upsilon_{\tilde{F}_e}(y)$

Definition 2.8 [1]: Let R be a ring and a non-empty set $K \subset 2^R\_\{\emptyset\}$ with two binary operation ' + ' and ' . ' then K is said to be HX ring of R if K is a ring with respect to the algebraic operation defined by

i- $A+B = \{a + b / a \in A$ and $b \in B\}$, which its null element is denoted by Q , and the negative element of A is denoted by – A .
ii- $AB = \{ab / a \in A$ and $b \in B\}$,
iii- A (B + C) = AB + AC and (B + C) A = BC + CA

Definition 2.9: Let R be a ring . (F, E) be anti fuzzy soft set defined on R and $K \subset 2^R\_\{\emptyset\}$ be HX ring on R. We define anti fuzzy soft HX set $(G^F, E): G^F: E \to I^K$ on K as follows :

For all $e \in E$, $G_e^F(A) = \min\{F_e(x) / \text{for all } x \in A \subseteq R\}$.

Definition 2.10: Let R be a ring . Let $(I\tilde{F}, E)$ be an anti intuitionistic fuzzy soft set defined on R. let $K \subset 2^R\_\{\emptyset\}$ be HX ring on R .We defined anti intuitionistic fuzzy soft HX set $(IG^{\tilde{F}}, E)$ on K as follows:

Where For all $e \in E$, $G_e^{\mu_{\tilde{F}}}(A) = \min\{\mu_{\tilde{F}_e}(x) / \text{for all } x \in A \subseteq R\}$ and
$G_e^{\upsilon_{\tilde{F}}}(A) = \max\{\upsilon_{\tilde{F}_e}(x) / \text{for all } x \in A \subseteq R\}$.

Definition 2.11: Let R be a ring , (F, E) be a fuzzy soft ring defined on R and $\mathrm{K} \subset 2^{\mathrm{R}}\_\{\emptyset\}$ be HX ring . A fuzzy soft subset $\left(\mathrm{G}^{\tilde{\mathrm{F}}}, \mathrm{E}\right) \equiv \left\{\mathrm{G}_{\mathrm{e}}^{\tilde{\mathrm{F}}}, \mathrm{e} \in \mathrm{E}\right\}$ of K is called anti fuzzy soft HX ring on K or fuzzy soft ring induced by F if the following conditions are satisfied for all A , B $\in$ K and e $\in$ E

i- $\mathrm{G}_{\mathrm{e}}^{\tilde{\mathrm{F}}}(\mathrm{A}-\mathrm{B}) \leq \max\left\{\mathrm{G}_{\mathrm{e}}^{\tilde{\mathrm{F}}}(\mathrm{A}), \mathrm{G}_{\mathrm{e}}^{\tilde{\mathrm{F}}}(\mathrm{B})\right\}$

ii- $\mathrm{G}_{\mathrm{e}}^{\tilde{\mathrm{F}}}(\mathrm{AB}) \leq \max\left\{\mathrm{G}_{\mathrm{e}}^{\tilde{\mathrm{F}}}(\mathrm{A}), \mathrm{G}_{\mathrm{e}}^{\tilde{\mathrm{F}}}(\mathrm{B})\right\}$,

where $\mathrm{G}_{\mathrm{e}}^{\tilde{\mathrm{F}}}(\mathrm{A}) = \min\{\tilde{\mathrm{F}}_{\mathrm{e}}(\mathrm{x}) \,/\, \text{for all } \mathrm{x} \in \mathrm{A} \subseteq \mathrm{R}\}$.

Definition 2.12 [3]: Let R be a ring . Let $(\mathrm{I}\tilde{\mathrm{F}}, \mathrm{E})$ be an intuitionistic fuzzy soft ring on R and a nonempty set $\mathrm{K} \subset 2^{\mathrm{R}}\_\{\emptyset\}$ be a HX ring . An intuitionistic fuzzy soft subset $\mathrm{M} = \langle \mathrm{A}, \mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{A}), \mathrm{G}_{\mathrm{e}}^{\upsilon_{\tilde{\mathrm{F}}}}(\mathrm{A})\rangle$ of a HX ring K is said to be an anti intuitionistic fuzzy soft HX ring$\left(\mathrm{I}\tilde{\mathrm{G}}\mathrm{SHX}\right)$ of K if the following conditions are satisfied for all $\mathrm{A}, \mathrm{B} \in \mathrm{K}$.

