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On Traces of Generalized (α,β) Permuting 3- Derivations on Lie ideals

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Abstract

Derivation plays an essential and important role in the integration of analysis, algebraic geometry, quantum physics and in many different contexts in various parts of mathematics. The relationship between usual derivations and its generalizations on the one side with commuting and centralizing on the other has been widely studied by many mathematicians. In this paper, the concepts of (α,β) permuting 3-derivation and generalized (α,β) permuting 3-derivation are presented and studied as a generalization of the concept of permuting 3-derivation. Also the traces of these concepts as commuting and centralizing on Lie ideals of prime rings are studied with more details.

MSC: 17B45

1. Introduction

Everywhere in this discussion $\mathcal{Z}(\mathcal{R})$ is denoted the center of an associative ring $\mathcal{R}$ and $[x_1,x_2]=x_1\ x_2-x_2\ x_1$ will indicate the commutator of $x_1,x_2\in\mathcal{R}$ [1]. A prime ring is a ring in which $x_1\mathcal{R}x_2=(0)$ implies $x_1=0$ or $x_2=0$ for $x_1,x_2\in\mathcal{R}$ [2]. Algebra, functional analysis and quantum physics are related with concept of derivation. An additive mapping d of $\mathcal{R}$ into itself such that for all x, $y\in\mathcal{R}$ $d(xy)=d(x)\,y+x\,d(y)$ is called a derivation of $\mathcal{R}$ [3]. An additive mapping $\mathcal{F}$of $\mathcal{R}$ itself is called generalized derivation if there exists a derivation d of $\mathcal{R}$ such that $\mathcal{F}(xy)=\mathcal{F}(x)\,y+x\,d(y)$, for all x , $y\in\mathcal{R}$ [4]. The terms of derivation and generalized derivation are identical whenever $F=d$. Maksa in 1987 presented the trem of a symmetric bi- derivation [5], a bi-aditive mapping d of $\mathcal{R}\times\mathcal{R}$ into $\mathcal{R}$ is said to be bi- derivation if $d(x,\ yz)=d(x,y)z+y\ d(x,y)$, $d(x\ \ y,z)=d(x,\ z)y+x\ d(y,z)$, for all x , $y,z\in\mathcal{R}$. In 1989 J. Vukman [6,7] examined symmetric bi- derivation s on prime rings. Any ring $\mathcal{R}$ that satisfies na=0 where n is a non-zero integer with $a\in\mathcal{R}$ implies a=0 is said to be n-torsion-free [2].
A subgroup ($\mathcal{U}$,+) of $\mathcal{R}$ is called Lie ideal if $[\mathcal{U},r]\subseteq\mathcal{U}$ whenever $u\in\mathcal{U}$, $r\in\mathcal{R}$ [3]. A Lie ideal $\mathcal{U}$ of $\mathcal{R}$ is called a square closed Lie ideal of $\mathcal{R}$ if u^2 in $\mathcal{U}$, for every $u\in\mathcal{U}$, [3]. In addition if $\mathcal{U}$ is not contained in $\mathcal{Z}(\mathcal{R})$ is called an admissible Lie ideal of $\mathcal{R}$ [8]. A map d of $\mathcal{R}$ is called commuting (resp. centralizing) on $\mathcal{U}$ if $[d(x),x]=0$ for all $x\in\mathcal{U}$ (resp.$[d(x),x]\in\mathcal{Z}(\mathcal{R})$, for all $x\in\mathcal{U}$), [9]. The history of centralizing and commuting mapping is due to Divinsky 1955, [10]. Posner initiated several aspects of a study of derivations as commuting and centralizing of prime ring, [11]. In 2007, Park and Jung introduced the concept of permuting 3-derivation and they are studied this concept as centralizing and commuting, [9]. Many papers have expressed their interests of the permuting 3-derivation [12], [13], [14], [15]. In this paper, the concept of generalized (α,β) permuting 3-derivation is introduced and the traces of it as commuting and centralizing are studied and the commutatively of Lie ideal under certain conditions. In this paper, $\mathcal{U}$ is represent a Lie ideal of $\mathcal{R}$, unless otherwise mentioned, also α , β are endomorphisms of $\mathcal{R}$.

