On Some Special Classes of Neutrosophic Crisp Topological Spaces

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ABSTRACT

Neutrosophic crisp sets have recently gained great importance in mathematics and in all pure and exact mathematical subdisciplines due to their significant role in solving real-life problems through the modeling of mathematical systems, which has contributed to addressing many practical problems across various fields. Based on neutrosophic crisp sets, the concept of a neutrosophic crisp topological space has been introduced as a generalization of classical topology.

In this academic and scientific research on neutrosophic crisp topological spaces, five neutrosophic crisp topological spaces are constructed, namely the , the Neutrosophic Crisp Topological Space generated by a basis $(Ɲ_{U}ÇƮ_{\beta})$, the Neutrosophic Crisp semi-open ${}^{1}_{(1,2)}$Topological Space $(Ɲ_{U}ÇƮ_{ş_1})$, the Neutrosophic Crisp semi-open ${}^{2}_{(1,2)}$Topological Space $(Ɲ_{U}ÇƮ_{ş_2})$, the Neutrosophic Crisp pre-open ${}^{1}_{(1,2)}$ Topological Space$(Ɲ_{U}ÇƮ_{ƥ_1})$, and the Neutrosophic Crisp pre-open ${}^{2}_{(1,2)}$ Topological Space$(Ɲ_{U}ÇƮ_{ƥ_2})$. These four neutrosophic crisp topological spaces are constructed through new definitions of the closure of a neutrosophic crisp set and the interior of a neutrosophic crisp set. Furthermore, all classical topological theorems, results, and relations that hold in these five neutrosophic crisp topological spaces are proved, and illustrative examples are provided for those that do not hold.

1.Introduction

A. A. Salama, Valeri Kroumov and Florentin Smarandache $[1-2]$introduced the concept of Neutrosophic crisp sets and defined the neutrosophic crisp topological space. They also presented three types of neutrosophic crisp sets as follows:

Let $Ɓ^{ռ} = <Ɓ_1, Ɓ_2, Ɓ_3>$ be a neutrosophic crisp set

Type I: $Ɓ_1 \cap Ɓ_2 = \emptyset$, $Ɓ_1 \cap Ɓ_3 = \emptyset$, $Ɓ_2 \cap Ɓ_3 = \emptyset$.

Type II: $Ɓ_1 \cap Ɓ_2 = \emptyset$, $Ɓ_1 \cap Ɓ_3 = \emptyset$, $Ɓ_2 \cap Ɓ_3 = \emptyset$ *and* $Ɓ_1 \cup Ɓ_2 \cup Ɓ_3 = Ӽ$

Type III: $Ɓ_1 \cap Ɓ_2 \cap Ɓ_3 = \emptyset$, *and* $Ɓ_1 \cup Ɓ_2 \cup Ɓ_3 = Ӽ$

They also defined four types of neutrosophic crisp empty sets and four types of neutrosophic crisp universal sets as follows:

$$\emptyset_1^{ռ} =< \emptyset, \emptyset, Ӽ > \quad , \quad \emptyset_2^{ռ} =< \emptyset, Ӽ, \emptyset > \quad , \quad \emptyset_3^{ռ} =< \emptyset, Ӽ, Ӽ > \quad , \quad \emptyset_4^{ռ} =< \emptyset, \emptyset, \emptyset >.$$
$$Ӽ_1^{ռ} =< Ӽ, \emptyset, \emptyset, > \quad , \quad Ӽ_2^{ռ} =< Ӽ, Ӽ, \emptyset > \quad , \quad Ӽ_3^{ռ} =< Ӽ, \emptyset, Ӽ > \quad , \quad Ӽ_4^{ռ} =< Ӽ, Ӽ, Ӽ >.$$

In addition, they introduced three types of complements. Let $Ꜧ^{ռ} =< Ꜧ_1, Ꜧ_2, Ꜧ_3 >$ be a neutrosophic crisp set in $Ӽ$. The complements are defined as follows:

Type I : $(Ꜧ^{ռ})^{c_1} =< {Ꜧ_1}^{c}, {Ꜧ_2}^{c}, {Ꜧ_3}^{c} >$, Type II: $(Ꜧ^{ռ})^{c_2} =< Ꜧ_3, Ꜧ_2, Ꜧ_1 >$,

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Type III: $(Ħ^{n})^{c_3} =< Ħ_3, Ħ_2{}^{c}, Ħ_1 >$.

Several researchers have contributed to the development of neutrosophic crisp sets and neutrosophic crisp topological spaces by introducing new relations, results, and theorems. In [3], new concepts of neutrosophic crisp closed sets were introduced, including g-neutrosophic crisp closed sets, αg −neutrosophic crisp closed sets, gα −neutrosophic crisp closed sets, and gαg −neutrosophic crisp closed sets, together with their fundamental properties in neutrosophic crisp topological spaces. Moreover, new notions such as gαg −neutrosophic crisp closure and gαg −neutrosophic crisp interior were defined, and some of their properties were investigated.

In [4], the general approach for generating any stable neutrosophic crisp topology using a basis or the concept of stable neutrosophic crisp interior was clarified. This interior is closed under finite intersections but not under finite unions. Conversely, the stable neutrosophic crisp exterior is closed under finite unions but not under finite intersections. Necessary and sufficient conditions for both finite unions and finite intersections to be closed were obtained based on the concept of confused crisp sets.

In this work, five neutrosophic crisp topological spaces are constructed, namely: the Neutrosophic Crisp Topological Space generated by a basis ($N_U ÇT_{\beta}$), the Neutrosophic Crisp semi-open ${}^{1}_{(1,2)}$Topological Space ($N_U ÇT_{s_1}$), the Neutrosophic Crisp semi-open ${}^{2}_{(1,2)}$Topological Space ($N_U ÇT_{s_2}$), the Neutrosophic Crisp pre-open ${}^{1}_{(1,2)}$ Topological Space ($N_U ÇT_{p_1}$), and the Neutrosophic Crisp pre-open ${}^{2}_{(1,2)}$ Topological Space ($N_U ÇT_{p_2}$). All classical topological relations, results, and theorems that hold in these five neutrosophic crisp topological spaces are proved, and illustrative examples are provided for those that do not hold.

List of symbols

Symbol	Descriptions
$N_U Ç_{s}$	Neutrosophic Crisp Sets
$N_U ÇT_{(1,2)}$-space	Neutrosophic Crisp Topological Spaces
$N_U ÇÇ$–*set*	Neutrosophic Crisp Closed Set
$N_U ÇO$−*set*	Neutrosophic Crisp Open Set
$N_U ÇÇL_{(\mathbf{1,2})}$	Neutrosophic Crisp Closure
$N_U ÇLnt_{(\mathbf{1,2})}$	Neutrosophic Crisp Interior
$N_U \boldsymbol{\beta} s_{i}$	The family of all Neutrosophic Crisp semi – open ${}^{i}_{(1,2)}$, $i = 1,2$
$N_U \boldsymbol{\beta} p_{i}$	The family of all Neutrosophic Crisp pre – open ${}^{i}_{(1,2)}$, $i = 1,2$

2.Preliminaries

Definition 2.1 [5]:
Let $Ɓ^{n} = < Ɓ_1, Ɓ_2, Ɓ_3 >$ and $Ǩ^{n} =< Ǩ_1, Ǩ_2, Ǩ_3 >$ be two neutrosophic crisp sets in Ӽ. Their union is realized according to the following definition:

Type I: $Ɓ^{n} \cup_1 Ǩ^{n} = < Ɓ_1 \cup Ǩ_1, Ɓ_2 \cup Ǩ_2, Ɓ_3 \cap Ǩ_3 >$

Type II: $Ɓ^{n} \cup_2 Ǩ^{n} = < Ɓ_1 \cup Ǩ_1, Ɓ_2 \cap Ǩ_2, Ɓ_3 \cap Ǩ_3 >$

Similarly, their intersection is defined as

Type I: $Ɓ^{n} \cap_1 Ǩ^{n} = < Ɓ_1 \cap Ǩ_1, Ɓ_2 \cap Ǩ_2, Ɓ_3 \cup Ǩ_3 >$
Type II: $Ɓ^{n} \cap_2 Ǩ^{n} = < Ɓ_1 \cap Ǩ_1, Ɓ_2 \cup Ǩ_2, Ɓ_3 \cup Ǩ_3 >$

Definition 2.2 [5]:
For any two neutrosophic crisp sets $Ɓ^{n} = < Ɓ_1, Ɓ_2, Ɓ_3 >$ and $Ǩ^{n} = < Ǩ_1, Ǩ_2, Ǩ_3 >$ in Ӽ, We define the relations between the two subsets as
Type I: $Ɓ^{n} \subseteq_1 Ǩ^{n} \Leftrightarrow Ɓ_1 \subseteq Ǩ_1, Ɓ_2 \subseteq Ǩ_2, Ǩ_3 \subseteq Ɓ_3$
Type II: $Ɓ^{n} \subseteq_2 Ɓ^{n} \Leftrightarrow Ɓ_1 \subseteq Ǩ_1, Ǩ_2 \subseteq Ɓ_2, Ǩ_3 \subseteq Ɓ_3$

Theorem 2.3 [1]:

Consider $\{Ɓ_j^{n} : j \in J\}$ as a collection of neutrosophic crisp subsets within universe in Ӽ. Then

1. The intersection $\cap_{j\in J} Ɓ_j^{n}$ The following two definitions can be used:
   i. $\bigcap_{1j} Ɓ_j^{n} = <\cap Ɓ_{j1}, \cap Ɓ_{j2}, \cup Ɓ_{j3} >$ ii. $\bigcap_{2j} Ɓ_j^{n} = <\cap Ɓ_{j1}, \cup Ɓ_{j2}, \cup Ɓ_{j3} >$
2. The union $\cup_{j\in J} Ɓ_j^{n}$ The following two definitions can be used:
   i. $\bigcup_{1j} Ɓ_j^{n} = <\cup Ɓ_{j1}, \cup Ɓ_{j2}, \cap Ɓ_{j3} >$ ii. $\bigcup_{2j} Ɓ_j^{n} = <\cup Ɓ_{j1}, \cap Ɓ_{j2}, \cap Ɓ_{j3} >$

Theorem 2.4 [5]:

Consider $Ɓ^{n}$ and $Ǩ^{n}$ as two neutrosophic crisp sets of arbitrary type within the universe Ӽ. Then

i. $(Ɓ^{n} \cap_1 Ǩ^{n})^{c_2} = (Ɓ^{n})^{c_2} \cup_2 \left(Ǩ^{n}\right)^{c_2}$, $(Ɓ^{n} \cup_2 Ǩ^{n})^{c_2} = (Ɓ^{n})^{c_2} \cap_1 (Ǩ^{n})^{c_2}$
ii. $(Ɓ^{n} \cap_1 Ǩ^{n})^{c_2} = (Ɓ^{n})^{c_2} \cup_2 \left(Ǩ^{n}\right)^{c_2}$, $(Ɓ^{n} \cup_2 Ǩ^{n})^{c_2} = (Ɓ^{n})^{c_2} \cap_1 (Ǩ^{n})^{c_2}$
iii. $(Ɓ^{n} \subseteq_1 Ǩ^{n})^{c_2} = \left(Ǩ^{n}\right)^{c_2} \subseteq_2 (Ɓ^{n})^{c_2}$, $(Ɓ^{n} \subseteq_2 Ǩ^{n})^{c_2} = (Ǩ^{n})^{c_2} \subseteq_1 (Ɓ^{n})^{c_2}$

Definition 2.5 [6]:
We say that (Ӽ, Ʈ) forms a Neutrosophic Crisp Topological ($Ɲ_UÇƮ_{(1,2)}$-space) when the following criteria hold:

i. $\emptyset_1^{n} = < \emptyset, \emptyset, Ӽ > \in Ʈ$, and $Ӽ_1^{n} = < Ӽ, \emptyset, \emptyset > \in Ʈ$.
ii. For any $\mathcal{B}^{n}, Ǩ^{n} \in Ʈ$, the set $\mathcal{B}^{n} \cap_1 Ǩ^{n} \in Ʈ$.
iii. If $\mathcal{B}^{n}{}_j \in Ʈ\ \forall j \in J$, $then \cup_{j2} \mathcal{B}^{n}{}_j \in Ʈ$.

For any $\mathcal{B}^{n} \in Ʈ$, it is classified as a Neutrosophic crisp open set ($Ɲ_U$ÇO$-set$), and $(\mathcal{B}^{n})^{c_2}$ is a Neutrosophic crisp closed set ($Ɲ_U$ÇÇ$-set$).

Remarks 2.6:

i. A $Ɲ_UÇƮ^1_{(1,2)}$–space is a topological space equipped with Type I neutrosophic crisp sets.
ii. A $Ɲ_UÇƮ^2_{(1,2)}$–space is a topological space equipped with Type II neutrosophic crisp sets.
iii. A $Ɲ_UÇƮ^3_{(1,2)}$–space is a topological space equipped with Type III neutrosophic crisp sets.

Example 2.7:
Let Ӽ = $\{a, b, c, d\}$, and Ʈ = $\{\emptyset_1^{n}, Ӽ_1^{n}, \mathcal{B}^{n}, Ǩ^{n}, \mathcal{M}^{n}\}$, $\mathcal{B}^{n} = < \{a\}, \{d\}, \{c\} >$, $Ǩ^{n} = < \{a\}, \{b\}, \{c\} >$, $\mathcal{M}^{n} = < \{a\}, \emptyset, \{c\} >$ are $Ɲ_UÇ_ș$ from Type I. Therefore, (Ӽ, Ʈ) is a $Ɲ_UÇƮ^1_{(1,2)}$ –space.