i- $\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{A}-\mathrm{B}) \leq \max\left\{\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{A}), \mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{B})\right\}$

ii- $\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{AB}) \leq \max\left\{\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{A}), \mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{B})\right\}$

iii- $\mathrm{G}_{\mathrm{e}}^{\upsilon_{\tilde{\mathrm{F}}}}(\mathrm{A}-\mathrm{B}) \geq \min\left\{\mathrm{G}_{\mathrm{e}}^{\upsilon_{\tilde{\mathrm{F}}}}(\mathrm{A}), \mathrm{G}_{\mathrm{e}}^{\upsilon_{\tilde{\mathrm{F}}}}(\mathrm{B})\right\}$

iv- $\mathrm{G}_{\mathrm{e}}^{\upsilon_{\tilde{\mathrm{F}}}}(\mathrm{AB}) \geq \min\left\{\mathrm{G}_{\mathrm{e}}^{\upsilon_{\tilde{\mathrm{F}}}}(\mathrm{A}), \mathrm{G}_{\mathrm{e}}^{\upsilon_{\tilde{\mathrm{F}}}}(\mathrm{B})\right\}$,

where $\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{A}) = \min\left\{\mu_{\tilde{\mathrm{F}}_{\mathrm{e}}}(\mathrm{x}) \,/\, \text{for all } \mathrm{x} \in \mathrm{A} \subseteq \mathrm{R}\right\}$

$\mathrm{G}_{\mathrm{e}}^{\upsilon_{\tilde{\mathrm{F}}}}(\mathrm{A}) = \max\{\upsilon_{\tilde{\mathrm{F}}_{\mathrm{e}}}(\mathrm{x}) \,/\, \text{for all } \mathrm{x} \in \mathrm{A} \subseteq \mathrm{R}\}$.

Theorem 2.13 : If M_1 and M_2 be two anti intuitionistic fuzzy soft HX rings of HX ring K, then $\mathrm{M}_1 \cap \mathrm{M}_2$ is also anti intuitionistic fuzzy soft HX rings of HX ring K.

Proof:

Let $\mathrm{M}_1 = \left\{\langle \mathrm{A}, \mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{A}), \mathrm{G}_{\mathrm{e}}^{\upsilon_{\tilde{\mathrm{F}}}}(\mathrm{A}) / \mathrm{A} \in \mathrm{K} \rangle\right\}$ and

$\mathrm{M}_2 = \left\{\langle \mathrm{B}, \mathrm{W}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{B}), \mathrm{W}_{\mathrm{e}}^{\upsilon_{\tilde{\mathrm{F}}}}(\mathrm{B}) / \mathrm{B} \in \mathrm{K} \rangle\right\}$ be two anti intuitionistic fuzzy soft HX rings of HX ring K .

i-
$$\begin{aligned}\left(\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}} \cap \mathrm{W}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}\right)(\mathrm{A}-\mathrm{B}) &= \min\left\{\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{A}-\mathrm{B}), \mathrm{W}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{A}-\mathrm{B})\right\} \\ &\leq \min\left\{\max\{\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{A}), \mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{B})\}, \max\{\mathrm{W}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{A}), \mathrm{W}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{B})\}\right\} \\ &= \max\left\{\min\{\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{A}), \mathrm{W}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{A})\}, \min\{\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{B}), \mathrm{W}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{B})\}\right\} \\ &= \max\left\{\left(\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}} \cap \mathrm{W}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}\right)(\mathrm{A}), \left(\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}} \cap \mathrm{W}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}\right)(\mathrm{B})\right\}.\end{aligned}$$

Hence,
$\left(\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}} \cap \mathrm{W}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}\right)(\mathrm{A}-\mathrm{B}) \leq \max\left\{\left(\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}} \cap \mathrm{W}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}\right)(\mathrm{A}), \left(\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}} \cap \mathrm{W}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}\right)(\mathrm{B})\right\}$

ii-
$$\begin{aligned}\left(\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}} \cap \mathrm{W}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}\right)(\mathrm{AB}) &= \min\left\{\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{AB}), \mathrm{W}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{AB})\right\} \\ &\leq \min\left\{\max\{\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{A}), \mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{B})\}, \max\{\mathrm{W}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{A}), \mathrm{W}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{B})\}\right\} \\ &= \max\left\{\min\{\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{A}), \mathrm{W}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{A})\}, \min\{\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{B}), \mathrm{W}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}(\mathrm{B})\}\right\} \\ &= \max\left\{\left(\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}} \cap \mathrm{W}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}\right)(\mathrm{A}), \left(\mathrm{G}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}} \cap \mathrm{W}_{\mathrm{e}}^{\mu_{\tilde{\mathrm{F}}}}\right)(\mathrm{B})\right\}.\end{aligned}$$

Hence,

$\left(G_e^{\mu_{\tilde{F}}} \cap W_e^{\mu_{\tilde{F}}}\right)(AB) \leq \max\left\{\left(G_e^{\mu_{\tilde{F}}} \cap W_e^{\mu_{\tilde{F}}}\right)(A), \left(G_e^{\mu_{\tilde{F}}} \cap W_e^{\mu_{\tilde{F}}}\right)(B)\right\}$

iii- $\left(G_e^{\upsilon_{\tilde{F}}} \cap W_e^{\upsilon_{\tilde{F}}}\right)(A-B) = \max\left\{G_e^{\upsilon_{\tilde{F}}}(A-B), W_e^{\upsilon_{\tilde{F}}}(A-B)\right\}$
$\geq \max\left\{\min\{G_e^{\upsilon_{\tilde{F}}}(A), G_e^{\upsilon_{\tilde{F}}}(B)\}, \min\{W_e^{\upsilon_{\tilde{F}}}(A), W_e^{\upsilon_{\tilde{F}}}(B)\}\right\}$
$= \min\left\{\max\{G_e^{\upsilon_{\tilde{F}}}(A), W_e^{\upsilon_{\tilde{F}}}(A)\}, \max\{G_e^{\upsilon_{\tilde{F}}}(B), W_e^{\upsilon_{\tilde{F}}}(B)\}\right\}$
$= \min\left\{\left(G_e^{\upsilon_{\tilde{F}}} \cap W_e^{\upsilon_{\tilde{F}}}\right)(A), \left(G_e^{\upsilon_{\tilde{F}}} \cap W_e^{\upsilon_{\tilde{F}}}\right)(B)\right\}.$