2. (α,β) Permuting 3- Derivations on Lie ideals

In this section the concept of (α,β) permuting 3-derivation is introduced and studied as commuting and centralizing of Lie ideals.

The starting point for key results will be with the following concepts and results:
Lemma (2.1) [8]: Let $\mathcal{R}$ be a prime 2-torsion free ring and $a\ \mathcal{U}\ b$ =0 such that $\mathcal{U}$ is non central, then either a=0 or b=0, whenever a,$b\in\mathcal{R}$.

Lemma (2.2) [3]: In a prime 2-torsion free ring, every nonzero admissible contains a nonzero ideal.

Lemma (2.3) [16]: If $\mathcal{U}$ is a commutative of a prime 2-torsion ring $\mathcal{R}$ is contained in $\mathcal{Z}(\mathcal{R})$.

Definition (2.4) [9]: A mapping d: $\mathcal{R}\times\mathcal{R}\times\mathcal{R}\to\mathcal{R}$ is permuting if for any elements $x_1, x_2, x_3 \in \mathcal{R}$ and for any permutation $\{\sigma(1), \sigma(2), \sigma(3)\}$, the relation $d(x_1, x_2, x_3) = d(x_{\sigma(1)}, x_{\sigma(2)}, x_{\sigma(3)})$ is satisfied.

Definition (2.5) [9]: A 3-derivation d: $\mathcal{R}\times\mathcal{R}\times\mathcal{R}\to\mathcal{R}$ is called permuting 3-derivation if d is permuting. That is, the following equations are equal to each other: $d(x_1 x, x_2, x_3) = d(x_1, x_2, x_3)x + x_1 d(x, x_2, x_3)$, $d(x_1, x_2 x, x_3) = d(x_1, x_2, x_3)x + x_2 d(x_1, x, x_3)$ and $d(x_1, x_2, x_3 x) = d(x_1, x_2, x_3)x + x_3 d(x_1, x_2, x)$, for all $x_1, x_2, x_3, x \in \mathcal{R}$.

Definition (2.6), [9]: The trace δ_d of d of $\mathcal{R}\times\mathcal{R}\times\mathcal{R}$ into $\mathcal{R}$ is given by $\delta_d(x) = d(x, x, x)$, for each $x \in \mathcal{R}$.

Lemma (2.7), [9]: Let d be a derivation of a prime ring $\mathcal{R}$ and a be any element of $\mathcal{R}$ such that $a\, d(x)=0$, for all $x\in\mathcal{R}$. Then either $a=0$ or $d=0$.

Lemma (2.8) [12]: For a 3!-torsion free ring $\mathcal{R}$ and δ_d is the trace of permuting3-derivation map d of $\mathcal{U}\times\mathcal{U}\times\mathcal{U}$. The following hold for each $x, y \in \mathcal{U}$ the following hold

(1) If δ_d is commuting on $\mathcal{U}$, then $3[d(x, y, y), y] + [\delta_d(y), x]=0$.

(2) If δ_d is centralizing on $\mathcal{U}$, then $3[d(x,y,y),y] + [\delta_d(y), x]$ belong to $Z(\mathcal{R})$.

Now the definition of (α,β) permuting 3-derivation can be given as follows

Definition (2.9): A (α,β) permuting 3-derivation is a 3-additive map d: $\mathcal{U}\times\mathcal{U}\times\mathcal{U}\to\mathcal{R}$ in which $d(x y, z, w) = d(x, z, w)\alpha(y) + \beta(x)\, d(y, z, w)$, for all $x, y, z, w \in \mathcal{U}$.

Every permuting $(1,1)$ 3-derivation is a permuting 3-derivation, where 1 is the identity endomorphism.