Remark 2.8:
Let $\mathcal{B}^{n}{}_{j}$ be a family of $N_U ÇO - set$ in a $N_U ÇT^2_{(1,2)}$–space. Then for each j, the set $\mathcal{B}^{n}{}_{j}$ is one of the following forms: $\mathcal{B}^{n}{}_{j} =< \mathcal{B}_{1j}, \emptyset, \mathcal{B}^c_{1j} >$ or $\mathcal{B}^{n}{}_{j} =< \mathcal{B}^c_{3j}, \emptyset, \mathcal{B}_{3j} >$
Proof
Let $\mathcal{B}^{n}{}_{j} =< \mathcal{B}_{1j}, \mathcal{B}_{2j}, \mathcal{B}_{3j} > \neq < \mathcal{B}_{1j}, \emptyset, \mathcal{B}^c_{1j} >$.
Since, $X^{n}_1 = < X, \emptyset, \emptyset > \in T$.
So, $\mathcal{B}^{n}{}_{j} \cap_1 X^{n}_1 = < \mathcal{B}_{1j}, \emptyset, \mathcal{B}_{3j} >. \mathcal{B}_{1j} \cup_j \mathcal{B}_{3j} = X \; \forall$ j.
However, this is not necessarily true under our assumption (since, $\mathcal{B}_{3j} \neq \mathcal{B}^c_{1j}$)
This contradiction that fact that $\mathcal{B}^{n}{}_{j}$ is a $N_U ÇO - set$ and should preserve the required structure intersection.
Therefore, it must be that $\mathcal{B}_{3j} = \mathcal{B}^c_{1j}$, $\mathcal{B}^{n}{}_{j} = < \mathcal{B}_{1j}, \emptyset, \mathcal{B}^c_{1j} >$.

Corollary 2.9 [6]:
Let (X, T) be a $N_U ÇT_{(1,2)}$ –space, and let Ψ = { Ӈ: Ӈ is $N_U ÇÇ-set$} satisfy the following conditions: i. If $\check{K}^{n}{}_{j} \in \Psi \;\; \forall j \in J$, $then$ $\cap_{1j} \check{K}^{n}{}_{j} \in \Psi$. ii. If $\mathcal{B}^{n}$, $\check{K}^{n} \in \Psi$,then $\mathcal{B}^{n} \cup_2 \check{K}^{n} \in \Psi$.

3. Closure of Neutrosophic Crisp Sets $(N_U ÇÇL_{(1,2)})$

This section presents a new type of closure of neutrosophic crisp sets, and a comprehensive study of its properties in relation to classical topology is conducted. Most properties of classical topology do not apply to all neutrosophic crisp sets.

Definition 3.1:
Let (X, T) be a $N_U ÇT_{(1,2)}$–space, then the closure of neutrosophic crisp set $Ʉ^{n} = < Ʉ_1, Ʉ_2, Ʉ_3 >$, denoted by $N_U ÇÇL_{(1,2)}(Ʉ^{n})$, and $N_U ÇÇL_{(1,2)}(Ʉ^{n}) = \{\cap_{1j} Ӈ^{n}{}_{j} : Ӈ^{n}{}_{j} =< Ӈ_{1j}, Ӈ_{2j}, Ӈ_{3j} >\} \ni Ӈ^{n}{}_{j}$ are $N_U ÇÇ-sets$ $\forall j$ and $Ʉ_1 \subseteq (Ӈ_{3j})^c \; \forall j$, $Ʉ_2 \subseteq (Ӈ_{2j})^c \; \forall j$, $Ʉ_3{}^c \subseteq Ӈ_{1j} \; \forall j$.

- From definition $N_U ÇÇL_{(1,2)}(Ʉ^{n})$ is a $N_U ÇÇ-set$ (by corollary 2.9).
- Always $N_U ÇÇL_{(1,2)}(Ʉ^{n}) = \cap_{1j} Ӈ^{n}{}_{j} = <\cap_j Ӈ_{j1}, \emptyset, \cup_j Ӈ_{j3} >$. Because X^{n}_1 $is\ a$ $N_U ÇÇ-set$ and satisfies the conditions in definition 3.1.

Example 3.2:
Let X = $\{a, b, c, d\}$, and T = $\{\emptyset^{n}_1, X^{n}_1, Ɓ^{n}, Ƙ^{n}, \mathcal{M}^{n}\}$, $Ɓ^{n}$ = < $\{a\}$, $\{d\}$, $\{c\}$ >, $Ƙ^{n}$ = < $\{a\}$, $\{b\}$, $\{c\}$>, $\mathcal{M}^{n}$ = < $\{a\}$, ∅, $\{c\}$ > be $N_U Ç_{\varsigma}$ from Type I. Then (X, T) is a $N_U ÇT^1_{(1,2)}$–space. Therefore:

i. $N_U ÇÇL_{(1,2)}(Ɓ^{n} = <$ {a},{d},{c} >) $= X^{n}_1 = < X, \emptyset, \emptyset >$.
ii. $N_U ÇÇL_{(1,2)}$ (A^{n} = < {b}, ∅, ∅ >) = $X^{n}_1 = < X, \emptyset, \emptyset >$.
iii. $N_U ÇÇL_{(1,2)}$ ($\mathcal{M}^{n C_2}$ = < {c}, ∅, {a} >) = $X^{n}_1 = < X, \emptyset, \emptyset >$.
iv. $N_U ÇÇL_{(1,2)}$ ($\mathcal{M}^{n}$ = < $\{a\}$, ∅, $\{c\}$>) = $X^{n}_1 = < X, \emptyset, \emptyset >$.
v. $N_U ÇÇL_{(1,2)}$ ($Ƙ^{n C_2}$ = < {c}, {b}, {a} >) = $X^{n}_1 = < X, \emptyset, \emptyset >$.
vi. $N_U ÇÇL_{(1,2)}$ (T^{n} = < $\{b\}, \{c\}, \{a, b, d\}$>) = $\mathcal{M}^{n C_2}$ = < {c}, ∅, {a} >.
vii. $N_U ÇÇL_{(1,2)}$ ($\emptyset^{n}_1$ = <∅, ∅, X >) = $\emptyset^{n}_1$ = < ∅, ∅, X>.
viii. $N_U ÇÇL_{(1,2)}$ (X^{n}_1 = < X, ∅, ∅ >) = $X^{n}_1 = < X, \emptyset, \emptyset >$.
ix. $N_U ÇÇL_{(1,2)}$ ($\mathcal{L}^{n}$ = < ∅, $\{b\}, \{a\}$ >) = $X^{n}_1 = < X, \emptyset, \emptyset >$.

Example 3.3:
Let X = $\{a, b, c\}$, and T = $\{\emptyset^{n}_1, X^{n}_1, A^{n}, B^{n}, C^{n}\}$, A^{n} = < $\{a$, b$\}$, ∅, $\{c\}$ >, B^{n} = < $\{a$, c$\}$, ∅, {b} >,

$C^{ɳ}$ = < {a}, ∅, {b, c} > be Ɲ$_U$Ç$_ş$ from type II. Then (Ӽ, Ʈ) is a Ɲ$_U$ÇƮ$^2_{(1,2)}$–space. Therefore:

i. Ɲ$_U$ÇÇL$_{(1,2)}$($A^{ɳ}$ = < {a, b}, ∅, {c} >) = Ӽ$^{ɳ}_1$ = < Ӽ, ∅, ∅ >.
ii. Ɲ$_U$ÇÇL$_{(1,2)}$ ($O^{ɳ}$ = < {b}, ∅, ∅>) = Ӽ$^{ɳ}_1$ = < Ӽ, ∅, ∅ >.
iii. Ɲ$_U$ÇÇL$_{(1,2)}$ ($C^{ɳ^{C_2}}$ = < {b, c}, ∅, {a} >) = $C^{ɳ^{C_2}}$ = < {b, c}, ∅, {a} >.
iv. Ɲ$_U$ÇÇL$_{(1,2)}$ ($C^{ɳ}$ = < {a}, ∅, {b, c} >) = Ӽ$^{ɳ}_1$ = < Ӽ, ∅, ∅ >.
v. Ɲ$_U$ÇÇL$_{(1,2)}$ ($B^{ɳ^{C_2}}$ = < {b}, ∅, {a, c}>) = $B^{ɳ^{C_2}}$ = < {b}, ∅, {a, c}>.
vi. Ɲ$_U$ÇÇL$_{(1,2)}$ ($U^{ɳ}$ = < {c}, {b}, {a}>) = $C^{ɳ^{C_2}}$ = < {b, c}, ∅, {a} >.
vii. Ɲ$_U$ÇÇL$_{(1,2)}$ ($A^{ɳ^{C_2}}$ = < {c}, ∅, {a, b} >) = $A^{ɳ^{C_2}}$ =< {c}, ∅, {a, b} >.
viii. Ɲ$_U$ÇÇL$_{(1,2)}$ (∅$^{ɳ}_1$ = <∅, ∅, Ӽ >) = ∅$^{ɳ}_1$ = < ∅, ∅, Ӽ >.
ix. Ɲ$_U$ÇÇL$_{(1,2)}$ (Ӽ$^{ɳ}_1$ = < Ӽ, ∅, ∅ >) = Ӽ$^{ɳ}_1$ = < Ӽ, ∅, ∅ >.

Examples 3.4:
Let Ӽ = {a, b, c}, and Ʈ = {∅$^{ɳ}_1$, Ӽ$^{ɳ}_1$, $Q^{ɳ}$, $W^{ɳ}$, $E^{ɳ}$, $\mathcal{R}^{ɳ}$}, $Q^{ɳ}$ = < {a, b}, {c}, Ӽ >, $W^{ɳ}$ = < Ӽ, {b}, {c}>, $E^{ɳ}$ = < {a, b}, ∅, Ӽ >, $\mathcal{R}^{ɳ}$ = < Ӽ, ∅, {c} > be Ɲ$_U$Ç$_ş$ from type III. Then (Ӽ, Ʈ) is a Ɲ$_U$ÇƮ$^3_{(1,2)}$–space. Therefore:

i. Ɲ$_U$ÇÇL$_{(1,2)}$($Q^{ɳ}$ = < {a, b}, {c}, Ӽ >) = Ӽ$^{ɳ}_1$ = < Ӽ, ∅, ∅ >.
ii. Ɲ$_U$ÇÇL$_{(1,2)}$ ($A^{ɳ}$ = < {b}, ∅, ∅>) = Ӽ$^{ɳ}_1$ = < Ӽ, ∅, ∅ >.
iii. Ɲ$_U$ÇÇL$_{(1,2)}$ ($E^{ɳ^{C_2}}$ = < Ӽ, ∅, {a, b} >) = Ӽ$^{ɳ}_1$ = < Ӽ, ∅, ∅ >.
iv. Ɲ$_U$ÇÇL$_{(1,2)}$ ($E^{ɳ}$ = < {a, b}, ∅, Ӽ >) = Ӽ$^{ɳ}_1$ = < Ӽ, ∅, ∅ >.
v. Ɲ$_U$ÇÇL$_{(1,2)}$ ($W^{ɳ^{C_2}}$ = < {c}, {b}, Ӽ > = $E^{ɳ^{C_2}}$ = < Ӽ, ∅, {a, b} >.
vi. Ɲ$_U$ÇÇL$_{(1,2)}$ ($J^{ɳ}$ = < {c}, {b}, {a, c}>) = $E^{ɳ^{C_2}}$ =< Ӽ, ∅, {a, b} >.
vii. Ɲ$_U$ÇÇL$_{(1,2)}$ ($Q^{ɳ^{C_2}}$ = < Ӽ, {c},{a, b} >) = Ӽ$^{ɳ}_1$ = < Ӽ, ∅, ∅ >.
viii. Ɲ$_U$ÇÇL$_{(1,2)}$ ($D^{ɳ}$= <{b}, {c}, Ӽ >) = Ӽ$^{ɳ}_1$ = < Ӽ, ∅, ∅ >.
ix. Ɲ$_U$ÇÇL$_{(1,2)}$ (∅$^{ɳ}_1$ = <∅, ∅, Ӽ >) = ∅$^{ɳ}_1$ = < ∅, ∅, Ӽ >.
x. Ɲ$_U$ÇÇL$_{(1,2)}$ (Ӽ$^{ɳ}_1$ = < Ӽ, ∅, ∅ >) = Ӽ$^{ɳ}_1$ = < Ӽ, ∅, ∅ >.

Corollary 3.5:
Let (Ӽ, Ʈ) be a Ɲ$_U$ÇƮ$^t_{(1,2)}$–space, t=1,2,3

i. Ɲ$_U$ÇÇL$_{(1,2)}$(Ӽ$^{ɳ}_1$) = Ӽ$^{ɳ}_1$ = < Ӽ, ∅, ∅ >   ii. Ɲ$_U$ÇÇL$_{(1,2)}$(∅$^{ɳ}_1$) = ∅$^{ɳ}_1$ = < ∅, ∅, Ӽ>

proof (i)
Let {Ħ$^{ɳ}{}_j$ = < Ħ$_{1j}$, Ħ$_{2j}$, Ħ$_{3j}$ >: $j \in J$} be the family of Ɲ$_U$ÇÇ – set such that
Ӽ ⊆ (Ħ$_{j3}$)^c ∀ j,   ∅ ⊆ (Ħ$_{j2}$)^c ∀ j,   ∅^{c} ⊆ Ħ$_{j1}$∀ j.
Thus, Ɲ$_U$ÇÇL$_{(1,2)}$(Ӽ$^{ɳ}_1$) = ∩$_{1j}$ Ħ$^{ɳ}{}_j$ =<∩$_j$ Ħ$_{1j}$, ∩$_j$ Ħ$_{2j}$, ∪$_j$ Ħ$_{3j}$ >.
Since Ӽ ⊆ (Ħ$_{3j}$)^c. So, Ħ$_{3j}$ ⊆ ∅. Therefore. Ħ$_{3j}$ = ∅ ∀ j. Hence, ∪$_j$ Ħ$_{3j}$ = ∅.