Hence,
$\left(G_e^{\upsilon_{\tilde{F}}} \cap W_e^{\upsilon_{\tilde{F}}}\right)(A-B) \geq \min\left\{\left(G_e^{\upsilon_{\tilde{F}}} \cap W_e^{\upsilon_{\tilde{F}}}\right)(A), \left(G_e^{\upsilon_{\tilde{F}}} \cap W_e^{\upsilon_{\tilde{F}}}\right)(B)\right\}$

iv- $\left(G_e^{\upsilon_{\tilde{F}}} \cap W_e^{\upsilon_{\tilde{F}}}\right)(AB) = \max\left\{G_e^{\upsilon_{\tilde{F}}}(AB), W_e^{\upsilon_{\tilde{F}}}(AB)\right\}$
$\geq \max\left\{\min\{G_e^{\upsilon_{\tilde{F}}}(A), G_e^{\upsilon_{\tilde{F}}}(B)\}, \min\{W_e^{\upsilon_{\tilde{F}}}(A), W_e^{\upsilon_{\tilde{F}}}(B)\}\right\}$
$= \min\left\{\max\{G_e^{\upsilon_{\tilde{F}}}(A), W_e^{\upsilon_{\tilde{F}}}(A)\}, \max\{G_e^{\upsilon_{\tilde{F}}}(B), W_e^{\upsilon_{\tilde{F}}}(B)\}\right\}$
$= \min\left\{\left(G_e^{\upsilon_{\tilde{F}}} \cap W_e^{\upsilon_{\tilde{F}}}\right)(A), \left(G_e^{\upsilon_{\tilde{F}}} \cap W_e^{\upsilon_{\tilde{F}}}\right)(B)\right\}.$

Hence,
$\left(G_e^{\upsilon_{\tilde{F}}} \cap W_e^{\upsilon_{\tilde{F}}}\right)(AB) \geq \min\left\{\left(G_e^{\upsilon_{\tilde{F}}} \cap W_e^{\upsilon_{\tilde{F}}}\right)(A), \left(G_e^{\upsilon_{\tilde{F}}} \cap W_e^{\upsilon_{\tilde{F}}}\right)(B)\right\}$.

Theorem 2.14: If M_1 and M_2 be two anti intuitionistic fuzzy soft HX rings of HX ring K, then $M_1 \cup M_2$ is also anti intuitionistic fuzzy soft HX rings of HX ring K.

Proof:

Let $M_1 = \left\{\langle A, G_e^{\mu_{\tilde{F}}}(A), G_e^{\upsilon_{\tilde{F}}}(A) / A \in K \rangle\right\}$ and

$M_2 = \left\{\langle B, W_e^{\mu_{\tilde{F}}}(B), W_e^{\upsilon_{\tilde{F}}}(B) / B \in K \rangle\right\}$ be two anti intuitionistic fuzzy soft HX rings of HX ring K .

v- $\left(G_e^{\mu_{\tilde{F}}} \cup W_e^{\mu_{\tilde{F}}}\right)(A-B) = \min\left\{G_e^{\mu_{\tilde{F}}}(A-B), W_e^{\mu_{\tilde{F}}}(A-B)\right\}$
$\leq \min\left\{\max\{G_e^{\mu_{\tilde{F}}}(A), G_e^{\mu_{\tilde{F}}}(B)\}, \max\{W_e^{\mu_{\tilde{F}}}(A), W_e^{\mu_{\tilde{F}}}(B)\}\right\}$
$= \max\left\{\min\{G_e^{\mu_{\tilde{F}}}(A), W_e^{\mu_{\tilde{F}}}(A)\}, \min\{G_e^{\mu_{\tilde{F}}}(B), W_e^{\mu_{\tilde{F}}}(B)\}\right\}$

$= \max\left\{\left(G_e^{\mu_{\tilde{F}}} \cup W_e^{\mu_{\tilde{F}}}\right)(A), \left(G_e^{\mu_{\tilde{F}}} \cup W_e^{\mu_{\tilde{F}}}\right)(B)\right\}.$