Example (2.10): Let $\mathcal{U} = \left\{\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix} : a, b \in \mathcal{S}, \mathcal{S} \text{ is a commutative ring}\right\}$ and $\mathcal{R} = \left\{\begin{pmatrix}a & b\\ 0 & c\end{pmatrix} : a, b, c \in \mathcal{S}\right\}$. Define $d: \mathcal{U}\times\mathcal{U}\times\mathcal{U}\to\mathcal{R}$ by

$d\left(\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix}, \begin{pmatrix}0 & c\\ 0 & d\end{pmatrix}, \begin{pmatrix}0 & e\\ 0 & f\end{pmatrix}\right) = \begin{pmatrix}0 & ace\\ 0 & 0\end{pmatrix}$, for all $\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix}, \begin{pmatrix}0 & c\\ 0 & d\end{pmatrix}, \begin{pmatrix}0 & e\\ 0 & f\end{pmatrix} \in \mathcal{U}$; and define α, β: $\mathcal{R}\to\mathcal{R}$ are defined by $\alpha\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix} = \begin{pmatrix}0 & a\\ 0 & b\end{pmatrix}$ and $\beta\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix} = \begin{pmatrix}0 & 0\\ 0 & a\end{pmatrix}$, for all $\begin{pmatrix}0 & a\\ 0 & b\end{pmatrix} \in \mathcal{U}$.

It is simple matter to see that d is (α, β) permuting 3-derivation on $\mathcal{U}\times\mathcal{U}\times\mathcal{U}$.

In the following results, $\mathcal{U}$ is an admissible Lie ideal of $\mathcal{R}$, unless otherwise mentioned.

Lemma (2.11): Let be 2-torsion free prime ring $\mathcal{R}$, α be a monomorphism of $\mathcal{R}$ and $[\alpha(x), y]=0$, for all $x, y \in \mathcal{U}$. Then $\mathcal{U}=0$.

Proof: By Lemma (2.2), $\mathcal{U}$ contains a non zero ideal I of $\mathcal{R}$ and this implies that $[\alpha(x), y]=0$, for all $x, y \in I$.

Replace x by $\alpha^{-1}(x) z$, $z \in I$, in above equation and use it to get

$[x, y]\alpha(z) = o$, for all $x, y, z \in I$.

By Lemma (2.7), either $[x, y]=0$ or $\alpha(z)=0$.

If $[x, y]=0$, for all $x, y \in I$. Then, by Lemma (2.3) and simple computations, this leads to $\mathcal{U}$ is central but this a contradiction with hypothesis. Hence $\mathcal{U}=0$.

Theorem (2.12): Let $\mathcal{R}$ be a prime3!- torsion free ring and $\mathcal{U}\neq 0$. If there exists a permuting (α,β) 3-derivation $d:\mathcal{U}\times\mathcal{U}\times\mathcal{U}\to\mathcal{R}$ such that the trace δ_d of d is commuting on $\mathcal{U}$ and β is monomorphism and α is an automorphism with $\beta^2\beta^2=\beta$. Then *d=0*.

Proof: The commuting of δ_d on $\mathcal{U}$ gives $[\delta_d(x),x]=o$, for each $x\in\mathcal{U}$ … (1)

By applying Lemma(2.8),

$3[d(x,y,y),y]+[\delta_d(y),x]=0$, for all x , $y\in\mathcal{U}$. ...(2)

Replace x by $2y\,x$ in equation (2) and use equation (1) to get

$o=2\,y\,[\delta_d(y),x]+6\delta_d(y)\,[\alpha(x),y]+6\beta(y)\,[d(x,y,y),y]+6\,[\beta(y),y\,]d(x,y,y)$.

$o=2\beta(y)\beta([\delta_d(y),x])+6\beta(\delta_d(y))\,\beta\,[\alpha(x),y]+6\beta(y)\,\beta([d(x,y,y),y])+6\,\beta([\beta(y),y]\beta(d(x,y,y))$

$o=\beta(y)\,\beta(2[\delta_d(y),x]+6\,[d(x,y,y),y])+6\beta(\delta_d(y)\beta\,[\alpha(x),y])+6\,\beta(\beta(y)\,y-y\,\beta(y))\beta(d(x,y,y))$.