Since Ӽ$^{ɳ}_1$ is a Ɲ$_U$ÇÇ–set. Therefore, ∩$_j$ Ħ$_{2j}$ = ∅.

Since ∅^{c} ⊆ Ħ$_{1j}$. Thus, Ӽ ⊆ Ħ$_{1j}$ ∀ j. Therefore, Ӽ ⊆ ∩$_j$ Ħ$_{1j}$ and ∩$_j$ Ħ$_{1j}$ ⊆ Ӽ. So, Ӽ =∩$_j$ Ħ$_{1j}$.

Hence, Ɲ$_U$ÇÇL$_{(1,2)}$(Ӽ$^{ɳ}_1$) = Ӽ$^{ɳ}_1$.

The proof of (ii) can be carried out in a manner analogous to the proof of (i).

Theorem 3.6:

Let (Ӽ, Ʈ) be a ƝᵤÇƮ$^{t}_{(1,2)}$–space, t=1,2,3. If $A^{ƞ} =< A_1, A_2, A_3 > \subseteq_i O^{ƞ} =< O_1, O_2, O_3 >, i = 1,2$.

Then ƝᵤÇÇĿ$_{(\mathbf{1,2})}(A^{ƞ}) \subseteq_i$ ƝᵤÇÇĿ$_{(\mathbf{1,2})}(O^{ƞ})$, i=1,2, ∋ $A^{ƞ}$, $O^{ƞ}$ are any ƝᵤÇ$_{ş}$.

Proof:

Suppose ƝᵤÇÇĿ$_{(\mathbf{1,2})}(A^{ƞ}) = \bigcap_{1j} \mathcal{F}^{ƞ}_{j} = <\cap_j \mathcal{F}_{1j}, \cap_j \mathcal{F}_{2j}, \cup_j \mathcal{F}_{3j} > \ni \mathcal{F}^{ƞ}_{j}$ are ƝᵤÇÇ–$sets$ $\forall j$, and
$A_1 \subseteq (\mathcal{F}_{3j})^c \ \forall j, \quad A_2 \subseteq (\mathcal{F}_{2j})^c \ \forall j, \quad {A_3}^c \subseteq \mathcal{F}_{j1} \forall j.$
Also let ƝᵤÇÇĿ$_{(\mathbf{1,2})}(O^{ƞ}) = \bigcap_{1j} \mathcal{H}^{ƞ}_{j} =<\cap_j \mathcal{H}_{1j}, \cap_j \mathcal{H}_{2j}, \cup_j \mathcal{H}_{3j} >\ni \mathcal{H}^{ƞ}_{j}$ are ƝᵤÇÇ–$sets$ $\forall j$,
$O_1 \subseteq (\mathcal{H}_{3j})^c \ \forall j, \quad O_2 \subseteq (\mathcal{H}_{2j})^c \ \forall j, \quad {O_3}^c \subseteq \mathcal{H}_{1j} \ \forall j.$
Since $O_1 \subseteq (\mathcal{H}_{3j})^c \ \forall j$, $A_1 \subseteq O_1$, and $A_1 \subseteq (\mathcal{F}_{3j})^c \ \forall j$. Thus, $\{{\mathcal{H}_{3j}}^c\} \subseteq \{{\mathcal{F}_{3j}}^c\}$.
Therefore, $\{\mathcal{H}_{3j}\} \subseteq \{\mathcal{F}_{3j}\}$. Hence, $\cup_j \mathcal{H}_{j3} \subseteq \cup_j \mathcal{F}_{j3}$. ….(1)
Since $\cap_j \mathcal{F}_{2j} = \cap_j \mathcal{H}_{2j} = \emptyset$. Hence, $\cap_j \mathcal{F}_{2j} \subseteq \cap_j \mathcal{H}_{2j}$ and $\cap_j \mathcal{H}_{2j} \subseteq \cap_j \mathcal{F}_{2j}$. ….(2)

Since ${O_3}^c \subseteq \mathcal{H}_{1j} \ \forall j$, ${A_3}^c \subseteq {O_3}^c$, and ${A_3}^c \subseteq \mathcal{F}_{1j} \forall j$. Therefore, $\{\mathcal{H}_{1j}\} \subseteq \{\mathcal{F}_{1j}\}$.

Hence, $\cap_j \mathcal{F}_{1j} \subseteq \cap_j \mathcal{H}_{1j}$ ...(3)

From (1) , (2) and (3) we get ƝᵤÇÇĿ$_{(\mathbf{1,2})}(A^{ƞ}) \subseteq_i$ ƝᵤÇÇĿ$_{(\mathbf{1,2})}(O^{ƞ})$, $i = 1,2$

Theorem 3.7:

Let (Ӽ, Ʈ) be a ƝᵤÇƮ$^{2}_{(1,2)}$–space and $Ʉ^{ƞ} = < Ʉ_1, Ʉ_2, Ʉ_3 >$ be any ƝᵤÇ$_{ş}$. Then $Ʉ^{ƞ} \subseteq_2$ ƝᵤÇÇĿ$_{(\mathbf{1,2})}(Ʉ^{ƞ})$.

Proof : Let $\{Ƕ^{ƞ}_{j}=< Ƕ_{1j}, Ƕ_{2j}, Ƕ_{3j}>: j \in J\}$ be the family of ƝᵤÇÇ–sets such that
$Ʉ_1 \subseteq (Ƕ_{3j})^c \ \forall$ j, $\quad Ʉ_2 \subseteq (Ƕ_{2j})^c \ \forall$ j, $\quad {Ʉ_3}^c \subseteq Ƕ_{1j} \ \forall$ j $\in$ J.
Thus,
ƝᵤÇÇĿ$_{(\mathbf{1,2})}$ $(Ʉ^{ƞ}) = \bigcap_{1j} Ƕ^{ƞ}_{j} = <\cap_j Ƕ_{1j}, \cap_j Ƕ_{2j}, \cup_j Ƕ_{3j} >=<\cap_j Ƕ_{1j}, \emptyset, \cup_j (Ƕ_{1j})^c >$. By remark 2.8
$Ƕ^{ƞ}_{j}=< Ƕ_{1j}, Ƕ_{2j}, Ƕ_{3j}> = < Ƕ_{1j},\emptyset , {Ƕ_{1j}}^c> \forall j \in J$ (by Remark 2.8)
Since $Ʉ_1 \subseteq (Ƕ_{3j})^c \ \forall$ j.

Therefore, $Ʉ_1 \subseteq Ƕ_{1j} \ \forall j$. Hence, $Ʉ_1 \subseteq \cap_j Ƕ_{1j} \ \forall j$. Also, it's clear that $Ʉ_2 \supseteq \emptyset$.

Since ${Ʉ_3}^c \subseteq Ƕ_{1j} \forall$j and $Ʉ_3 \supseteq {Ƕ_{1j}}^c \ \forall j$. Therefore, $Ʉ_3 \supseteq \cup_j (Ƕ_{1j})^c$.

Hence, $Ʉ^{ƞ} \subseteq_2$ ƝᵤÇÇĿ$_{(\mathbf{1,2})}(Ʉ^{ƞ})$.

Remarks 3.8:

- Replacing the type-II inclusion with the Type-I inclusion causes Theorem 3.7 to fail, as demonstrated in Example 3.3, where

  $U^{ƞ} = < \{c\}, \{b\}, \{a\} > \not\subseteq_1$ ƝᵤÇÇĿ$_{(\mathbf{1,2})}$ $(U^{ƞ}) = C^{ƞ^{C_2}}$ = < {b,c}, ∅, {a} >.

- Substituting ƝᵤÇƮ$^{2}_{(1,2)}$–space with the ƝᵤÇƮ$^{1}_{(1,2)}$–space causes Theorem 3.7 to fail, as shown in Example 3.2, where $T^{ƞ} = < \{b\}, \{c\}, \{a, b\} > \not\subseteq_i$ ƝᵤÇÇĿ$_{(\mathbf{1,2})}$ $(T^{ƞ}) = \mathcal{M}^{ƞ^{C_2}} =< \{c\}, \emptyset, \{a\} >$, $i = 1,2$.

- Substituting $\mathcal{N}_U ÇT^2_{(1,2)}$–space with the $\mathcal{N}_U ÇT^3_{(1,2)}$–space causes Theorem 3.7 to fail, as shown in Example 3.4, where $J^{n} = < \{c\}, \{b\}, \{a, c\} > \not\subseteq_i \mathcal{N}_U ÇÇL_{(1,2)}(J^{n}) = E^{n^{C_2}} = < X, \emptyset, \{a, b\} >, i = 1,2$.

Corollary 3.9 : Let (X, T) be a $\mathcal{N}_U ÇT^2_{(1,2)}$–space, and $\mho^{n}$ be any $\mathcal{N}_U Ç_{ş}$. Then

i. $\mho^{n} \cap_1 \mathcal{N}_U ÇÇL_{(1,2)}(\mho^{n}) = \mho^{n}$

ii. $\mho^{n} \cup_2 \mathcal{N}_U ÇÇL_{(1,2)}(\mho^{n}) = \mathcal{N}_U ÇÇL_{(1,2)}(\mho^{n})$

Proof: It's clear by Theorem 3.7.

Theorem 3.10:
Let (X, T) be a $\mathcal{N}_U ÇT^2_{(1,2)}$–space. Then $\check{K}^{n} = < \check{K}_1, \check{K}_2, \check{K}_3 >$ is a $\mathcal{N}_U ÇÇ$– set if and only if $\mathcal{N}_U ÇÇL_{(1,2)}(\check{K}^{n}) = \check{K}^{n}$.

Proof: If $\mathcal{N}_U ÇÇL_{(1,2)}(\check{K}^{n}) = \check{K}^{n}$, then $\check{K}^{n}$ is a $\mathcal{N}_U ÇÇ$– set by Definition 3.1.

Conversely,
let $\{Ħ^{n}_j = < Ħ_{1j}, Ħ_{2j}, Ħ_{3j} > : j \in J\}$ be the family of $\mathcal{N}_U ÇÇ$ – set such that

$\check{K}_1 \subseteq (Ħ_{3j})^c\ \forall$ j, $\check{K}_2 \subseteq (Ħ_{2j})^c\ \forall$ j, ${\check{K}_3}^c \subseteq Ħ_{1j} \forall$ j $\in$ J.

Thus,

$\mathcal{N}_U ÇÇL_{(1,2)}(\check{K}^{n}) = \cap_{1j} Ħ^{n}_j = <\cap_j Ħ_{1j}, \cap_j Ħ_{2j}, \cup_j Ħ_{3j} > = <\cap_j Ħ_{1j}, \emptyset, \cup_j (Ħ_{1j})^c >$ by Remark 2.8.

Since $\check{K}_1 \subseteq (Ħ_{3j})^c$. So, $\check{K}_1 \subseteq Ħ_{1j}$.

Therefore, $\check{K}_1 \subseteq \cap Ħ_{j1}\ \forall$ j $\in$ J, and $\check{K}_1 \supseteq \cap Ħ_{1j}\ \forall$ j $\in$ J.

Hence, $\check{K}_1 = \cap Ħ_{1j}\ \forall$ j $\in$ J. Since $\check{K}_2 = Ħ_{2j} = \emptyset$ .Hence, $\check{K}_2 = \cap Ħ_{2j} \forall$ j $\in$ J ($\check{K}^{n}$ is $\mathcal{N}_U ÇÇ$ – set).

Since ${\check{K}_3}^c \subseteq Ħ_{1j}$. So, ${\check{K}_3}^c \subseteq (Ħ_{3j})^c$.Thus, $\check{K}_3 \supseteq Ħ_{3j}\ \forall$ j $\in$ J.

So, $\check{K}_3 \supseteq \cup Ħ_{3j}\ \forall$ j $\in$ J and $\check{K}_3 \subseteq \cup Ħ_{3j}\ \forall$ j $\in$ J .

Therefore, $\check{K}_3 = \cup Ħ_{3j}\ \forall$ j $\in$ J. Hence, $\mathcal{N}_U ÇÇL_{(1,2)}(\check{K}^{n}) = \check{K}^{n}$.

Remark 3.11:

- Substituting $\mathcal{N}_U ÇT^2_{(1,2)}$–space with the $\mathcal{N}_U ÇT^1_{(1,2)}$–space causes Theorem 3.10 to fail, as shown in Example 3.2, where $\mathcal{N}_U ÇÇL_{(1,2)}(\mathcal{M}^{n^{C_2}}) = X^{n}_1 \neq \mathcal{M}^{n^{C_2}}$.
- Substituting $\mathcal{N}_U ÇT^2_{(1,2)}$–space with the $\mathcal{N}_U ÇT^3_{(1,2)}$–space causes Theorem 3.10 to fail, as shown in Example 3.4, where , $\mathcal{N}_U ÇÇL_{(1,2)}(W^{n^{C_2}}) = E^{n^{C_2}} \neq W^{n^{C_2}}$.

Corollary 3.12:

Let (X, T) be a $\mathcal{N}_U ÇT^2_{(1,2)}$–space and let $\mho^{n}$ be any $\mathcal{N}_U Ç_{ş}$.

Then $\mathcal{N}_U ÇÇL_{(1,2)}[\mathcal{N}_U ÇÇL_{(1,2)}(\mho^{n})] = \mathcal{N}_U ÇÇL_{(1,2)}(\mho^{n})$

Proof: It's clear by Theorem 3.10.