Hence,
$\left(G_e^{\mu_{\tilde{F}}} \cup W_e^{\mu_{\tilde{F}}}\right)(A-B) \leq \max\left\{\left(G_e^{\mu_{\tilde{F}}} \cup W_e^{\mu_{\tilde{F}}}\right)(A), \left(G_e^{\mu_{\tilde{F}}} \cup W_e^{\mu_{\tilde{F}}}\right)(B)\right\}$

vi- $\left(G_e^{\mu_{\tilde{F}}} \cup W_e^{\mu_{\tilde{F}}}\right)(AB) = \min\left\{G_e^{\mu_{\tilde{F}}}(AB), W_e^{\mu_{\tilde{F}}}(AB)\right\}$
$\leq \min\left\{\max\{G_e^{\mu_{\tilde{F}}}(A), G_e^{\mu_{\tilde{F}}}(B)\}, \max\{W_e^{\mu_{\tilde{F}}}(A), W_e^{\mu_{\tilde{F}}}(B)\}\right\}$
$= \max\left\{\min\{G_e^{\mu_{\tilde{F}}}(A), W_e^{\mu_{\tilde{F}}}(A)\}, \min\{G_e^{\mu_{\tilde{F}}}(B), W_e^{\mu_{\tilde{F}}}(B)\}\right\}$

$= \max\left\{\left(G_e^{\mu_{\tilde{F}}} \cup W_e^{\mu_{\tilde{F}}}\right)(A), \left(G_e^{\mu_{\tilde{F}}} \cup W_e^{\mu_{\tilde{F}}}\right)(B)\right\}.$

Hence,
$\left(G_e^{\mu_{\tilde{F}}} \cup W_e^{\mu_{\tilde{F}}}\right)(AB) \leq \max\left\{\left(G_e^{\mu_{\tilde{F}}} \cup W_e^{\mu_{\tilde{F}}}\right)(A), \left(G_e^{\mu_{\tilde{F}}} \cup W_e^{\mu_{\tilde{F}}}\right)(B)\right\}$

vii- $\left(G_e^{\upsilon_{\tilde{F}}} \cup W_e^{\upsilon_{\tilde{F}}}\right)(A-B) = \max\left\{G_e^{\upsilon_{\tilde{F}}}(A-B), W_e^{\upsilon_{\tilde{F}}}(A-B)\right\}$
$\geq \max\left\{\min\{G_e^{\upsilon_{\tilde{F}}}(A), G_e^{\upsilon_{\tilde{F}}}(B)\}, \min\{W_e^{\upsilon_{\tilde{F}}}(A), W_e^{\upsilon_{\tilde{F}}}(B)\}\right\}$

$$= \min\{\max\{G_e^{\upsilon_{\tilde{F}}}(A), W_e^{\upsilon_{\tilde{F}}}(A)\}, \max\{G_e^{\upsilon_{\tilde{F}}}(B), W_e^{\upsilon_{\tilde{F}}}(B)\}\}$$
$$= \min\{(G_e^{\upsilon_{\tilde{F}}} \cup W_e^{\upsilon_{\tilde{F}}})(A), (G_e^{\upsilon_{\tilde{F}}} \cup W_e^{\upsilon_{\tilde{F}}})(B)\}.$$

Hence,

$$(G_e^{\upsilon_{\tilde{F}}} \cup W_e^{\upsilon_{\tilde{F}}})(A - B) \geq \min\{(G_e^{\upsilon_{\tilde{F}}} \cup W_e^{\upsilon_{\tilde{F}}})(A), (G_e^{\upsilon_{\tilde{F}}} \cup W_e^{\upsilon_{\tilde{F}}})(B)\}$$

viii- $(G_e^{\upsilon_{\tilde{F}}} \cup W_e^{\upsilon_{\tilde{F}}})(AB) = \max\{G_e^{\upsilon_{\tilde{F}}}(AB), W_e^{\upsilon_{\tilde{F}}}(AB)\}$

$$\geq \max\{\min\{G_e^{\upsilon_{\tilde{F}}}(A), G_e^{\upsilon_{\tilde{F}}}(B)\}, \min\{W_e^{\upsilon_{\tilde{F}}}(A), W_e^{\upsilon_{\tilde{F}}}(B)\}\}$$
$$= \min\{\max\{G_e^{\upsilon_{\tilde{F}}}(A), W_e^{\upsilon_{\tilde{F}}}(A)\}, \max\{G_e^{\upsilon_{\tilde{F}}}(B), W_e^{\upsilon_{\tilde{F}}}(B)\}\}$$
$$= \min\{(G_e^{\upsilon_{\tilde{F}}} \cup W_e^{\upsilon_{\tilde{F}}})(A), (G_e^{\upsilon_{\tilde{F}}} \cup W_e^{\upsilon_{\tilde{F}}})(B)\}.$$

Hence,

$$(G_e^{\upsilon_{\tilde{F}}} \cup W_e^{\upsilon_{\tilde{F}}})(AB) \geq \min\{(G_e^{\upsilon_{\tilde{F}}} \cup W_e^{\upsilon_{\tilde{F}}})(A), (G_e^{\upsilon_{\tilde{F}}} \cup W_e^{\upsilon_{\tilde{F}}})(B)\}$$

Theorem 2.15: M is an anti intuitionistic fuzzy soft HX ring of HX ring K if and only if M^c is an intuitionistic fuzzy soft HX ring of HX ring K .

