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Soft Regularity and Soft Singularity in SBA

By

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Abstract: The material in this paper is mainly related to the soft Banach algebra. The aim of this paper is to provide basic information about the soft unitary element in SBA. Another types of soft elements are soft regular and soft singular, these concepts are illustrated by some results.

Keywords: soft Banach algebra , unitary soft element , soft regular element , soft singular element.
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1.Introduction:

The concept of soft sets was first introduced by Molodtsov [7] in 1999 as a general mathematical tool for dealing with uncertain objects. After presentation of the operations of soft sets [4] , [5] the properties and applications of soft set theory have been studied increasingly [1] ,[6] , [12] and [13] and [9]. In 2011, Shabir and Naz [10] initiated the study of soft topological spaces. In 2014 Samanta S. et al. [11] introduced the concept of soft normed space on a soft linear spaces and soft Banach space. Recently, in 2015 , Thakur R. et al. [14] , initiated the study of soft Banach algebra. Applications of soft Banach algebra investigated by Petroudi S. et al. [8]. In this paper we introduce and study the notions soft regularity and soft singularity and give basic definitions and theorems about it.

2. Preliminaries:

Definition (2.1) [14]:

Let V be an algebra over a scalar field $\mathbb{R}$ and let E be the parameter set and F_E be a soft set over V. Now, F_E is called soft algebra (for short SA) of V over $\mathbb{R}(E)$ if $F(e)$ is a sub algebra of V for all $e \in E$. It is very easy to see that in a SA the soft elements satisfy the properties:

(i) $(\tilde{x}^e\tilde{y}^e)\tilde{z}^e = \tilde{x}^e(\tilde{y}^e\tilde{z}^e)$.

(ii) $\tilde{x}^e(\tilde{y}^e\tilde{+}\tilde{z}^e) = \tilde{x}^e\tilde{y}^e\tilde{+}\tilde{x}^e\tilde{z}^e$; $(\tilde{y}^e\tilde{+}\tilde{z}^e)\tilde{x}^e = \tilde{y}^e\tilde{x}^e\tilde{+}\tilde{z}^e\tilde{x}^e$.

(iii) $\tilde{\alpha}(\tilde{x}^e\tilde{y}^e) = (\tilde{\alpha}\tilde{x}^e)\tilde{y}^e = \tilde{x}^e(\tilde{\alpha}\tilde{y}^e)$, where for all $\tilde{x}^e, \tilde{y}^e, \tilde{z}^e \tilde{\in} F_E$ and for any soft scalar $\tilde{\alpha}$.

If F_E is also soft Banach space w.r.t. a soft norm that satisfies the inequality $\|\tilde{x}^e\tilde{y}^e\| \leq \|\tilde{x}^e\|\|\tilde{y}^e\|$ and F_E is contains the unitary element $\tilde{u}^e$ such that $\tilde{x}^e\tilde{u}^e = \tilde{u}^e\tilde{x}^e = \tilde{x}^e$ with $\|\tilde{u}^e\| = \bar{1}$, then is called soft Banach algebra (for short SBA).

3. Main Results:

Definition (3.1) [14]:

Let $\mathfrak{A}$ is a (SBA) with $\tilde{u}^e$. Then $\tilde{x}^e \tilde{\in} \mathfrak{A}$ is said to be soft regular, if $\tilde{x}^e$ is invertible (i.e. there a soft element ${\tilde{x}^e}^{-1}$ called the inverse of $\tilde{x}^e$, such that $\tilde{x}^e(\tilde{x}^e)^{-1} = (\tilde{x}^e)^{-1}\tilde{x}^e = \tilde{u}^e$). A non-soft regular element is called soft singular (it's not invertible).

Remark (3.2) [14]:

Clearly, $\tilde{u}^e$ is invertible. If $\tilde{x}^e$ is invertible, then the inverse is a unique. Further, if $\tilde{x}^e$ and $\tilde{y}^e$ are both invertible, then $\tilde{x}^e\tilde{y}^e$ is invertible and $(\tilde{x}^e\tilde{y}^e)^{-1} = (\tilde{y}^e)^{-1}(\tilde{x}^e)^{-1}$.

Notation (3.3):

(i) $G_E = \{\tilde{x}^e \tilde{\in} \mathfrak{A} : \tilde{x}^e \text{ is soft regular}\}$.

(ii) $S_E = \{\tilde{x}^e \tilde{\in} \mathfrak{A} : \tilde{x}^e \text{ is soft singular}\}$.

Remarks (3.4) [14]:

It is clear that G_E is a soft group under the soft multiplication.

Theorem (3.5) [8]:

Let $\mathfrak{A}$ be a (SBA) with $\tilde{u}^e$.

Then every $\tilde{x}^e \tilde{\in} \mathfrak{A}$ for $\|\tilde{u}^e \tilde{-} \tilde{x}^e\| \tilde{\leq} \bar{1}$ is soft regular and $\tilde{x}^{e^{-1}} = \sum_{n=0}^{\infty}(\tilde{u}^e \tilde{-} \tilde{x}^e)^n$.