Remark 3.13:

- Substituting $\mathrm{N_U ÇT^2_{(1,2)}}$–space with the $\mathrm{N_U ÇT^1_{(1,2)}}$–space causes Corollary 3.12 to fail, as shown in Example 3.2, where $\mathrm{N_U ÇÇL_{(1,2)}}[\mathrm{N_U ÇÇL_{(1,2)}}(T^{n} = < \{b\},\{c\},\{a,b,d\} >)] = X_1^{n} \neq \mathrm{N_U ÇÇL_{(1,2)}}(T^{n})$.
- Substituting $\mathrm{N_U ÇT^2_{(1,2)}}$–space with the $\mathrm{N_U ÇT^3_{(1,2)}}$–space causes Corollary 3.12 to fail, as shown in Example 3.4, where , $\mathrm{N_U ÇÇL_{(1,2)}}[\mathrm{N_U ÇÇL_{(1,2)}}(J^{n} = < \{c\},\{b\}, \{a,c\} >)] = X_1^{n} \neq \mathrm{N_U ÇÇL_{(1,2)}}(J^{n})$.

Theorem 4.14:

Let (X, T) be a $\mathrm{N_U ÇT^t_{(1,2)}}$–space, $t = 1,2,3$, and $\mathrm{U}^{n} = < \mathrm{U}_1, \mathrm{U}_1, \mathrm{U}_1 >$, $\mathfrak{R}^{n} = <\mathfrak{R}_1, \mathfrak{R}_2, \mathfrak{R}_3>$ be any $\mathrm{N_U Ç_{ş}}$. Then $\mathrm{N_U ÇÇL_{(1,2)}}[\mathrm{U}^{n} \cup_2 \mathfrak{R}^{n}] = \mathrm{N_U ÇÇL_{(1,2)}}(\mathrm{U}^{n}) \cup_2 \mathrm{N_U ÇÇL_{(1,2)}}(\mathfrak{R}^{n})$.

Proof: By definition, $\mathrm{U}^{n} \cup_2 \mathfrak{R}^{n} = < \mathrm{U}_1 \cup \mathfrak{R}_1, \mathrm{U}_2 \cap \mathfrak{R}_2, \mathrm{U}_3 \cap \mathfrak{R}_3 >$.
Since $\mathrm{U}^{n} \subseteq_2 \mathrm{U}^{n} \cup_2 \mathfrak{R}^{n}$. So, $\mathrm{N_U ÇÇL_{(1,2)}}(\mathrm{U}^{n}) \subseteq_i \mathrm{N_U ÇÇL_{(1,2)}}[\mathrm{U}^{n} \cup_2 \mathfrak{R}^{n}]$, $i = 1,2$.
Since $\mathfrak{R}^{n} \subseteq_2 \mathrm{U}^{n} \cup_2 \mathfrak{R}^{n}$. Thus, $\mathrm{N_U ÇÇL_{(1,2)}}(\mathfrak{R}^{n}) \subseteq_i \mathrm{N_U ÇÇL_{(1,2)}}[\mathrm{U}^{n} \cup_2 \mathfrak{R}^{n}]$, $i = 1,2$.
Therefore, $\mathrm{N_U ÇÇL_{(1,2)}}(\mathrm{U}^{n}) \cup_2 \mathrm{N_U ÇÇL_{(1,2)}}(\mathfrak{R}^{n}) \subseteq_i \mathrm{N_U ÇÇL_{(1,2)}}[\mathrm{U}^{n} \cup_2 \mathfrak{R}^{n}]$, $i = 1,2$ ... (i)
Now, $\mathrm{N_U ÇÇL_{(1,2)}}[\mathrm{U}^{n} \cup_2 \mathfrak{R}^{n}] = \cap_{1j} \mathcal{J}^{n}{}_j = <\cap_j \mathcal{J}_{1j}, \emptyset, \cup_j \mathcal{J}_{3j} >$ and
$\{\mathcal{J}^{n}{}_j = < \mathcal{J}_{1j}, \mathcal{J}_{2j}, \mathcal{J}_{3j}> : j \in J\}$ be the family of $\mathrm{N_U ÇÇ} - set$ such that
$\mathrm{U}_1 \cup \mathfrak{R}_1 \subseteq (\mathcal{J}_{3j})^c\ \forall j \in J$, $\mathrm{U}_2 \cap \mathfrak{R}_2 \subseteq \mathcal{J}_{j2}{}^c\ \forall j \in J$, $(\mathrm{U}_3 \cap \mathfrak{R}_3)^c \subseteq \mathcal{J}_{1j}\ \forall j \in J$.
Let $\{\mathcal{H}^{n}{}_j = < \mathcal{H}_{1j}, \mathcal{H}_{2j}, \mathcal{H}_{3j}> : j \in J\}$ be the family of $\mathrm{N_U ÇÇ} - set$
such that $\mathrm{U}_1 \subseteq (\mathcal{H}_{3j})^c\ \forall j \in J$, $\mathrm{U}_2 \subseteq (\mathcal{H}_{2j})^c\ \forall j \in J$, $\mathrm{U}_3{}^c \subseteq \mathcal{H}_{1j} \forall j \in J$.
So, $\mathrm{N_U ÇÇL_{(1,2)}}(\mathrm{U}^{n}) = \cap_{1j} \mathcal{H}^{n}{}_j = <\cap_j \mathcal{H}_{1j}, \emptyset, \cup_j \mathcal{H}_{3j} >$ and let $\{\mathcal{L}^{n}{}_j = < \mathcal{L}_{1j}, \mathcal{L}_{2j}, \mathcal{L}_{3j}> : j \in J\}$ be the family of $\mathrm{N_U ÇÇ} - set$ such that
$\mathfrak{R}_1 \subseteq (\mathcal{L}_{3j})^c\ \forall j \in J$, $\mathfrak{R}_2 \subseteq (\mathcal{L}_{2j})^c\ \forall j \in J$, $\mathfrak{R}_3{}^c \subseteq \mathcal{L}_{1j}\ \forall j \in J$.
Thus, $\mathrm{N_U ÇÇL_{(1,2)}}(\mathfrak{R}^{n}) = \cap_{1j} \mathcal{L}^{n}{}_j = <\cap_j \mathcal{L}_{1j}, \emptyset, \cup_j \mathcal{L}_{3j} >$.
Hence, $\mathrm{N_U ÇÇL_{(1,2)}}(\mathrm{U}^{n}) \cup_2 \mathrm{N_U ÇÇL_{(1,2)}}(\mathfrak{R}^{n}) = < (\cap_j \mathcal{H}_{1j}) \cup (\cap_j \mathcal{L}_{1j}), \emptyset, (\cup_j \mathcal{H}_{3j}) \cap (\cup_j \mathcal{L}_{3j}) >$.
Since, $\mathrm{U}_1 \cup \mathfrak{R}_1 \subseteq (\mathcal{J}_{3j})^c$. Therefore, $\mathrm{U}_1 \subseteq (\mathcal{J}_{3j})^c$ and $\mathfrak{R}_1 \subseteq (\mathcal{J}_{3j})^c$.
But $\mathrm{U}_1 \subseteq (\mathcal{H}_{3j})^c$ and $\mathfrak{R}_1 \subseteq (\mathcal{L}_{3j})^c$.
Thus, $\{\mathcal{J}_{3j}{}^c : j \in J\} = \{\mathcal{H}_{3j}{}^c : j \in J\} \cap \{\mathcal{L}_{3j}{}^c : j \in J\}$.
So, $\cap_j \mathcal{J}_{3j}{}^c = \cap_j \{\mathcal{L}_{3j}{}^c \cap \mathcal{H}_{3j}{}^c : j \in J\} \subseteq \cap_j \mathcal{H}_{3j}{}^c \cup (\cap_j \mathcal{L}_{3j}{}^c)$.
Hence, $\cup_j \mathcal{J}_{3j} \supseteq (\cup_j \mathcal{H}_{3j}) \cap (\cup_j \mathcal{L}_{3j})$. … (1)
It's clear. $\cap_j \mathcal{J}_{2j} = \cap_j \mathcal{H}_{2j} = \cap_j \mathcal{L}_{2j} = \emptyset$. … (2)
Since $(\mathrm{U}_3 \cap \mathfrak{R}_3)^c \subseteq \mathcal{J}_{1j}\ \forall j \in J$. Therefore, $\mathrm{U}_3{}^c \cup \mathfrak{R}_3{}^c \subseteq \mathcal{J}_{1j}\ \forall j \in J$.
Thus, $\mathrm{U}_3{}^c \subseteq \mathcal{J}_{1j}$ and $\mathfrak{R}_3{}^c \subseteq \mathcal{J}_{1j}\ \forall j$.
But $\mathrm{U}_3{}^c \subseteq \mathcal{H}_{1j}\ \forall j \in J$ and $\mathfrak{R}_3{}^c \subseteq \mathcal{L}_{1j} \forall j \in J$ .So, $\{\mathcal{J}_{1j} : j \in J\} = \{\mathcal{H}_{1j} : j \in J\} \cap \{\mathcal{L}_{1j} : j \in J\}$.
Hence, $\cap_j \mathcal{J}_{1j} = \cap_j \{\mathcal{H}_{1j} \cap \mathcal{L}1_j : j \in J\} \subseteq (\cap_j \mathcal{H}_{1j}) \cup (\cap_j \mathcal{L}_{1j})$ ... (3)
From (1),(2) and (3) *we get*
$\mathrm{N_U ÇÇL_{(1,2)}}[\mathrm{U}^{n} \cup_2 \mathfrak{R}^{n}] \subseteq_i \mathrm{N_U ÇÇL_{(1,2)}}(\mathrm{U}^{n}) \cup_2 \mathrm{N_U ÇÇL_{(1,2)}}(\mathfrak{R}^{n})$ ... (ii)
From (i), (ii) we get $\mathrm{N_U ÇÇL_{(1,2)}}[\mathrm{U}^{n} \cup_2 \mathfrak{R}^{n}] = \mathrm{N_U ÇÇL_{(1,2)}}(\mathrm{U}^{n}) \cup_2 \mathrm{N_U ÇÇL_{(1,2)}}(\mathfrak{R}^{n})$.

Remark 3.15:

- If the type-II union is replaced by the type-I intersection in the $\mathrm{N_U ÇT}^1_{(1,2)}$–space, Theorem 3.14 does not hold true, as shown in Example 3.2:

  $\mathrm{N_U ÇÇL}_{(1,2)}[B^{n} \cap_1 T^{n}] = \emptyset_1^{n} \neq \mathrm{N_U ÇÇL}_{(1,2)}(B^{n}) \cap_1 \mathrm{N_U ÇÇL}_{(1,2)}(T^{n}) = E^{n^{C_2}}$.

- If the type-II union is replaced by the type-I intersection in the $\mathrm{N_U ÇT}^2_{(1,2)}$–space, Theorem 3.14 does not hold true, as shown in Example 3.3:

  $\mathrm{N_U ÇÇL}_{(1,2)}[U^{n} \cap_1 A^{n}] = \emptyset_1^{n} \neq \mathrm{N_U ÇÇL}_{(1,2)}(U^{n}) \cap_1 \mathrm{N_U ÇÇL}_{(1,2)}(A^{n}) = C^{n^{C_2}}$.

- If the type-II union is replaced by the type-I intersection in the $\mathrm{N_U ÇT}^3_{(1,2)}$–space, Theorem 3.14 does not hold true, as shown in Example 3.4:

  $\mathrm{N_U ÇÇL}_{(1,2)}[J^{n} \cap_1 Q^{n}] = \emptyset_1^{n} \neq \mathrm{N_U ÇÇL}_{(1,2)}(J^{n}) \cap_1 \mathrm{N_U ÇÇL}_{(1,2)}(Q^{n}) = E^{n^{C_2}}$.

4. Interior of Neutrosophic Crisp Sets $\left(\mathrm{N_U ÇLnt}_{(1,2)}\right)$

This section presents a new type of interior of neutrosophic crisp sets, and a comprehensive study of its properties in relation to classical topology is conducted. Most properties of classical topology do not apply to all neutrosophic crisp sets.

Definition 4.1:
Let (Ӿ, Ʈ) be a $\mathrm{N_U ÇT}_{(1,2)}$–space, then the interior of neutrosophic crisp set $Ʉ^{n} = < Ʉ_1, Ʉ_2, Ʉ_3 >$, denoted by $\mathrm{N_U ÇLnt}_{(1,2)}(Ʉ^{n})$ and $\mathrm{N_U ÇLnt}_{(1,2)}(Ʉ^{n}) = \cup_{2j}\{ \mathfrak{S}^{n}_j : \mathfrak{S}^{n}_j =< \mathfrak{S}_{1j}, \mathfrak{S}_{2j}, \mathfrak{S}_{3j} >\} \ni \mathfrak{S}^{n}_j$ are $\mathrm{N_U ÇO}-set$ $\forall j$, and $(\mathfrak{S}_{j1})^c \supseteq Ʉ_3$ $\forall$ j $\in J$, $(\mathfrak{S}_{j2})^c \supseteq Ʉ_2 \forall$ j $\in J$, $(\mathfrak{S}_{j3})^c \subseteq Ʉ_1 \forall$ j $\in J$.

- From definition $\mathrm{N_U ÇLnt}_{(1,2)}(Ʉ^{n})$ *is* a $\mathrm{N_U ÇO}$ –*set*.
- Always $\mathrm{N_U ÇLnt}_{(1,2)}(Ʉ^{n}) = \cup_{2j} \mathfrak{S}^{n}_j = <\cup_j \mathfrak{S}_{1j}, \emptyset, \cap_j \mathfrak{S}_{3j} >$. Because $\emptyset_1^{n}$ *is a* $\mathrm{N_U ÇO}$ – *set* and satisfies the conditions in definition 4.1.