By using equation (2) and since $\beta^2\beta^2=\beta$, the last equation is reduced to

$o=6\beta\,(\delta_d(y)\,[\alpha(x),y]$.

Because $\mathcal{R}$ is 3!-torsion free and β is monomorphism, then

$\delta_d(y)\,[\alpha(x),y]=o$, for all $x,y\in\mathcal{U}$

Let substitute $x=2xz$ in the last equation and by using it, then

$o=2\,\delta_d(y)\,\alpha(x)[\alpha(z),y]$ and this implies that $\alpha^{-1}(\delta_d(y))\,x\,[z,\alpha^{-1}(y)]=0$.

Using Lemma (2.1), either $\delta_d(y)=0$ or $[\alpha(z),y]=0$

Applying lemma (2.2) and since $\mathcal{U}$ is a nonzero Lie ideal of $\mathcal{R}$, then $\delta_d(y)=0$, for all $y\in\mathcal{U}$. ... (3)

Linearize the last equation on y and using it (twice time) to get

$d(x,z,y)=0$, for all $x,y,z\in\mathcal{U}$. …(4)

By hypothesis and by Lemma (2.2), there exists a nonzero ideal I of $\mathcal{U}$ such that

$d(x,y,z)=0$, for all $x,y,z\in I$. …(5)

Replace x by , $rx\in\mathcal{R}$ in equation (5) and use it to get

$0=d(rx,y,z)=d(r,y,z)\alpha(x)+\beta(r)\,d(x,y,z)$

The primness of $\mathcal{R}$ and $\mathcal{U}\neq 0$ gives $d(r,y,z)=0$ on $\mathcal{R}\times\mathcal{U}\times\mathcal{U}$. …(6)

Similarly, replace y by $2s\,y$ and z by z, $t,s\in\mathcal{R}$ in equation (6) and hypothesis to get $d=0$.

Theorem (2.13): Let $\mathcal{R}$ be 3!-torsion free prime ring and d is (α,β) a permuting 3-derivation on $\mathcal{U}\times\mathcal{U}\times\mathcal{U}$ wiyh centralizing trace δ_d on $\mathcal{U}$ with α,β are and commuting and $[\alpha(u),\beta(u)]\neq 0$, for all $u\in\in\mathcal{U}$ .Then δ_d is commuting on $\mathcal{U}$.
Proof: since δ_d is commuting and use Lemma (2.8) gives $Z(\mathcal{R})\ni[\delta_d(y),x]+3[d(x,y,y),y]$ for all $x,y\in\mathcal{U}$. …(7)

Putting $2y^2$ instead of x in equation (7) to get

$Z(\mathcal{R})\ni 2y[\delta_d(y),y]+6\delta_d(y)[\alpha(y),y]+6[\delta_d(y),y]\,\alpha(y)$

$+ 6[\beta(y),y]\ \delta_d(y) + 6\beta(y)\ [\delta_d(y)\ ,y]$

This means that

$$0 = 2[y,x][\delta_d(y),y] + 6[\delta_d(y),x]\ [\alpha(y),y] + 6\delta_d(y)\ [[\alpha(y),y],x]$$

$$+ 6\ [[\delta_d(y),y],x]\ \alpha(y) + 6\ [\delta_d(y),\ y]\ [\alpha(y),x]$$

$$+ 6[\ [\beta(y),y],x]\ \delta_d(y) + 6[\beta(y),y]\ [\delta_d(y),x]$$

$$+ 6\ [\beta(y),x]\ [\delta_d(y), y] + 6\ \beta(y)[\ [\delta_d(y),y],x].$$

$$= 2[\ y,x][\ \delta_d(y),y] + 6[\alpha\ (y), x][\delta_d(y),y] + 6[[\beta(y),x]\ [\delta_d(y),y].$$

$$= ([y,x] + 3\ [\alpha(y),x] + 3[\beta(y),x])\ [\delta_d(y),y].$$

Hypothesis and Lemma (2.1) gives either $[\delta_d(y),y]=0$ or $[y+ 3\ \alpha\ (y)+3\beta(y),x] =0$.