Proof:

Suppose that $M = \{\langle A, G_e^{\mu_{\tilde{F}}}(A), G_e^{\upsilon_{\tilde{F}}}(A) / A \in K\rangle\}$ an anti intuitionistic fuzzy soft HX ring of HX ring K .

i- $(G_e^{\mu_{\tilde{F}}})^c(A - B) = 1 - G_e^{\mu_{\tilde{F}}}(A - B) \geq 1 - \max\{G_e^{\mu_{\tilde{F}}}(A), G_e^{\mu_{\tilde{F}}}(B)\}$

$$= 1 - \max\left\{1 - (G_e^{\mu_{\tilde{F}}})^c(A), 1 - (G_e^{\mu_{\tilde{F}}})^c(B)\right\}$$
$$= \min\left\{(G_e^{\mu_{\tilde{F}}})^c(A), (G_e^{\mu_{\tilde{F}}})^c(B)\right\}$$

Hence, $(G_e^{\mu_{\tilde{F}}})^c(A - B) \geq \min\left\{(G_e^{\mu_{\tilde{F}}})^c(A), (G_e^{\mu_{\tilde{F}}})^c(B)\right\}$.

ii- $(G_e^{\mu_{\tilde{F}}})^c(AB) = 1 - G_e^{\mu_{\tilde{F}}}(AB) \geq 1 - \max\{G_e^{\mu_{\tilde{F}}}(A), G_e^{\mu_{\tilde{F}}}(B)\}$

$$= 1 - \max\left\{1 - (G_e^{\mu_{\tilde{F}}})^c(A), 1 - (G_e^{\mu_{\tilde{F}}})^c(B)\right\}$$
$$= \min\left\{(G_e^{\mu_{\tilde{F}}})^c(A), (G_e^{\mu_{\tilde{F}}})^c(B)\right\}.$$

Hence, $(G_e^{\mu_{\tilde{F}}})^c(AB) \geq \min\left\{(G_e^{\mu_{\tilde{F}}})^c(A), (G_e^{\mu_{\tilde{F}}})^c(B)\right\}$

iii- $(G_e^{\upsilon_{\tilde{F}}})^c(A - B) = 1 - G_e^{\upsilon_{\tilde{F}}}(A - B) \leq 1 - \min\{G_e^{\upsilon_{\tilde{F}}}(A), G_e^{\upsilon_{\tilde{F}}}(B)\}$

$$= 1 - \min\left\{1 - (G_e^{\upsilon_{\tilde{F}}})^c(A), 1 - (G_e^{\upsilon_{\tilde{F}}})^c(B)\right\}$$
$$= \max\left\{(G_e^{\upsilon_{\tilde{F}}})^c(A), (G_e^{\upsilon_{\tilde{F}}})^c(B)\right\}$$

Hence, $(G_e^{\upsilon_{\tilde{F}}})^c(A - B) \leq \max\left\{(G_e^{\upsilon_{\tilde{F}}})^c(A), (G_e^{\upsilon_{\tilde{F}}})^c(B)\right\}$

iv- $(G_e^{\upsilon_{\tilde{F}}})^c(AB) = 1 - G_e^{\upsilon_{\tilde{F}}}(A - B) \leq 1 - \min\{G_e^{\upsilon_{\tilde{F}}}(A), G_e^{\upsilon_{\tilde{F}}}(B)\}$

$$= 1 - \min\left\{1 - (G_e^{\upsilon_{\tilde{F}}})^c(A), 1 - (G_e^{\upsilon_{\tilde{F}}})^c(B)\right\}$$
$$= \max\left\{(G_e^{\upsilon_{\tilde{F}}})^c(A), (G_e^{\upsilon_{\tilde{F}}})^c(B)\right\}.$$

Hence, $(G_e^{\upsilon_{\tilde{F}}})^c(AB) \leq \max\left\{(G_e^{\upsilon_{\tilde{F}}})^c(A), (G_e^{\upsilon_{\tilde{F}}})^c(B)\right\}$

Hence, $M^c = \{\langle A, (G_e^{\mu_{\tilde{F}}})^c(A), (G_e^{\upsilon_{\tilde{F}}})^c(A)/A \in K\rangle\}$ be an intuitionistic fuzzy soft a HX rings of K.

Conversely, to prove by similarity above.

Definition 2.16 [3]: $M = \{\langle A, G_e^{\mu_{\tilde{F}}}(A), G_e^{\upsilon_{\tilde{F}}}(A)/A \in K\rangle\}$ be an intuitionistic fuzzy soft subset of HX rings K. We define the following " necessity" and "possibility" operations

$$\square M = \left\{\langle A, G_e^{\mu_{\tilde{F}}}(A), 1 - G_e^{\mu_{\tilde{F}}}(A)\rangle/A \in K\right\}$$
$$\diamond M = \left\{\langle A, 1 - G_e^{\upsilon_{\tilde{F}}}(A), G_e^{\upsilon_{\tilde{F}}}(A)\rangle/A \in K\right\}$$

Theorem 2.17: If M is an anti intuitionistic fuzzy soft HX ring of a HX ring K then

i- $\square$M is an anti intuitionistic fuzzy soft HX ring of HX ring K.
ii- $\diamond$ M is an anti intuitionistic fuzzy soft HX ring of HX ring K.