Corollary (3.6):

Let $\mathfrak{A}$ be a (SBA) with $\tilde{u}^e$. Then For $\tilde{x}^e \tilde{\in} \mathfrak{A}$, then $\left\|\tilde{x}^{e^{-1}}\right\| \tilde{\leq} \frac{1}{\tilde{u}^e \tilde{-} \|\tilde{u}^e \tilde{-} \tilde{x}^e\|}$.

Proof: we know

$$\tilde{x}^{e^{-1}} = \sum_{n=0}^{\infty}(\tilde{u}^e \tilde{-} \tilde{x}^e)^n \Rightarrow \left\|\tilde{x}^{e^{-1}}\right\| = \left\|\sum_{n=0}^{\infty}(\tilde{u}^e \tilde{-} \tilde{x}^e)^n\right\| \tilde{\leq} \sum_{n=0}^{\infty}\|\tilde{u}^e \tilde{-} \tilde{x}^e\|^n = \frac{1}{\tilde{u}^e \tilde{-} \|\tilde{u}^e \tilde{-} \tilde{x}^e\|} .$$

Definition (3.7)

A (SBA) $\mathfrak{A}$ is a soft topological algebra (for short STA) , if $\mathfrak{A}$ is a soft topological linear space , see [Samanta] and to every $\tilde{x}^e, \tilde{y}^e \tilde{\in} \mathfrak{A}$ and every soft nbhd $\tilde{N}_E(\tilde{x}^e\tilde{y}^e)$, there are soft nbhds $\tilde{N}_E(\tilde{x}^e)$ and $\tilde{N}_E(\tilde{y}^e)$ such that:

(i) $\tilde{x}^e\tilde{N}_E(\tilde{y}^e) \tilde{\subseteq} \tilde{N}_E(\tilde{x}^e\tilde{y}^e)$.

(ii) $\tilde{N}_E(\tilde{x}^e)\tilde{y}^e \tilde{\subseteq} \tilde{N}_E(\tilde{x}^e\tilde{y}^e)$.

Remark (3.8):

(i) $\tilde{\Theta}^e \tilde{\in} S_E$.

(ii) If $\mathfrak{A}$ is a (SBA) , then $\mathfrak{A}$ is (STA).

Theorem (3.9) [14]:

Let $\mathfrak{A}$ be a (SBA). Then G_E is soft open subset of $\mathfrak{A}$ and therefore S_E is soft closed subset of $\mathfrak{A}$.

Theorem (3.10):

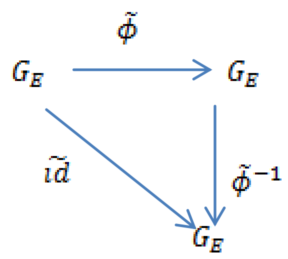
Let $\mathfrak{A}$ be a (SBA) . Then a soft mapping $\tilde{\phi} : G_E \to G_E$ defined by: $\tilde{\phi}(\tilde{x}^e) = \tilde{x}^{e^{-1}}$, for all $\tilde{x}^e \tilde{\in} \mathfrak{A}$ is soft homeomorphism.

Proof: To show that $\tilde{\phi}$ is soft well define and soft injective.

Let $\tilde{x}^e, \tilde{y}^e \tilde{\in} G_E$ and $\tilde{x}^e = \tilde{y}^e \Rightarrow \tilde{x}^{e^{-1}} = \tilde{y}^{e^{-1}} \Rightarrow \tilde{\phi}(\tilde{x}^e) = \tilde{\phi}(\tilde{y}^e)$. Thus $\tilde{\phi}$ is soft well defined.

And suppose that $\tilde{\phi}(\tilde{x}^e) = \tilde{\phi}(\tilde{y}^e) \Rightarrow \tilde{x}^{e^{-1}} = \tilde{y}^{e^{-1}} \Rightarrow \tilde{x}^e = \tilde{y}^e$ then $\tilde{\phi}$ is soft injective.

Now, to show that $\tilde{\phi}$ is soft surjective. Let $\tilde{y}^{e} \tilde{\in} G_E$, there is $\tilde{y}^{e^{-1}} \tilde{\in} G_E$ with $\tilde{\phi}(\tilde{y}^{e^{-1}}) = (\tilde{y}^{e^{-1}})^{-1} = \tilde{y}^{e}$. Hence $\tilde{\phi}$, is soft bijective. Finally to sow that $\tilde{\phi}$ and $\tilde{\phi}^{-1}$ are soft continuous by using the commutative diagram:



This implies that $\tilde{\phi}$ is soft homeomorphism.

Definition (4.1) [8]:

Let $\mathfrak{A}$ be a $(\boldsymbol{SBA})$ with $\tilde{u}^{e}$ and $\tilde{x}^{e} \tilde{\in} \mathfrak{A}$.