Now, if $[y+ 3\ \alpha\ (y)+3\beta\ (y), x]=0$, then by taken $x= \alpha\ (y)$ to get

$[\beta(y),\ \alpha\ (y)]=0$ and this is a contradiction. Hence, $[\delta_d(y), y] = 0$.

The following remark is immediately from Theorem (2.13).

Remark (2.14)**:** In Theorem (2.13) , if α=-β. Then δ_d is commuting on $\mathcal{U}$.

Theorem (2.15): Let $\mathcal{R}$ be a prime 3!- torsion free ring and $\mathcal{U}$ be square closed of $\mathcal{R}$. $\mathcal{R}$ is a commutative, if d is a (α,β) permuting 3- derivation on $\mathcal{U} \times \mathcal{U} \times \mathcal{U}$ such that α is commuting on $\mathcal{U}$ and the trace δ_d of d is centerlizing on $\mathcal{U}$ and α is automorphism with β is monomorphsim such that $\beta^2\beta^2 = \beta$ and $[\beta(u), \alpha(u)]\neq 0$, for every $u \in \mathcal{U}$. .

Proof: If $\mathcal{U}$ is a non-commutative. By Theorem (2.14) and Theorem (2.13and because $\mathcal{U}$ is an admissible of $\mathcal{R}$, then the mapping d =0 which is a contradiction .

3. **Generalized (α,β) Permuting 3-derivations on Lie ideals**

In this section the concept of generalized (α,β) permuting 3-derivation is given and it studied as commuting and centralizing on Lie ideals.

Definition (3.1): A 3-additive mapping $\mathcal{F}$: $\mathcal{U} \times \mathcal{U} \times \mathcal{U} \to \mathcal{R}$ is called a generalized a (α, β) permuting 3-derivation if there exists a (α,β) permuting 3-derivation d: $\mathcal{U}\times \mathcal{U} \times \mathcal{U} \to \mathcal{R}$ such that the following relation is satisfied for all $x,y,z \in \mathcal{U}$:

$\mathcal{F}\ (\ x y,z, w\ ) = \mathcal{F}(x, z, w)\alpha\ (y) +\beta\ (x)\ d\ (y,z, w\ )$, for all $x\ ,y,z,\ w \in \mathcal{U}$.

If $d= \mathcal{F}$ then the definition of generalized (α,β) permuting 3-derivation and the definition (α,β) permuting 3-derivation are identical.

Example (3.2): Let$\mathcal{U}=\{\begin{pmatrix}0 & a\\0 & b\end{pmatrix} : a,b \in S, S$ is a commutative ring $\}$ and $\mathcal{R} =\left\{\begin{pmatrix}a & b\\0 & c\end{pmatrix} : a,b,c \in S\right\}$.

Define $\mathcal{F} :\mathcal{U} \times\mathcal{U} \times \mathcal{U} \to \mathcal{R}$ by

$\mathcal{F}\left(\begin{pmatrix}0 & a\\0 & b\end{pmatrix}, \begin{pmatrix}0 & c\\0 & d\end{pmatrix}, \begin{pmatrix}0 & e\\0 & f\end{pmatrix}\right) =\begin{pmatrix}0 & 0\\0 & b\,d\,f\end{pmatrix}$, for all $\begin{pmatrix}0 & a\\0 & b\end{pmatrix},\begin{pmatrix}0 & c\\0 & d\end{pmatrix},\begin{pmatrix}0 & e\\0 & f\end{pmatrix} \in \mathcal{U}$. Also α,β: $\mathcal{R}\to\mathcal{R}$ are defined by α is the identity mapping and β $\left(\begin{pmatrix}0 & a\\0 & b\end{pmatrix}\right)= \begin{pmatrix}0 & 0\\0 & a\end{pmatrix}$, for all $\begin{pmatrix}0 & a\\0 & b\end{pmatrix} \in \mathcal{U}$.