Proof :

i-Let $\square M = \left\{\langle A, G_e^{\mu_{\tilde{F}}}(A), (G_e^{\mu_{\tilde{F}}})^c(A)\rangle/A \in K\right\}$ and

$M = \left\{\langle A, G_e^{\mu_{\tilde{F}}}(A), G_e^{\upsilon_{\tilde{F}}})(A)\rangle/A \in K\right\}$ be an anti intuitionistic fuzzy soft HX ring of K

Now, $(G_e^{\mu_{\tilde{F}}})^c(A - B) = 1 - G_e^{\mu_{\tilde{F}}}(A - B) \geq 1 - \max\ \{G_e^{\mu_{\tilde{F}}}(A), G_e^{\mu_{\tilde{F}}}(B)\}$

$$= 1 - \max\left\{1 - (G_e^{\mu_{\tilde{F}}})^c(A), 1 - (G_e^{\mu_{\tilde{F}}})^c(B)\right\}$$
$$= \min\left\{(G_e^{\mu_{\tilde{F}}})^c(A), (G_e^{\mu_{\tilde{F}}})^c(B)\right\}$$

$(G_e^{\mu_{\tilde{F}}})^c(AB) = 1 - G_e^{\mu_{\tilde{F}}}(AB) \geq 1 - \max\ \{G_e^{\mu_{\tilde{F}}}(A), G_e^{\mu_{\tilde{F}}}(B)\}$

$$= 1 - \max\left\{1 - (G_e^{\mu_{\tilde{F}}})^c(A), 1 - (G_e^{\mu_{\tilde{F}}})^c(B)\right\}$$
$$= \min\left\{(G_e^{\mu_{\tilde{F}}})^c(A), (G_e^{\mu_{\tilde{F}}})^c(B)\right\}$$

Hence, $G_e^{\mu_{\tilde{F}}}((A - B) \geq \min\{G_e^{\mu_{\tilde{F}}}(A), G_e^{\mu_{\tilde{F}}}(B)\}$,

Therefore, $\square$M is an intuitionistic anti fuzzy soft HX ring of K.

ii- To prove by similarity (i)

Definition 2.18 [3]: Let K_1 and K_2 be any two HX rings. Then the function $f: K_1 \longrightarrow K_2$ is said to be a homomorphism if it satisfies the following axioms :

i- $f(A + B) = f(A) + f(B)$ and
ii- $f(AB) = f(A)\, f(B)$ for all $A, B \in K_1$.

Definition 2.19 [3]: Let K_1 and K_2 be any two HX rings. Then the function $f: K_1 \longrightarrow K_2$ is said to be an anti homomorphism if it satisfies the following axioms :

iii- $f(A + B) = f(A) + f(B)$ and
iv- $f(AB) = f(B)f(A)$ for all $A, B \in K_1$.

Definition 2.20: Let R_1 and R_2 be any two ring let $K_1 \subset 2^{R_1} - \{\emptyset\}$ and $K_2 \subset 2^{R_2} - \{\emptyset\}$ be two HX rings .

i- let $A = \{\langle x, \mu_{\tilde{F}(A)}(x), \upsilon_{\tilde{F}(A)}(x)\rangle / x \in R_1\}$ be any anti intuitionistic fuzzy soft sets on R_1, $C = \{\langle (U, G_e^{\mu_{\tilde{F}}}(U), G_e^{\upsilon_{\tilde{F}}}(U))\rangle / U \in K_1\}$ be anti intuitionistic fuzzy soft HX sets on K_1 and Let f be a function from K_1 in to K_2. Then the image of C on K_1 under f is defined as :

$$f(G_e^{\mu_{\tilde{F}}}(U)) = \begin{cases} \min\{G_e^{\mu_{\tilde{F}}}(U) : U \in f^{-1}(V)\} & , \quad f^{-1}(V) \neq \emptyset\}, \\ 0 & otherwise \end{cases}$$

$$f(G_e^{\upsilon_{\tilde{F}}}(V)) = \begin{cases} \max\{G_e^{\upsilon_{\tilde{F}}}(V) : \{U \in f^{-1}(V)\} & , \quad f^{-1}(V) \neq \emptyset\} \\ 1 & otherwise \end{cases}$$

ii- $B = \{\langle y, \alpha_{F(B)}(y), \beta_{\tilde{F}(B)}(y)\rangle / y \in R_2\}$ be an anti intuitionistic fuzzy soft set on R_2 and

$D = \{\langle V, M_e^{\mu_{\tilde{F}}}(V), M_e^{\upsilon_{\tilde{F}}}(V)\rangle / V \in K_2\}$ be an anti intuitionistic fuzzy soft HX set on K_2, then pre – image of D under f is defined as:

$f^{-1}(M_e^{\mu_{\tilde{F}}})(U) = M_e^{\mu_{\tilde{F}}}(f(U))$ also $f^{-1}\left(\left(M_e^{\upsilon_{\tilde{F}}}(U)\right)\right) = M_e^{\upsilon_{\tilde{F}}}(f(U)) \quad U \in K_1$.