Then the soft set $\delta(\tilde{x}^{e}) = \{\tilde{\lambda} \tilde{\in} \mathbb{R}(E) : (\tilde{x}^{e} \simeq \tilde{\lambda}\tilde{u}^{e}) \text{ is soft singular}\}$ is called soft spectrum of $\tilde{x}^{e} \tilde{\in} \mathfrak{A}$.

Theorem (4.2):

A soft set $\delta(\tilde{x}^{e})$ is soft closed subset of $\mathbb{R}(E)$.

Proof:

Let $(\tilde{\lambda}_n)_{n\in\mathbb{N}}$ be a soft seq. in $\mathbb{R}(E)$ such that $\tilde{\lambda}_n \tilde{\to} \tilde{\lambda}$. Since $\tilde{\lambda}_n \tilde{\in} \delta(\tilde{x}^{e})$ for all $n \in \mathbb{N}$, then $\tilde{x}^{e} \simeq \tilde{\lambda}_n \tilde{u}^{e}$ is soft singular. Also, since $\tilde{\lambda}_n \tilde{\to} \tilde{\lambda}$, we have $\tilde{\lambda}_n \tilde{u}^{e} \tilde{\to} \tilde{\lambda}\tilde{u}^{e}$.

Define $\tilde{\Phi} : \mathbb{R}(E) \tilde{\to} \mathfrak{A}$ by $\tilde{\Phi}(\tilde{\lambda}) = \tilde{x}^{e} \simeq \tilde{\lambda}_n \tilde{u}^{e}$ for all $\tilde{\lambda} \tilde{\in} \mathbb{R}(E)$. Clear that is soft continuous. Since S_E be a soft closed subset of $\mathfrak{A}$, then:

$$\begin{aligned}\tilde{\Phi}^{-1}(S_E) &= \{\tilde{\lambda} \tilde{\in} \mathbb{R}(E) : \tilde{\Phi}(\tilde{\lambda}) \tilde{\in} S_E\} \\ &= \{\tilde{\lambda} \tilde{\in} \mathbb{R}(E) : \tilde{x}^{e} \simeq \tilde{\lambda}\tilde{u}^{e} \tilde{\in} S_E\} \\ &= \{\tilde{\lambda} \tilde{\in} \mathbb{R}(E) : \tilde{x}^{e} \simeq \tilde{\lambda}\tilde{u}^{e} \text{ is soft singular}\} = \delta(\tilde{x}^{e}).\end{aligned}$$

Thus $\delta(\tilde{x}^{e})$ is soft closed subset of $\mathbb{R}(E)$.

Theorem (4.3):

If $\tilde{\lambda} \tilde{\in} \delta(\tilde{x}^{e})$, $|\tilde{\lambda}| \tilde{\leq} \|\tilde{x}^{e}\|$, (i.e. $\delta(\tilde{x}^{e})$ is soft bounded subset of $\mathbb{R}(E)$)

Proof:

Suppose that if possible $|\tilde{\lambda}| \tilde{\nleq} \|\tilde{x}^{e}\|$. i.e. $|\tilde{\lambda}| \tilde{>} \|\tilde{x}^{e}\| \Rightarrow \frac{\|\tilde{x}^{e}\|}{|\tilde{\lambda}|} \tilde{<} \tilde{1} \Rightarrow \left\|\frac{\tilde{x}^{e}}{\tilde{\lambda}}\right\| \tilde{<} \tilde{1} \Rightarrow \frac{\|\tilde{x}^{e}\|}{|\tilde{\lambda}|} \tilde{<} \tilde{1} \Rightarrow \left\|\tilde{u}^{e} - (\tilde{u}^{e} \simeq \frac{\tilde{x}^{e}}{\tilde{\lambda}})\right\| \tilde{<} \tilde{1}$.

This implies that $(\tilde{u}^{e} \simeq \frac{\tilde{x}^{e}}{\tilde{\lambda}})$ is soft regular. Hence $\tilde{\lambda} \tilde{\notin} \delta(\tilde{x}^{e})$, contradiction.

Theorem (4.4):

$\delta(\tilde{x}^{\varepsilon})$ is soft compact subset of $\mathbb{R}(E)$.

Proof:

From soft Heine-Borel property and theorems (4.2) and (4.3) , we get the desired.

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الانتظام الواهن و التفرد الواهن في فضاء بناخ الجبري الواهن

المستخلص:

أن الهدف الرئيسي من هذا العمل هو تقديم نوع جديد من المتجهات الخطية الواهنة مثل المتجهات المنتظمة الواهنة و المتجهات المنفردة الواهنة في فضاء بناخ الجبري الواهن و دراسة بعض خواصها . كما قدمنا خلال هذا البحث مفهوم (مجموعة الطيف الواهنة) و العلاقة بينها وبين المتجهات المنتظمة الواهنة و المتجهات المنفردة الواهنة.