Example 4.2:
Let Ӿ = $\{a, b, c, d\}$and Ʈ = $\{\emptyset_1^{n}, Ӿ_1^{n}, B^{n}, K^{n}, \mathcal{M}^{n}\}$, $B^{n} = < \{a\}, \{d\}, \{c\} >$, $K^{n} = < \{a\}, \{b\}, \{c\} >$, $\mathcal{M}^{n} = < \{a\}, \emptyset, \{c\} >$ be $\mathrm{N_U Ç_s}$ from Type I. Then (Ӿ, Ʈ) is a $\mathrm{N_U ÇT}^1_{(1,2)}$–space. Therefore:

i. $\mathrm{N_U ÇLnt}_{(1,2)}(B^{n} = < \{a\}, \{d\}, \{c\} >) = \emptyset_1^{n} = < \emptyset, \emptyset, Ӿ >$.
ii. $\mathrm{N_U ÇLnt}_{(1,2)}(S^{n} = < \{b\}, \emptyset, \emptyset >) = \emptyset_1^{n} = < \emptyset, \emptyset, Ӿ >$.
iii. $\mathrm{N_U ÇLnt}_{(1,2)}(\mathcal{M}^{n^{C_2}} = < \{c\}, \emptyset, \{a\} >) = \emptyset_1^{n} = < \emptyset, \emptyset, Ӿ >$.
iv. $\mathrm{N_U ÇLnt}_{(1,2)}(\mathcal{M}^{n} = < \{a\}, \emptyset, \{c\}>) = \emptyset_1^{n} = < \emptyset, \emptyset, Ӿ >$.
v. $\mathrm{N_U ÇLnt}_{(1,2)}(K^{n^{C_2}} = < \{c\}, \{b\}, \{a\} >) = \emptyset_1^{n} = < \emptyset, \emptyset, Ӿ >$.
vi. $\mathrm{N_U ÇLnt}_{(1,2)}(L^{n} = < \{a, b, d\}, \{c\}, \{b\} >) = \mathcal{M}^{n} = < \{a\}, \emptyset, \{c\} >$.
vii. $\mathrm{N_U ÇLnt}_{(1,2)}(\emptyset_1^{n} = <\emptyset, \emptyset, Ӿ >) = \emptyset_1^{n} = < \emptyset, \emptyset, Ӿ >$.
viii. $\mathrm{N_U ÇLnt}_{(1,2)}(Ӿ_1^{n} = < Ӿ, \emptyset, \emptyset >) = Ӿ_1^{n} = < Ӿ, \emptyset, \emptyset >$.
ix. $\mathrm{N_U ÇLnt}_{(1,2)}(\mathcal{L}^{n} = < \{a\}, \{d\}, \emptyset >) = \emptyset_1^{n} =< \emptyset, \emptyset, Ӿ >$.

Example 4.3:
Let Ӿ = $\{a, b, c\}$. Ʈ = $\{\emptyset_1^{n}, Ӿ_1^{n}, A^{n}, B^{n}, C^{n}\}$, $A^{n} = < \{a, b\}, \emptyset, \{c\} >$, $B^{n} = < \{a, c\}, \emptyset, \{b\} >$,
$C^{n} = < \{a\}, \emptyset, \{b, c\} >$ be $\mathrm{N_U Ç_s}$ from Type II. Then (Ӿ, Ʈ) is a $\mathrm{N_U ÇT}^2_{(1,2)}$–space. Therefore:

i. $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}(A^{n} = <\{a,b\},\emptyset,\{c\}>) = A^{n} = <\{a,b\},\emptyset,\{c\}>$.
ii. $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}(O^{n} = <\{b\},\emptyset,\emptyset>) = \emptyset_1^{n} = <\emptyset,\emptyset,X>$.
iii. $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}({C^{n}}^{C_2} = <\{b,c\},\emptyset,\{a\}>) = \emptyset_1^{n} = <\emptyset,\emptyset,X>$.
iv. $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}(C^{n} = <\{a\},\emptyset,\{b,c\}>) = C^{n} = <\{a\},\emptyset,\{b,c\}>$.
v. $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}(B^{n} = <\{a,c\},\emptyset,\{b\}>) = B^{n} = <\{a,c\},\emptyset,\{b\}>$.
vi. $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}(U^{n} = <\{a\},\{b\},\{c\}>) = C^{n} = <\{a\},\emptyset,\{b,c\}>$.
vii. $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}({A^{n}}^{C_2} = <\{c\},\emptyset,\{a,b\}>) = \emptyset_1^{n} = <\emptyset,\emptyset,X>$.
viii. $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}(\emptyset_1^{n} = <\emptyset,\emptyset,X>) = \emptyset_1^{n} = <\emptyset,\emptyset,X>$.
ix. $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}(X_1^{n} = <X,\emptyset,\emptyset>) = X_1^{n} = <X,\emptyset,\emptyset>$.

Example 4.4:
Let $X = \{a, b, c\}$, $T = \{\emptyset_1^{n}, X_1^{n}, Q^{n}, W^{n}, E^{n}, \mathcal{R}^{n}\}$, $Q^{n} = <\{a,b\},\{c\},X>$, $W^{n} = <X,\{b\},\{c\}>$, $E^{n} = <\{a,b\},\emptyset,X>$, $\mathcal{R}^{n} = <X,\emptyset,\{c\}>$ be $\mathrm{N_U Ç_ş}$ from Type III . (X, T) is a $\mathrm{N_U ÇT}^3_{(1,2)}$–space. Therefore:
i. $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}(Q^{n} = <\{a,b\},\{c\},X>) = \emptyset_1^{n} = <\emptyset,\emptyset,X>$.
ii. $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}(A^{n} = <\{b\},\emptyset,\emptyset>) = E^{n} = <\{a,b\},\emptyset,X>$.
iii. $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}({E^{n}}^{C_2} = <X,\emptyset,\{a,b\}>) = \emptyset_1^{n} = <\emptyset,\emptyset,X>$.
iv. $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}(E^{n} = <\{a,b\},\emptyset,X>) = \emptyset_1^{n} = <\emptyset,\emptyset,X>$.
v. $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}({W^{n}}^{C_2} = <\{c\},\{b\},X>) = \emptyset_1^{n} = <\emptyset,\emptyset,X>$.
vi. $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}(S^{n} = <\{a,c\},\{b\},\{c\}>) = E^{n} = <\{a,b\},\emptyset,X>$.
vii. $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}({Q^{n}}^{C_2} = <X,\{c\},\{a,b\}>) = \emptyset_1^{n} = <\emptyset,\emptyset,X>$.
viii. $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}(\emptyset_1^{n} = <\emptyset,\emptyset,X>) = \emptyset_1^{n} = <\emptyset,\emptyset,X>$.
ix. $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}(X_1^{n} = <X,\emptyset,\emptyset>) = X_1^{n} = <X,\emptyset,\emptyset>$.
x. $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}(D^{n} = <X,\{c\},\{b\}>) = \emptyset_1^{n} = <\emptyset,\emptyset,X>$.

Corollary 4.5:
Let (X, T) be *a* $\mathrm{N_U ÇT}^t_{(1,2)}$–space, t=1,2,3. Then

i. $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}(X_1^{n}) = X_1^{n} = <X,\emptyset,\emptyset>$ ii. $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}(\emptyset_1^{n}) = \emptyset_1^{n} = <\emptyset,\emptyset,X>$

Proof (i):

Consider $\{\mathfrak{S}^{n}{}_j = <\mathfrak{S}_{1j}, \mathfrak{S}_{2j}, \mathfrak{S}_{3j}> : j \in J\}$ as the family of $\mathrm{N_U ÇO}$ –*set*s satisfying:
$(\mathfrak{S}_{1j})^c \supseteq \emptyset\ \forall$ j, $(\mathfrak{S}_{2j})^c \supseteq \emptyset\ \forall$ j, $(\mathfrak{S}_{3j})^c \subseteq X\ \forall$ j.

Thus, $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}(X_1^{n}) = \cup_{2j}\,\mathfrak{S}^{n}{}_j = <\cup_j \mathfrak{S}_{1j}, \cap_j \mathfrak{S}_{2j}, \cap_j \mathfrak{S}_{3j}>$.
Since $(\mathfrak{S}_{1j})^c \supseteq \emptyset$. So, $\mathfrak{S}_{1j} \subseteq X\ \forall j$.Therefore, $\cup_j \mathfrak{S}_{1j} \subseteq X$ and $\cup_j \mathfrak{S}_{1j} \supseteq X$. Hence, $\cup_j \mathfrak{S}_{j1} = X\ \forall j$.

Since $\emptyset_1^{n}$ is $\mathrm{N_U ÇO}$ – *set* by definition 4.1.

Hence, $\cap_j \mathfrak{S}_{2j} = \emptyset$. Because X_1^{n} is $\mathrm{N_U ÇO}$ – set by definition 4.1.

Therefore, $\cap_j \mathfrak{S}_{j3} = \emptyset$. Hence, $\mathrm{N_U ÇLn t}_{(\mathbf{1,2})}(X_1^{n}) = X_1^{n} = <X,\emptyset,\emptyset>$

The proof of (ii) can be carried out in a manner analogous to the proof of (i).

Theorem 4.6:

Let (Ӽ, Ʈ) be a Ꞑ$_U$ÇƮ$^{2}_{(1,2)}$–space and let Ʉ^{n} = <Ʉ$_1$, Ʉ$_2$, Ʉ$_3$> be any Ꞑ$_U$Ç$_{ş}$. Then Ꞑ$_U$ÇLꞑŧ$_{(1,2)}$(Ʉ^{n}) $\subseteq_1$ Ʉ^{n}.

Proof : Consider $\{\mathfrak{S}^{n}{}_j = <\mathfrak{S}_{1j}, \mathfrak{S}_{2j}, \mathfrak{S}_{3j}> = <\mathfrak{S}_{1j}, \emptyset, \mathfrak{S}_{1j}{}^{c}> : j \in J\}$ as the family of Ꞑ$_U$ÇO –*set*s satisfying: $(\mathfrak{S}_{1j})^c \supseteq$ Ʉ$_3$ ∀ j, $(\mathfrak{S}_{2j})^c \supseteq$ Ʉ$_2$ ∀ j, $\mathfrak{S}_{3j}{}^{c} \subseteq$ Ʉ$_1$ ∀ j.
Thus, Ꞑ$_U$ÇLꞑŧ$_{(1,2)}$(Ʉ^{n}) = $\cup_{2j}\, \mathfrak{S}^{n}{}_j = <\cup_j \mathfrak{S}_{1j},\ \cap_j \mathfrak{S}_{2j},\ \cap_j \mathfrak{S}_{3j}> = <\cup_j \mathfrak{S}_{1j},\ \emptyset,\ \cap_j (\mathfrak{S}_{1j})^c>$
by Remark 2.8.
Since $(\mathfrak{S}_{1j})^c \supseteq$ Ʉ$_3$ ∀ j . So, $\mathfrak{S}_{3j} \supseteq$ Ʉ$_3$ ∀ j . Therefore, $\cap_j \mathfrak{S}_{3j} \supseteq$ Ʉ$_3$.

Since $\cap \mathfrak{S}_{2j} = \emptyset$. Therefore, Ʉ$_2 \supseteq \cap \mathfrak{S}_{2j} = \emptyset$.

Since $\mathfrak{S}_{3j}{}^{c} \subseteq$ Ʉ$_1$ ∀ j. *So,* $\mathfrak{S}_{1j} \subseteq$ Ʉ$_1$ ∀ j. Therefore, $\cup_j \mathfrak{S}_{1j} \subseteq$ Ʉ$_1$.

Hence, Ꞑ$_U$ÇLꞑŧ$_{(1,2)}$(Ʉ^{n}) $\subseteq_1$ Ʉ^{n}.

Remarks 4.7:

- Replacing the type-II inclusion with the type-I inclusion causes Theorem 4.6 to fail, as shown in Example 4.3,where Ꞑ$_U$ÇLꞑŧ$_{(1,2)}$(U^{n}) = C^{n} =< $\{a\}$, ∅, {b,c} > $\nsubseteq_2$ U^{n} = < $\{a\}, \{b\}$, {c} >.
- Substituting Ꞑ$_U$ÇƮ$^{2}_{(1,2)}$–space with the Ꞑ$_U$ÇƮ$^{1}_{(1,2)}$–space causes Theorem 4.6 to fail, as shown in Example 4.2, where Ꞑ$_U$ÇLꞑŧ$_{(1,2)}$ (L^{n}) = $\mathcal{M}^{n}$ =< {a}, ∅, {c} > $\nsubseteq_i$ L^{n} = < $\{a, b, d\}, \{c\}, \{b\}$ >, i = 1,2.
- Substituting Ꞑ$_U$ÇƮ$^{2}_{(1,2)}$–space with the Ꞑ$_U$ÇƮ$^{3}_{(1,2)}$–space causes Theorem 4.6 to fail, as shown in Example 4.4, where Ꞑ$_U$ÇLꞑŧ$_{(1,2)}$ (S^{n}) = E^{n} = < {a, b}, ∅, Ӽ > $\nsubseteq_i$ S^{n} =< $\{a, c\}, \{b\}$, {c} >, i = 1,2.

Theorem 4.8:

Let (Ӽ, Ʈ) be *a* Ꞑ$_U$ÇƮ$^{2}_{(1,2)}$–space. Then Ƀ^{n} =< Ƀ$_1$, Ƀ$_2$, Ƀ$_3$ > is a Ꞑ$_U$ÇO– *set* if and only if Ꞑ$_U$ÇLꞑŧ$_{(1,2)}$(Ƀ^{n}) = Ƀ^{n}.
Proof: If Ꞑ$_U$ÇLꞑŧ$_{(1,2)}$ (Ƀ^{n}) = Ƀ^{n}. Then Ƀ^{n} Ꞑ$_U$ÇO – *set* by definition 4.1.