Theorem 2.21: Let R_1 and R_2 be any two rings, Let $A = \{\langle x, \mu_{\tilde{F}(A)}(x), \upsilon_{\tilde{F}(A)}(x)/\rangle x \in R_1\}$ and $B = \{\langle y, \alpha_{F(B)}(y), \beta_{\tilde{F}(B)}(y)\rangle / y \in R_2\}$ be any two anti intuitionistic fuzzy soft sets on R_1 and R_2 respectively and f be a onto homomorphism from K_1 to K_2:

i- Let $D = \{\langle V, M_e^{\mu_{\tilde{F}}}(V), M_e^{\upsilon_{\tilde{F}}}(V)\rangle / V \in K_2\}$ be anti intuitionistic fuzzy soft sets on K_2. If D intuitionistic anti fuzzy soft HX ring of K_2 then $f^{-1}(D)$ is a anti intuitionistic fuzzy soft HX ring of K_1.

ii- Let $C = \{\langle U, G_e^{\mu_{\tilde{F}}}(\mathrm{U}), G_e^{\upsilon_{\tilde{F}}}(U)\rangle / U \in K_1\}$ be anti intuitionistic fuzzy soft sets on K_1 . If C anti intuitionistic fuzzy soft HX ring of K_1 then $f(C)$ is a anti intuitionistic fuzzy soft HX ring of K_2.

Proof:

i- Let D be anti intuitionistic fuzzy soft HX ring of K_2 .

Let $V = f(U)$, $W = f(T) \in K_2$ Where $U, T \in K_1$

$[f^{-1}(M_e^{\mu_{\tilde{F}}})](U-T) = M_e^{\mu_{\tilde{F}}}[(f(U-T)]$

$= M_e^{\mu_{\tilde{F}}}[f(U) - f(T)]$ (f *is homomorphism*)

$\leq \max\{M_e^{\mu_{\tilde{F}}}(f(U), M_e^{\mu_{\tilde{F}}}(f(\mathrm{T})\} \quad = \max\{f^{-1}(M_e^{\mu_{\tilde{F}}}(U), f^{-1}(M_e^{\mu_{\tilde{F}}}(T)\}$.

Hence , $[f^{-1}(M_e^{\mu_{\tilde{F}}})](U-T) \leq \max\{f^{-1}(M_e^{\mu_{\tilde{F}}}(U), f^{-1}(M_e^{\mu_{\tilde{F}}}(T)\}$.

$[f^{-1}(M_e^{\mu_{\tilde{F}}})](UT) = M_e^{\mu_{\tilde{F}}}[(f(UT)]$

$= M_e^{\mu_{\tilde{F}}}[f(U)f(T)]$ (f *is homomorphism*)

$\leq \max\{M_e^{\mu_{\tilde{F}}}(f(U), M_e^{\mu_{\tilde{F}}}(f(T)\} \quad = \max\{f^{-1}(M_e^{\mu_{\tilde{F}}}(U), f^{-1}(M_e^{\mu_{\tilde{F}}}(T)\}$.

Hence , $\left[f^{-1}\left(M_e^{\mu_{\tilde{F}}}\right)\right](UT) \leq max\ \{f^{-1}\ (M_e^{\mu_{\tilde{F}}}(U), f^{-1}\ \left(M_e^{\mu_{\tilde{F}}}(T)\right\}$.

$f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)(U-T) =\ M_e^{v_{\tilde{F}}}[f(U-T)] =\ M_e^{v_{\tilde{F}}}[f(U)-f(T)]$

$\geq min\{\ M_e^{v_{\tilde{F}}}(f(U)), M_e^{v_{\tilde{F}}}(f(T))\}$

$= min\ \{\left[f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)\right](U), [f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)](T)\}$.

Hence , $f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)(U-T) \geq min\ \{\left[f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)\right](U), [f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)](T)\}$.

$f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)(UT) =\ M_e^{v_{\tilde{F}}}[f(UT) =\ M_e^{v_{\tilde{F}}}[f(U)-f(T)]$

$\geq min\{\ M_e^{v_{\tilde{F}}}(f(U)), M_e^{v_{\tilde{F}}}(f(T)$

$= min\ \{\left[f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)\right](U), [f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)](T)\}$

Hence, $f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)(UT) \geq min\ \{\left[f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)\right](U), [f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)](T)\}$.

Therefore, $f^{-1}(D)$ is a anti intuitionistic fuzzy soft HX ring of K_1.

ii- It is clear.