Then $\mathcal{F}$ is a generalized (α,β) a permuting 3-derivation, since there exists (α,β) a permuting 3- derivation d which is defined as example (2.11).

Theorem (3.3): Let $\mathcal{R}$ be a prime 3!- torsion free ring and $\mathcal{F}$: $\mathcal{U} \times \mathcal{U} \times \mathcal{U} \rightarrow \mathcal{R}$ be a generalized(α,β) a permuting 3-derivation and associated with (α,β) permuting 3- derivation d such that the traces $\delta_{\mathcal{F}}$ and δ_d of $\mathcal{F}$ and d respectively and α is commuting and α is an automorphsim and $\beta$is monomorphism with $\beta^2 = \beta$, then $\mathcal{F}$=0.

Proof: Since $\delta_{\mathcal{F}}$ is commuting and using Lemma (2.8), then for all $x,y \in \mathcal{U}$; $3[\mathcal{F}(x,y,y),y]+[\delta_{\mathcal{F}}(y),x] = 0$.

…(8)

Replace $2yx$ instead of x in equation (8) and hypothesis to obtain

$0=y[\delta_{\mathcal{F}}(y),x]+3\delta_{\mathcal{F}}(y)[\alpha(x),y]$ …(9)

Multiply equation (8) by y from left and compare it with equation (9) to get

$y[\mathcal{F}(x,y,y),y] - 3\delta_{\mathcal{F}}(y)[\alpha(x),y]=0$ …(10)

Let $x=2x\ y$ substituted in equation (10) and by assumption

$[\delta_{\mathcal{F}}(y),x]y+3[\mathcal{F}(x,y,y),y]\alpha(y)=0$ …(11)

Let $x=2\ y\ x$ in equation (10), to obtain $y[\delta_{\mathcal{F}}(y)\ ,x]+3\delta_{\mathcal{F}}(y)[\alpha(x),y]=0$ …(12)

Multiply equation (8) by α (y) from right and using equation (12) to get

$0=[\delta_{\mathcal{F}}(y),x]y-[\delta_{\mathcal{F}}(y),x]\alpha(y)$

$=[\delta\_F(y),x](y-\alpha(y))$

Lemma (2.1) and last equation gives, for all $y \in \mathcal{U}$ either $\delta_{\mathcal{F}}(y) \in \mathcal{Z}(\mathcal{R})$ or $\alpha(y)=y$.

If $\alpha(y)=y$, for all $y \in \mathcal{U}$, then by simple computation this equation implies that $\alpha(y)=y$, for all $y \in \mathcal{R}$ and as proof of [12, Theorem (1.3.6)] then one can prove that $\mathcal{F}$=0.

If $\delta_{\mathcal{F}}(y) \in \mathcal{Z}(\mathcal{R})$, for all $\in \mathcal{U}$, then equation (12) gives

$3\delta_{\mathcal{F}}(y)[\alpha(x),y]=0$.

Since $\mathcal{R}$ is prime and by Lemma (2.12) then $\delta_{\mathcal{F}}(y)=0$. …(13)

Linearize equation (13) on y to get

$0=\delta_{\mathcal{F}}(x)+\delta_{\mathcal{F}}(y)+3\mathcal{F}(x,x,y)+3\mathcal{F}(x,y,y)$

By equation (13) and since $\mathcal{R}$ is 3!-torsion free, the last equation can be reduced to

$\mathcal{F}(x,x,y)+\mathcal{F}(x,y,y)=0$, for all $x,y \in \mathcal{U}$. ...(14)

Again linearize equation (14) on y and since $\mathcal{R}$ is 3!-torsion free, then

$0=\mathcal{F}$ on $\mathcal{U}\times\mathcal{U}\times\mathcal{U}$.