Conversely,
Let $\{\mathfrak{S}^{n}{}_j = <\mathfrak{S}_{1j}, \mathfrak{S}_{2j}, \mathfrak{S}_{3j}> = <\mathfrak{S}_{1j}, \emptyset, \mathfrak{S}_{1j}{}^{c}> : j \in J\}$ be the family of Ꞑ$_U$ÇO –*set*s such that $\mathfrak{S}_{1j}{}^{c} \supseteq$ Ƀ$_3$ ∀ j, $\mathfrak{S}_{2j}{}^{c} \supseteq$ Ƀ$_2$ ∀ j, $\mathfrak{S}_{3j}{}^{c} \subseteq$ Ƀ$_1$ ∀ j.
Thus, Ꞑ$_U$ÇLꞑŧ$_{(1,2)}$ (Ƀ^{n}) (Ƀ^{n}) = $\cup_{2j}\, \mathfrak{S}^{n}{}_j = <\cup_j \mathfrak{S}_{1j},\ \cap_j \mathfrak{S}_{2j},\ \cap_j \mathfrak{S}_{3j}> = <\cup_j \mathfrak{S}_{1j},\ \emptyset,\ \cap_j (\mathfrak{S}_{1j})^c>$
by Remark 2.8.

Since $\mathfrak{S}_{1j}{}^{c} \supseteq$ Ƀ$_3$ ∀ j, and Ƀ^{n} is a Ꞑ$_U$ÇO – *set*. Therefore, $\mathfrak{S}_{3j} \supseteq$ Ƀ$_3$, Ƀ$_3 \subseteq \cap_j \mathfrak{S}_{3j}$ and Ƀ$_3 \supseteq \cap_j \mathfrak{S}_{3j}$.

Hence, Ƀ$_3 = \cap_j \mathfrak{S}_{3j}$.

Also, it's clear that Ƀ$_2 = \cap_j \mathfrak{S}_{2j} = \emptyset$.

Since $\mathfrak{S}_{3j}{}^{c} \subseteq$ Ƀ$_1$ ∀ j, and Ƀ^{n} is a Ꞑ$_U$ÇO – *set*. Thus, $\mathfrak{S}_{1j} \subseteq$ Ƀ$_1$, $\cup_j \mathfrak{S}_{1j} \subseteq$ Ƀ$_1$ and $\cup_j \mathfrak{S}_{1j} \supseteq$ Ƀ$_1$.

Therefore, $\cup_j \mathfrak{S}_{1j} = \mathcal{B}_1$. Hence, $N_U ÇLnt_{(1,2)}(\mathcal{B}^{n}) = \mathcal{B}^{n}$.

Remarks 4.9:

- The validity of Theorem 4.8 fails in the $N_U ÇT^1_{(1,2)}$–space, as demonstrated in Example 4.2: $\mathcal{B}^{n} = < \{a\},\{d\},\{c\} >$ is a $N_U ÇO-set$. But ,$N_U ÇLnt_{(1,2)}(\mathcal{B}^{n}) = \emptyset_1^{n} \neq \mathcal{B}^{n}$.
- The validity of Theorem 4.8 fails in the $N_U ÇT^3_{(1,2)}$–space, as demonstrated in Example 4.4: $Q^{n} = < \{a,b\}, \{c\}, X >$ is a $N_U ÇO-set$. But ,$N_U ÇLnt_{(1,2)}(Q^{n}) = \emptyset_1^{n} \neq Q^{n}$.

Corollary 4.10:

Let (X, T) be a $N_U ÇT^2_{(1,2)}$–space and let $\mathcal{U}^{n}$ be any $N_U Ç_s$. Then:

$N_U ÇLnt_{(1,2)}[N_U ÇLnt_{(1,2)}(\mathcal{U}^{n})] = N_U ÇLnt_{(1,2)}(\mathcal{U}^{n})$.

Proof : Since $N_U ÇLnt_{(1,2)}(\mathcal{U}^{n})$ is a $N_U ÇO-$ set.

Hence, $N_U ÇLnt_{(1,2)}[N_U ÇLnt_{(1,2)}(\mathcal{U}^{n})] = N_U ÇLnt_{(1,2)}(\mathcal{U}^{n})$ by Theorem 4.8.

Theorem 4.11:

Let (X, T) be a $N_U ÇT^t_{(1,2)}$–space, t=1,2,3. And let $\mathcal{U}^{n} =< \mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3 >$ be any neutrosophic crisp set. Then

i. $N_U ÇLnt_{(1,2)}(\mathcal{U}^{n}) = [N_U ÇÇL_{(1,2)}(\mathcal{U}^{n^{c_2}})]^{c_2}$.

ii. $N_U ÇÇL_{(1,2)}(\mathcal{U}^{n}) = [N_U ÇLnt_{(1,2)}(\mathcal{U}^{n^{c_2}})]^{c_2}$.

iii. $[N_U ÇLnt_{(1,2)}(\mathcal{U}^{n})]^{c_2} = N_U ÇÇL_{(1,2)}(\mathcal{U}^{n^{c_2}})$.

iv. $[N_U ÇÇL_{(1,2)}(\mathcal{U}^{n})]^{c_2} = N_U ÇLnt_{(1,2)}(\mathcal{U}^{n^{c_2}})$.

Proof(i):

Consider $\{\mathfrak{S}^{n}_j = < \mathfrak{S}_{1j}, \mathfrak{S}_{2j}, \mathfrak{S}_{3j} > : j \in J\}$ as the family of $N_U ÇO-sets$ satisfying:

${\mathfrak{S}_{1j}}^c \supseteq \mathcal{U}_3 \ \forall$ j, ${\mathfrak{S}_{2j}}^c \supseteq \mathcal{U}_2 \ \forall$ j, ${\mathfrak{S}_{3j}}^c \subseteq \mathcal{U}_1 \ \forall$ j.

Hence, $N_U ÇLnt_{(1,2)}(\mathcal{U}^{n}) = \bigcup_{2j} \mathfrak{S}^{n}_j = < \cup \mathfrak{S}_{1j}, \cap \mathfrak{S}_{2j}, \cap \mathfrak{S}_{3j} >$. …(1)

Also $\{\mathfrak{S}^{n}{}_j^{c_2} = < \mathfrak{S}_{3j}, \mathfrak{S}_{2j}, \mathfrak{S}_{1j} > : j \in J\}$ be the family of $N_U ÇÇ-sets$ such that

$\mathcal{U}_3 \subseteq {\mathfrak{S}_{1j}}^c \ \forall j \in J$, $\mathcal{U}_2 \subseteq {\mathfrak{S}_{2j}}^c \ \forall j \in J$, $\mathcal{U}_1 \supseteq {\mathfrak{S}_{3j}}^c \forall j \in J$.

Therefore, $N_U ÇÇL_{(1,2)}(\mathcal{U}^{n^{c_2}} = < \mathcal{U}_3, \mathcal{U}_2, \mathcal{U}_1 >) = \bigcap_{1j} \mathfrak{S}^{n}{}_j^{c_2} = < \cap_j \mathfrak{S}_{3j}, \cap_j \mathfrak{S}_{2j}, \cup_j \mathfrak{S}_{1j} >$.

Hence, $(N_U ÇÇL^{\breve{\alpha}}_{(1,2)}(\mathcal{U}^{n^{c_2}}))^{c_2} = < \cup_j \mathfrak{S}_{1j}, \cap_j \mathfrak{S}_{2j}, \cap_j \mathfrak{S}_{3j} >$. …(2)

From (1) and (2) we get $N_U ÇLnt_{(1,2)}(\mathcal{U}^{n})$. $= [N_U ÇÇL_{(1,2)}(\mathcal{U}^{n^{c_2}})]^{c_2}$.

Proof(iv):

Let $\{\mathfrak{S}^{n}_j = < \mathfrak{S}_{1j}, \mathfrak{S}_{2j}, \mathfrak{S}_{3j} > : j \in J\}$ be the family of $N_U ÇO-sets$ such that

${\mathfrak{S}_{1j}}^c \supseteq \mathcal{U}_3 \ \forall$ j, ${\mathfrak{S}_{2j}}^c \supseteq \mathcal{U}_2 \ \forall$ j, ${\mathfrak{S}_{3j}}^c \subseteq \mathcal{U}_1 \ \forall$ j.

Hence, $N_U ÇLnt_{(1,2)}(\mathcal{U}^{n^{c_2}}) = \bigcup_{2j} \mathfrak{S}^{n}_j = < \cup \mathfrak{S}_{1j}, \cap \mathfrak{S}_{2j}, \cap \mathfrak{S}_{3j} >$. …(1)

Also $\{ \mathfrak{S}^{n}{}_{j}{}^{c_2} = < \mathfrak{S}_{3j}, \mathfrak{S}_{2j}, \mathfrak{S}_{1j} >: j \in J\}$ be the family of $N_U ÇÇ - sets$ such that

$Ꞓ_1 \subseteq \mathfrak{S}_{1j}{}^{c} \ \forall j \in J, \quad Ꞓ_2 \subseteq \mathfrak{S}_{2j}{}^{c} \ \forall j \in J, \quad Ꞓ_3 \supseteq \mathfrak{S}_{3j}{}^{c}$.
Therefore, $N_U ÇÇL_{(1,2)}(Ꞓ^{n}) = \cap_{1j} \mathfrak{S}^{n}{}_{j}{}^{c_2} = <\cap_j \mathfrak{S}_{3j}, \cap_j \mathfrak{S}_{2j}, \cup_j \mathfrak{S}_{1j}>$.
Hence, $(N_U ÇÇL_{(1,2)}(Ꞓ^{n}))^{c_2} = <\cup_j \mathfrak{S}_{1j}, \cap_j \mathfrak{S}_{2j}, \cap_j \mathfrak{S}_{3j}>$. ...(2)

From (1) and (2) we get $[N_U ÇÇL_{(1,2)}(Ꞓ^{n})]^{c_2} = N_U ÇLnt_{(1,2)}(Ꞓ^{n^{c_2}})$.

Corollary 4.12:

Let (Ꭓ, Ʈ) be a $N_U ÇƮ^{t}_{(1,2)}$–space, t=1,2,3. And let $Ꞓ^{n}, \mathfrak{R}^{n}$ be any neutrosophic crisp sets. Then

$N_U ÇLnt_{(1,2)}[Ꞓ^{n} \cap_1 \mathfrak{R}^{n}] = N_U ÇLnt_{(1,2)}(Ꞓ^{n}) \cap_2 N_U ÇLnt_{(1,2)}(\mathfrak{R}^{n})$

Proof: It is clear based on Theorem 3.14 and Theorem 4.11.

Remarks 4.13:

- Replacing the type-I intersection with the type-I union in the $N_U ÇƮ^{1}_{(1,2)}$–space causes Corollary 4.12 to fail, as shown in Example 4.2:
$$N_U ÇLnt_{(1,2)}[\mathcal{M}^{n^{C_2}} \cup_1 L^{n}] = Ꭓ_1^{n} \neq N_U ÇLnt_{(1,2)}(\mathcal{M}^{n^{C_2}}) \cup_1 N_U ÇLnt_{(1,2)}(L^{n}) = \mathcal{M}^{n}.$$
- Replacing the type-I intersection with the type-I union in the $N_U ÇƮ^{2}_{(1,2)}$—space causes Corollary 4.12 to fail, as shown in Example 4.3:
$$N_U ÇLnt_{(1,2)}[U^{n} \cup_1 A^{n}] = A^{n} \neq N_U ÇLnt_{(1,2)}(U^{n}) \cup_1 N_U ÇLnt_{(1,2)}(A^{n}) = C^{n}.$$
- Replacing the type-I intersection with the type-I union in the $N_U ÇƮ^{3}_{(1,2)}$–space causes Corollary 4.12 to fail, as shown in Example 4.4:
$$N_U ÇLnt_{(1,2)}[S^{n} \cup_1 Q^{n}] = Ꭓ_1^{n} \neq N_U ÇLnt_{(1,2)}(S^{n}) \cup_1 N_U ÇLnt_{(1,2)}(Q^{n}) = E^{n}.$$

Corollary 4.14:

Let (Ꭓ, Ʈ) be a $N_U ÇƮ^{t}_{(1,2)}$–space, t=1,2,3.If $A^{n} \subseteq_i O^{n}, i = 1,2$ then

$N_U ÇLnt_{(1,2)}(A^{n}) \subseteq_i N_U ÇLnt_{(1,2)}(O^{n}), i = 1,2.$

Proof**:** Since $A^{n} \subseteq_i O^{n}, i = 1,2$, so $O^{n^{C_2}} \subseteq_i A^{n^{C_2}}, i = 1,2$. by Theorem 2.4.iii
Thus, by Theorem 3.7, we get $N_U ÇÇL_{(1,2)}(O^{n^{C_2}}) \subseteq_i N_U ÇÇL_{(1,2)}(A^{n^{C_2}}), i = 1,2.$
Therefore, $[N_U ÇÇL_{(1,2)}(A^{n})^{c_2}]^{c_2} \subseteq_i N_U ÇÇL_{(1,2)}(O^{n})^{c_2}]^{c_2}$.
Hence, by Theorem 4.11, we get $N_U ÇLnt_{(1,2)}(A^{n}) \subseteq_i N_U ÇLnt_{(1,2)}(O^{n}), i = 1,2.$

Theorem 4.15:
Let (Ꭓ, Ʈ) be a $N_U ÇƮ^{2}_{(1,2)}$–space and let $Ꞓ^{n} = < Ꞓ_1, Ꞓ_2, Ꞓ_3 >$ is any neutrosophic crisp set, $then$ $N_U ÇLnt_{(1,2)}(Ꞓ^{n}) \subseteq_i N_U ÇÇL_{(1,2)}(Ꞓ^{n}), i = 1,2.$
Proof: It is clear based on Theorem 3.7 and Theorem 4.6.

5. A Neutrosophic Crisp Topology Space generated by Basis ($\mathrm{N_U Ç T}_{\beta}$)

In this section, a new definition of neutrosophic crisp topology is introduced by constructing the neutrsophic crisp topological space generated by a defined basis.