Theorem 2.22: Let R_1 and R_2 be any two rings, $A = \{\langle x\ , \mu_{\tilde{F}(x)}(x), \upsilon_{\tilde{F}(x)}(x)\rangle / x \in\ R_1\}$ and $B = \left\{\langle y\ , \alpha_{\tilde{F}(B)}(y), \beta_{\tilde{F}(B)}(y)\rangle / y \in R_2\right\}$ be any two anti intuitionistic fuzzy soft sets on R_1 and R_2 respectively . Let f be an onto anti- homomorphism from K_1 to K_2:

.

i- let $D = \{\langle V\ , M_e^{\mu_{\tilde{F}}}(V),\ M_e^{v_{\tilde{F}}}(V)\rangle\ /\ V \in\ K_2\ \}$ be anti intuitionistic fuzzy soft sets on $K_2$and . If D anti intuitionistic fuzzy soft HX ring of K_2 then $f^{-1}(D)$ is a anti intuitionistic fuzzy soft HX ring of K_1.

ii- Let $C = \{\langle U\ , G_e^{\mu_{\tilde{F}}}(U)\ , G_e^{\upsilon_{\tilde{F}}}(U)(U)\rangle / U \in\ K_1\}$. If C is anti intuitionistic fuzzy soft HX subring of K_1 then $f(C)$ is an anti intuitionistic fuzzy soft HX ring of K_2.

Proof:

i- let D anti intuitionistic fuzzy soft HX ring of K_2 .

let $V = f(U)$, $W = f(T)\ \in K_2$ where $U\ , T\ \in K_1$

$\left[f^{-1}\left(M_e^{\mu_{\tilde{F}}}\right)\right](U-T) = M_e^{\mu_{\tilde{F}}}[(f(U-T)]$

$= M_e^{\mu_{\tilde{F}}}[f(T)-f(U)]$ (f is an anti homomorphism)

$\leq max\ \{M_e^{\mu_{\tilde{F}}}(f(T), M_e^{\mu_{\tilde{F}}}(\ f(U)\}$

$= max\ \{M_e^{\mu_{\tilde{F}}}(f(U), M_e^{\mu_{\tilde{F}}}(\ f(T)\}$

$= max\ \{[f^{-1}\ (M_e^{\mu_{\tilde{F}}})](U), [f^{-1}\ (M_e^{\mu_{\tilde{F}}})](T)\}$

Hence, $\left[f^{-1}\left(M_e^{\mu_{\tilde{F}}}\right)\right](U-T) \leq max\ \{[f^{-1}\ (M_e^{\mu_{\tilde{F}}})](U), [f^{-1}\ (M_e^{\mu_{\tilde{F}}})(T)]\}$.

$\left[f^{-1}\left(M_e^{\mu_{\tilde{F}}}\right)\right](UT) = M_e^{\mu_{\tilde{F}}}[(f(UT)]$

$= M_e^{\mu_{\tilde{F}}}[f(U)f(T)]$ (f is an anti homomorphism)

$\leq max\ \{M_e^{\mu_{\tilde{F}}}(f(T), f(U)\}\ = max\ \{M_e^{\mu_{\tilde{F}}}(f(U), f(T)\}$

$= max\ \{[f^{-1}\ (M_e^{\mu_{\tilde{F}}})](U), [f^{-1}\ (M_e^{\mu_{\tilde{F}}})](T)\}$

Hence, $\left[f^{-1}\left(M_e^{\mu_{\tilde{F}}}\right)\right](UT) \leq max\ \{[f^{-1}\ (M_e^{\mu_{\tilde{F}}})](U), [\ f^{-1}\ (M_e^{\mu_{\tilde{F}}})](T)\}$.

$[f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)](U-T) =\ M_e^{v_{\tilde{F}}}[f(U-T)]$

$= M_e^{v_{\tilde{F}}}[f(T-U)]$ (f is an anti homomorphism

$\geq min\{\ M_e^{v_{\tilde{F}}}(f(T)), M_e^{v_{\tilde{F}}}(f(U))\}$

$= min\{\ M_e^{v_{\tilde{F}}}(f(U)), M_e^{v_{\tilde{F}}}(f(T))\}$

$= min\ \{\left[f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)\right](U), [f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)](T)\}$

Hence,$[f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)](U-T) \geq min\ \{\left[f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)\right](U), [f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)](T)\}$.

$f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)(UT) =\ M_e^{v_{\tilde{F}}}[f(UT)]$

$= M_e^{v_{\tilde{F}}}[f(T)f(U)]$ (f is an anti- homomorphism)

$\geq min\{\ M_e^{v_{\tilde{F}}}(f(T)), M_e^{v_{\tilde{F}}}(f(U))\}$

$= min\{\ M_e^{v_{\tilde{F}}}(f(U)), M_e^{v_{\tilde{F}}}(f(T))\}$

$= min\ \{\left[f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)\right](U), [f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)](T)\}$

Hence , $f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)(U-T) \geq min\ \{\left[f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)\right](U), [f^{-1}\left(\ M_e^{v_{\tilde{F}}}\right)](T)\}$.

Therefore, $f^{-1}(D)$ is an anti intuitionistic fuzzy soft HX ring of K_1.

ii- It is clear.

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