Because of the hypothesis on $\mathcal{U}$ and Lemma (2.3) there is a a non zero ideal I of $\mathcal{U}$, then $\mathcal{F}$=0on $I\times I\times I$. ...(15)

As proof of Theorem (2.12) the proof is compete.

Theorem (3.4): Let $\mathcal{R}$ and $\mathcal{F}$ be as in Theorem (3.3). If δ_d and $\delta_{\mathcal{F}}$ are centerlizing with α, β are commuting suh that[$\alpha(y)$, $\beta(y)$]≠0 .Then $\delta_{\mathcal{F}}$ is commuting on $\mathcal{U}$.

Proof: Since $[\delta_{\mathcal{F}}(x), x]$ is central and Lemma (2.8) gives

$[\delta_{\mathcal{F}}(y), x] + 3\,[\mathcal{F}(x,y,y),y]$, for all $x, y \in \mathcal{U}$. …(16)

Replace $2y^2$ instead of x in equation (16)and by simple computations to get

$0=4[y,x][\delta_{\mathcal{F}}(y),y] + 6[\delta_{\mathcal{F}}(y),y][\alpha(y),y] + 6[\beta(y),x][\delta_d(y),y]$

Applying Theorem (2.14) leads to

$0= [2y + 3\alpha(y),x][\delta_{\mathcal{F}}(y),y]$

Since $\mathcal{R}$ is prime, the last equation gives us

Either $2y + 3\alpha(y) \in \mathcal{Z}(\mathcal{R})$ or $\delta_{\mathcal{F}}$ is commuting on $\mathcal{U}$.

If $2y + 3\alpha(y) \in \mathcal{Z}(\mathcal{R})$, then

$0= [2y + 3\alpha(y), \beta(y)] =[\alpha(y), \beta(y)]$ which is a contradiction.

Therefore $[\delta_{\mathcal{F}}(y), y]=0$, for all $y \in \mathcal{U}$.

Theorem (3.5): Let $\mathcal{R}$ be a prime 3!- torsion free ring and $\mathcal{U}$ be square closed. If $\mathcal{F} : \mathcal{U} \times \mathcal{U} \times \mathcal{U} \rightarrow \mathcal{R}$ is a (α,β) a nonzero generalized a permuting 3- derivation such that α is commuting on $\mathcal{U}$, and $\beta^2\beta^2 = \beta$, where δ_F is the trace of $\mathcal{F}$ is centerlizing on $\mathcal{U}, \alpha$ is automorphism, β is monomorphsim and α,β is commuting and $[\beta(u),\alpha(u)] \neq 0$, then $\mathcal{U}$ is commutative.

Proof: Take $\mathcal{U}$ is noncommutative, then it is an admissible by Theorem (3.4) δ_F is commuting and by Theorem (3.3) $\mathcal{F}$ =0 This is a contradiction Hence $\mathcal{U}$ is commutative.

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حول اثار تعميم مشتقات (α, β) التبادلية من النمط 3 على مثاليات لي

انوار خليل فرج
الجامعة التكنولوجية
قسم العلوم التطبيقية

صابرين جاسب شريف
الجامعة التكنولوجية
قسم العلوم التطبيقية

الخلاصة

الاشتقاق يلعب دورا أساسيا وهاما في دمج التحليل والهندسة الجبرية والفيزياء الكم وفي العديد من السياقات المختلفة في أجزاء مختلفة من الرياضيات. وقد درس العديد من علماء الرياضيات على نطاق واسع العلاقة بين الاشتقاقات الاعتيادية والتعميمات من جانب و التنقل والمركزية من جهة أخرى. في هذا البحث المفاهيم مشتقات (α، β) التبادلية من النمط 3 وتعمم مشتقات (α، β) التبادلية من النمط 3 قدمت ودرست كتعميم لمفهوم المشتقة التبادلية من النمط 3. كذلك تمت دراسة أثر هذه المفاهيم كالتنقل والمركزية على مثالي لي للحلقات الاولية وبينت المزيد من التفاصيل.