Definition 5.1:

Let **(X, T)** be A $\mathrm{N_U ÇT}^{t}_{(1,2)}$–space, t=1,2,3, be a neutrosophic crisp topological space. The basis of the neutrosophic crisp topological space is denoted by the family $\mathrm{N_U}\beta$**,** and it is defined as follows: $\mathrm{N_U}\beta = \{\, \mathrm{N_U ÇLnt}_{(1,2)}(\mathcal{M}^{n}) : \mathcal{M}^{n}$ *is a* neutrosophic crisp set$\}$. Then $\mathrm{N_U}\beta$ is called a neutrosophic crisp topological basis.

Definition 5.2:

The neutrosophic **c**risp topological space $\mathrm{N_U ÇT}_{\beta}$ was constructed. This is the smallest neutrosophic **c**risp topological space generated by the basis $\mathrm{N_U}\beta$, which is defined in Definition 5.1.

In Example 5.3, it is shown how to obtain the smallest neutrosophic **c**risp topological space $\mathrm{N_U ÇT}_{\beta}$ generated by the basis $\mathrm{N_U}\beta$.

Example 5.3:

Let X = $\{a, b\}$and T = $\{\emptyset^{n}_{1}, X^{n}_{1}, Q^{n}, W^{n}\}$, $Q^{n} =< \emptyset, \emptyset, \{a\} >$, $W^{n} = < \{a\}, \emptyset, \emptyset >$, be $\mathrm{N_U Ç}_{ş}$ from TypeI. Then (X, T) is a $\mathrm{N_U ÇT}^{1}_{(1,2)}$–space. Therefore:

1. $\mathrm{N_U ÇLnt}_{(1,2)}(Q^{n} = < \emptyset, \emptyset, \{a\} >) = \emptyset^{n}_{1}$.
2. $\mathrm{N_U ÇLnt}_{(1,2)} (W^{n} = < \{a\}, \emptyset, \emptyset >) = \emptyset^{n}_{1}$.
3. $\mathrm{N_U ÇLnt}_{(1,2)} (A^{n} = < \emptyset, \emptyset, \{b\} >) = \emptyset^{n}_{1}$
4. $\mathrm{N_U ÇLnt}_{(1,2)} (B^{n} =< \{b\}, \emptyset, \emptyset >) = Q^{n}$.
5. $\mathrm{N_U ÇLnt}_{(1,2)} (C^{n} =< \emptyset, \{b\}, \emptyset >) = \emptyset^{n}_{1}$.
6. $\mathrm{N_U ÇLnt}_{(1,2)} (D^{n} =< \emptyset, \{a\}, \emptyset >) = \emptyset^{n}_{1}$.
7. $\mathrm{N_U ÇLnt}_{(1,2)} (E^{n} =< \emptyset, X, \emptyset >) = \emptyset^{n}_{1}$.
8. $\mathrm{N_U ÇLnt}_{(1,2)} (F^{n} =< \{a\}, \{b\}, \emptyset >) = \emptyset^{n}_{1}$.
9. $\mathrm{N_U ÇLnt}_{(1,2)} (G^{n} =< \{b\}, \{a\}, \emptyset >) = Q^{n}$.
10. $\mathrm{N_U ÇLnt}_{(1,2)} (H^{n} =< \emptyset, \{b\}, \{a\} >) = \emptyset^{n}_{1}$.
11. $\mathrm{N_U ÇLnt}_{(1,2)} (I^{n} =< \emptyset, \{a\}, \{b\} >) = \emptyset^{n}_{1}$.
12. $\mathrm{N_U ÇLnt}_{(1,2)} (J^{n} =< \{a\}, \emptyset, \{b\} >) = \emptyset^{n}_{1}$.
13. $\mathrm{N_U ÇLnt}_{(1,2)} (K^{n} =< \{b\}, \emptyset, \{a\} >) = Q^{n}$.
14. $\mathrm{N_U ÇLnt}_{(1,2)} (O^{n} =< \emptyset, \emptyset, \emptyset >) = \emptyset^{n}_{1}$.
15. $\mathrm{N_U ÇLnt}_{(1,2)} (\emptyset^{n}_{1} = <\emptyset, \emptyset, X >) = \emptyset^{n}_{1}$.
16. $\mathrm{N_U ÇLnt}_{(1,2)} (X^{n}_{1} = < X, \emptyset, \emptyset >) = X^{n}_{1}$.

Thus, $\mathrm{N_U}\beta = \{\emptyset^{n}_{1}, X^{n}_{1}, Q^{n}\}$. Hence, $\mathrm{N_U ÇT}_{\beta} = \{\emptyset^{n}_{1}, X^{n}_{1}, Q^{n}\}$.

6. A Neutrosophic Crisp semi – open $^{i}_{(1,2)}$ Topological Space ($\mathrm{N_U ÇT}_{§_i}$), $i = 1, 2$.

In this section, a new definitions of two neutrosophic crisp topological spaces**,** $\mathrm{N_U ÇT}_{§_1}$ and $\mathrm{N_U ÇT}_{§_2}$, are introduced by constructing the neutrosophic crisp topological spaces generated by the sub-bases $\mathrm{N_U}\beta§_1$ and $\mathrm{N_U}\beta§_1$, respectively**.**

Definition 6.1:

Let (X, T) be *a* $\mathrm{N_U ÇT}^{t}_{(1,2)}$–space, t=1,2,3. Then $\mathcal{M}^{n} \subseteq_1 X^{n}_{1}$ is called neutrosophic crisp semi – open $^{1}_{(1,2)}$ if and only if $\mathcal{M}^{n} \subseteq_1 \mathrm{N_U ÇÇL}_{(1,2)}\left(\mathrm{N_U ÇLnt}_{(1,2)}(\mathcal{M}^{n})\right)$. The family of all neutrosophic crisp semi – open $^{1}_{(1,2)}$denoted by $\mathrm{N_U}\beta§_1$ such that

$Ɲ_{U}\beta §_1 = \left\{ \mathcal{M}^{n} \subseteq_1 Ẋ_1^{n} : \mathcal{M}^{n} \subseteq_1 \; Ɲ_{U}ÇÇL_{(1,2)}\left(Ɲ_{U}ÇLnt_{(1,2)}(\mathcal{M}^{n}) \right) \right\}$ and $Ɲ_{U}\beta §_1$ is a neutrosophic crisp topological sub-basis.

- The neutrosophic crisp semi – open $^{1}_{(1,2)}$ topological space $Ɲ_{U}ÇƮ_{§_1}$ was constructed. This is the smallest neutrosophic crisp topological space generated by the sub-basis $Ɲ_{U}\beta §_1$.

Definition 6.2:
Let (Ẋ, Ʈ) be a $Ɲ_{U}ÇƮ^{t}_{(1,2)}$–space, t=1,2,3. Then $\mathcal{M}^{n} \subseteq_2 Ẋ_1^{n} \mathcal{M}^{n}$ is called neutrosophic crisp semi – open $^{2}_{(1,2)}$ if and only if $\mathcal{M}^{n} \subseteq_2 Ɲ_{U}ÇÇL_{(1,2)}\left(Ɲ_{U}ÇLnt_{(1,2)}(\mathcal{M}^{n}) \right)$. The family of all neutrosophic crisp semi – open $^{2}_{(1,2)}$ denoted by $Ɲ_{U}\beta §_2$ such that
$Ɲ_{U}\beta §_2 = \left\{ \mathcal{M}^{n} \subseteq_2 Ẋ_1^{n} \ni \mathcal{M}^{n} \subseteq_2 \; Ɲ_{U}ÇÇL_{(1,2)}\left(Ɲ_{U}ÇLnt_{(1,2)}(\mathcal{M}^{n}) \right) \right\}$ and $Ɲ_{U}\beta §_2$ is a neutrosophic crisp topological sub-basis.

- The neutrosophic crisp semi – open $^{2}_{(1,2)}$ topological space $Ɲ_{U}ÇƮ_{§_2}$ was constructed. This is the smallest neutrosophic crisp topological space generated by the sub-basis $Ɲ_{U}\beta §_2$.

The following example illustrates how to obtain the smallest neutrosophic crisp topological spaces $Ɲ_{U}ÇƮ_{§_1}$ and $Ɲ_{U}ÇƮ_{§_2}$ generated by the sub- basis $Ɲ_{U}\beta §_1$ and the sub- basis $Ɲ_{U}\beta §_2$ respectively.

Example 6.3:
Let Ẋ = {a, b, c}and Ʈ = {$\emptyset_1^{n}, Ẋ_1^{n}, A^{n}, B^{n}, C^{n}, D^{n}$}, A^{n} = < {a}, {b}, {c} >, B^{n} = < {a}, ∅, {c} >, C^{n} = < {c}, ∅, {a, b} >, D^{n} = < {a, c}, ∅, ∅>. Then (Ẋ, Ʈ) is a $Ɲ_{U}ÇƮ^{1}_{(1,2)}$– space.

Therefore, $Ɲ_{U}\beta §_1$ = {E^{n}, $C^{n}, G^{n}, Ẋ_1^{n}, \emptyset_1^{n}$}, E^{n} = < {c}, ∅, {a} >, C^{n} = < {c}, ∅, {a, b} >, G^{n} = < {a, b}, ∅, {c}>.
$Ɲ_{U}\beta §_2$ = {E^{n}, $F^{n}, C^{n}, G^{n}, Ẋ_1^{n}, \emptyset_1^{n}$}, E^{n} =< {c}, {b}, {A} >, F^{n} = < {c}, ∅, {a} >, C^{n} = < {c}, ∅, {a, b}> , G^{n} = < {a, b}, ∅, {c}>.
Hence, $Ɲ_{U}ÇƮ_{§_1}$ = { $\emptyset_1^{n}, Ẋ_1^{n}, E^{n}$, C^{n}, G^{n}, Q^{n}}, E^{n} = < {c}, ∅, {a} >, C^{n} = < {c}, ∅, {a, b} >, G^{n} = < {a, b}, ∅, {c}>, Q^{n} =< ∅, ∅, {a, c} >.
$Ɲ_{U}ÇƮ_{§_2}$ = {$\emptyset_1^{n}, Ẋ_1^{n}, E^{n}$, $F^{n}, C^{n}, G^{n}, Q^{n}$}, E^{n} =< {c}, {b}, {a} >, F^{n} = < {c}, ∅, {a} >,

C^{n} = < {c}, ∅, {a, b} >, G^{n} = < {a, b}, ∅, {c}>, Q^{n} =< ∅, ∅, {a, c} >.

Theorem 6.4:

Every neutrosophic crisp semi – open $^{1}_{(1,2)}$ is a neutrosophic crisp semi – open $^{2}_{(1,2)}$.

Proof: Assume $\mathcal{M}^{n} =< \mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3 >$ is a semi–open $^{1}_{(1,2)}$.

Thus, $\mathcal{M}^{n} \subseteq_1 \; Ɲ_{U}ÇÇL_{(1,2)}\left(Ɲ_{U}ÇLnt_{(1,2)}(\mathcal{M}^{n}) \right) = \cap_{1j} \mathcal{F}^{n}_{j} =< \cap_j \mathcal{F}_{j1}, \emptyset, \cup_j \mathcal{F}_{j3} >$.

Therefore, $\mathcal{M}_1 \subseteq \cup_j \mathcal{F}_{j1}$, $\mathcal{M}_2 \subseteq \emptyset$, $\mathcal{M}_3 \supseteq \cap_j \mathcal{F}_{j3}$. Because, $\mathcal{M}_2 \subseteq \emptyset$ so $\mathcal{M}_2 = \emptyset$ thus,
$\mathcal{M}^{n} \subseteq_2 Ɲ_{U}ÇÇL_{(1,2)}\left(Ɲ_{U}ÇLnt_{(1,2)}(\mathcal{M}^{n}) \right)$. Hence, $\mathcal{M}^{n}$ is semi – open $^{2}_{(1,2)}$.

Remark 6.5:

The converse of Theorem 6.4 is not true, as illustrated by the following example:

Let Ӽ = $\{a, b\}$, Ţ = $\{\emptyset_1^{n}, Ӽ_1^{n}, Q^{n}, W^{n}\}$, $Q^{n} = <\emptyset, \emptyset, \{a\}>$, $W^{n} = <\{a\}, \emptyset, \emptyset>$ be $N_U Ç_ş$.
Therefore, (Ӽ, Ţ) is a $N_U ÇŢ^1_{(1,2)}$–space. Hence, $G^{n} = <\{b\}, \{a\}, \emptyset>$ is classified as semi – open $^2_{(1,2)}$, but it does not qualify as semi– open $^1_{(1,2)}$.

Theorem 6.6:

Let A^{n}_j be neutrosophic crisp semi– open $^s_{(1,2)}$sets, $s = 1,2, \forall j \in J$.Then $\cup_{2j} A^{n}_j$ is a neutrosophic crisp semi – open $^s_{(1,2)}$ set, $s = 1,2$.

Proof: Since A^{n}_j $\forall j \in J$ be neutrosophic crisp semi – open $^s_{(1,2)}$ set, $s = 1,2, \forall j \in J$.

Thus, $A^{n}_j \subseteq_i N_U ÇÇL_{(1,2)}(N_U ÇLnt_{(1,2)}(A^{n}_j))$ $i = 1,2, \forall j \in J$.Therefore,

$\cup_{2j} A^{n}_j \subseteq_i \cup_{2j} [N_U ÇÇL^{\breve{\alpha}}_{(1,2)} (N_U ÇLnt_{(1,2)}(A^{n}_j))] \subseteq_i [N_U ÇÇL^{\breve{\alpha}}_{(1,2)}[\cup_{2j} N_U ÇLnt_{(1,2)}(A^{n}_j))]$

$$\subseteq_i N_U ÇÇL_{(1,2)}\left[N_U ÇLnt_{(1,2)}\left(\cup_{2j} A^{n}_j\right)\right], i = 1,2.$$

Hence, $\cup_{2j} A^{n}_j \subseteq_i N_U ÇÇL_{(1,2)}[N_U ÇLnt_{(1,2)}(\cup_{2j} A^{n}_j)]$ $i = 1,2$.

Remarks 6.7:

- If A^{n} and B^{n} are two semi – open $^1_{(1,2)}$ sets, then $A^{n} \cap_1 B^{n}$ is not necessarily semi-open $^1_{(1,2)}$, as shown in Example 6.3. In particular, if $E^{n} = <\{c\}, \emptyset, \{a\}>$, and $G^{n} = <\{a, b\}, \emptyset, \{c\}>$, then $E^{n} \cap_1 G^{n} = <\emptyset, \emptyset, \{a, c\}>$ is not semi-open $^1_{(1,2)}$.
- If A^{n} and B^{n} are two semi – open $^2_{(1,2)}$ sets, then $A^{n} \cap_1 B^{n}$ is not necessarily semi-open $^2_{(1,2)}$, as shown in Example 6.3. In particular, if $E^{n} = <\{c\}, \emptyset, \{a\}>$, and $G^{n} = <\{a, b\}, \emptyset, \{c\}>$, then $E^{n} \cap_1 G^{n} = <\emptyset, \emptyset, \{a, c\}>$ is not semi-open $^2_{(1,2)}$.

Theorem 6.8:

If (Ӽ, Ţ) is a $N_U ÇŢ^2_{(1,2)}$–space. Then every neutrosophic crisp open set is a neutrosophic crisp semi– open $^s_{(1,2)}$, $s = 1,2$.

Proof : Let $B^{n} = <B_1, \emptyset, B_1{}^c>$ be a $N_U ÇO - set$. So, $N_U ÇLnt_{(1,2)}(B^{n}) = B^{n}$ by Theorem 4.8.
Thus, $N_U ÇÇL_{(1,2)}(N_U ÇLnt_{(1,2)}(B^{n})) = N_U ÇÇL_{(1,2)}(B^{n})$.

Therefore, $B^{n} = <B_1, \emptyset, B_1{}^c> \subseteq_i N_U ÇÇL_{(1,2)}(B^{n}), i = 1,2$.

Hence, $B^{n} \subseteq_i N_U ÇÇL_{(1,2)}(N_U ÇLnt_{(1,2)}(B^{n}))$. $i = 1,2$.

Remarks 6.9:

- Theorem 6.8 does not hold in the $N_U ÇŢ^1_{(1,2)}$–space , as illustrated by the following example:
  Let Ӽ = $\{a, b\}$, and Ţ = $\{\emptyset_1^{n}, Ӽ_1^{n}, Q^{n}, W^{n}\}$, $Q^{n} = <\emptyset, \emptyset, \{a\}>$, $W^{n} = <\{a\}, \emptyset, \emptyset>$ be $N_U Ç_ş$.
  Therefore, (Ӽ, Ţ) is a $N_U ÇŢ^1_{(1,2)}$–space. Hence, $Q^{n} = <\emptyset, \emptyset, \{a\}>$is classified as $N_U ÇO - set$, but it does not qualify as semi– open $^s_{(1,2)}$, $s = 1,2$.

- Theorem 6.8 does not hold in the $N_U ÇT^3_{(1,2)}$–space , as illustrated by the following example:
  Let $X = \{a, b\}$, and $T = \{\emptyset_1^n, X_1^n, A^n\}$, $A^n = <\{a\}, \emptyset, X>$ be a $N_U Ç_s$.
  Therefore, (X, T) is $a\, N_U ÇT^3_{(1,2)}$–space. Hence, $A^n = <\{a\}, \emptyset, X>$ is classified as $N_U ÇO - set$, but it does not qualify as semi– open $^{s}_{(1,2)}, s = 1,2$.

7. A Neutrosophic Crisp pre – open $^{i}_{(1,2)}$Topological Space ($N_U ÇT_{p_i}$), $i = 1, 2$.

In this section, two sharp neutrosophic topological spaces, $N_U ÇT_{p_1}$ and $N_U ÇT_{p_2}$, are constructed, which are generated by the sub-bases $N_U \beta p_1$ and $N_U \beta p_2$, respectively.

Definition 7.1:
Let (X, T) be a $N_U ÇT^t_{(1,2)}$–space, t=1,2,3. Then $\mathcal{M}^n \subseteq_1 X_1^n$ is called neutrosophic crisp pre– open $^1_{(1,2)}$ if and only if $\mathcal{M}^n \subseteq_1 N_U ÇInt_{(1,2)}\left(N_U ÇÇL_{(1,2)}(\mathcal{M}^n)\right)$. The family of all neutrosophic pre – open $^1_{(1,2)}$denoted by $N_U \beta p_1$ such that

$$N_U \beta p_1 = \left\{\mathcal{M}^n \subseteq_1 X_1^n : \mathcal{M}^n \subseteq_1 N_U ÇInt_{(1,2)}\left(N_U ÇÇL_{(1,2)}(\mathcal{M}^n)\right)\right\}.$$

- $N_U \beta p_1$ is called a neutrosophic crisp topological sub-basis.
- A Neutrosophic Crisp pre– open $^1_{(1,2)}$Topological Space ($N_U ÇT_{p_1}$)is the smallest sharp neutrosophic topological space generated by the sub-basis $N_U \beta p_1$.

Definition 7.2:
Let (X, T) be a $N_U ÇT^t_{(1,2)}$–space, t=1,2,3. Then $\mathcal{M}^n \subseteq_2 X_1^n$ is called neutrosophic crisp pre– open $^2_{(1,2)}$ if and only if $\mathcal{M}^n \subseteq_2 N_U ÇInt_{(1,2)}\left(N_U ÇÇL_{(1,2)}(\mathcal{M}^n)\right)$. The family of all neutrosophic pre – open $^2_{(1,2)}$denoted by $N_U \beta p_2$ such that

$$N_U \beta p_2 = \left\{\mathcal{M}^n \subseteq_1 X_1^n : \mathcal{M}^n \subseteq_2 N_U ÇInt_{(1,2)}\left(N_U ÇÇL_{(1,2)}(\mathcal{M}^n)\right)\right\}.$$

- $N_U \beta p_2$ is called a neutrosophic crisp topological sub-basis.
- A Neutrosophic Crisp pre– open $^2_{(1,2)}$Topological Space ($N_U ÇT_{p_2}$)is the smallest sharp neutrosophic topological space generated by the sub-basis $N_U \beta p_2$.

The following example illustrates how to obtain the smallest sharp neutrosophic topological spaces $N_U \beta p_1$ generated by the sub- basis $N_U \beta p_1$.

Example 7.3:
Let $X = \{a, b, c\}$, and $\mathcal{T} = \{\emptyset_1^n, X_1^n, H_{28}^n, H_3^n\}$, $H_{28}^n = <\{a\}, \{b\}, \{c\}>$, $H_3^n = <\{a\}, \emptyset, \{c\}>$.
Then $(X, \mathcal{T})$ is a $N_U Ç\mathcal{T}^1_{(1,2)}$–space. Therefore,

$$N_U \beta p_1 = \left\{\begin{matrix} H_1^n, H_2^n, H_3^n, H_4^n, H_5^n, H_6^n, H_7^n, H_8^n, H_9^n, H_{10}^n, H_{11}^n, H_{12}^n, H_{13}^n, H_{14}^n, H_{16}^n, H_{17}^n, H_{18}^n, \\ H_{20}^n, H_{21}^n, H_{22}^n, H_{23}^n, H_{24}^n H_{25}^n, H_{26}^n, H_{27}^n \end{matrix}\right\}$$

Hence,

$$\mathrm{N_U Ç}\mathcal{T}_{p_1} = \left\{ \begin{matrix} H^{n}_{1}, H^{n}_{2}, H^{n}_{3}, H^{n}_{4}, H^{n}_{5}, H^{n}_{6}, H^{n}_{7}, H^{n}_{8}, H^{n}_{9}, H^{n}_{10}, H^{n}_{11}, H^{n}_{12}, H^{n}_{13}, H^{n}_{14}, H^{n}_{16}, H^{n}_{17}, H^{n}_{18}, \\ H^{n}_{20}, H^{n}_{21}, H^{n}_{22}, H^{n}_{23}, H^{n}_{24} H^{n}_{25}, H^{n}_{26}, H^{n}_{27}, H^{n}_{19} \end{matrix} \right\}$$

And

$$\mathrm{N_U}\beta p_2 = \left\{ \begin{matrix} H^{n}_{1}, H^{n}_{2}, H^{n}_{3}, H^{n}_{4}, H^{n}_{5} H^{n}_{6}, H^{n}_{7}, H^{n}_{8}, H^{n}_{9}, \quad H^{n}_{10}, H^{n}_{11}, H^{n}_{12}, H^{n}_{13}, H^{n}_{14}, H^{n}_{16}, H^{n}_{17}, H^{n}_{18}, \\ H^{n}_{20}, H^{n}_{21}, H^{n}_{22}, H^{n}_{23}, H^{n}_{24} H^{n}_{25}, H^{n}_{26}, H^{n}_{27}, H^{n}_{28}, H^{n}_{29}, H^{n}_{30}, H^{n}_{31}, H^{n}_{32}, H^{n}_{33}, H^{n}_{34}, H^{n}_{35}, \\ H^{n}_{36}, H^{n}_{37}, H^{n}_{38}, H^{n}_{39}, H^{n}_{40} H^{n}_{41}, H^{n}_{42}, H^{n}_{43}, H^{n}_{44}, H^{n}_{45}, H^{n}_{46}, H^{n}_{47}, H^{n}_{48}, H^{n}_{49}, H^{n}_{50}, H^{n}_{51}, \\ H^{n}_{52}, H^{n}_{53}, H^{n}_{55}, H^{n}_{56}, H^{n}_{57} H^{n}_{58}, H^{n}_{59}, H^{n}_{60}, H^{n}_{61}, H^{n}_{62}, H^{n}_{63}, H^{n}_{64} \end{matrix} \right\}$$

$$\mathrm{N_U Ç}\mathcal{T}_{p_2} = \left\{ \begin{matrix} H^{n}_{1}, H^{n}_{2}, H^{n}_{3}, H^{n}_{4}, H^{n}_{5} H^{n}_{6}, H^{n}_{7}, H^{n}_{8}, H^{n}_{9}, \quad H^{n}_{10}, H^{n}_{11}, H^{n}_{12}, H^{n}_{13}, H^{n}_{14}, H^{n}_{16}, H^{n}_{17}, H^{n}_{18}, \\ H^{n}_{20}, H^{n}_{21}, H^{n}_{22}, H^{n}_{23}, H^{n}_{24} H^{n}_{25}, H^{n}_{26}, H^{n}_{27}, H^{n}_{28}, H^{n}_{29}, H^{n}_{30}, H^{n}_{31}, H^{n}_{32}, H^{n}_{33}, H^{n}_{34}, H^{n}_{35}, \\ H^{n}_{36}, H^{n}_{37}, H^{n}_{38}, H^{n}_{39}, H^{n}_{40} H^{n}_{41}, H^{n}_{42}, H^{n}_{43}, H^{n}_{44}, H^{n}_{45}, H^{n}_{46}, H^{n}_{47}, H^{n}_{48}, H^{n}_{49}, H^{n}_{50}, H^{n}_{51}, \\ H^{n}_{52}, H^{n}_{53}, H^{n}_{55}, H^{n}_{56}, H^{n}_{57} H^{n}_{58}, H^{n}_{59}, H^{n}_{60}, H^{n}_{61}, H^{n}_{62}, H^{n}_{63}, H^{n}_{64}, H^{n}_{19}, H^{n}_{54} \end{matrix} \right\}$$

Such that

$H^{n}_{1} =< \{a\}, \emptyset, \{b\} >$	$H^{n}_{2} =< \{b\}, \emptyset, \{a\} >$	$H^{n}_{3} =< \{a\}, \emptyset, \{c\} >$	$H^{n}_{4} =< \{c\}, \emptyset, \{a\} >$
$H^{n}_{5} =< \{b\}, \emptyset, \{c\} >$	$H^{n}_{6} =< \{c\}, \emptyset, \{b\} >$	$H^{n}_{7} =< \emptyset, \emptyset, \{a\} >$	$H^{n}_{8} =< \emptyset, \emptyset, \{b\} >$
$H^{n}_{9} =< \emptyset, \emptyset, \{c\} >$	$H^{n}_{10} =< \{a\}, \emptyset, \emptyset >$	$H^{n}_{11} =< \{b\}, \emptyset, \emptyset >$	$H^{n}_{12} =< \{c\}, \emptyset, \emptyset >$
$H^{n}_{13} =< \{a\}, \emptyset, \{b, c\} >$	$H^{n}_{14} =< \{b\}, \emptyset, \{a, c\} >$	$H^{n}_{15} =< \{c\}, \emptyset, \{a, b\} >$	$H^{n}_{16} =< \{a, b\}, \emptyset, \{c\} >$
$H^{n}_{17} =< \{a, c\}, \emptyset, \{b\} >$	$H^{n}_{18} =< \{b, c\}, \emptyset, \{a\} >$	$H^{n}_{19} =< \emptyset, \emptyset, \{a, b\} >$	$H^{n}_{20} =< \{a, b\}, \emptyset, \emptyset >$
$H^{n}_{21} =< \{a, c\}, \emptyset, \emptyset >$	$H^{n}_{22} =< \{b, c\}, \emptyset, \emptyset >$	$H^{n}_{23} =< \emptyset, \emptyset, \{a, c\} >$	$H^{n}_{24} =< \emptyset, \emptyset, \{b, c\} >$
$H^{n}_{25} =< \emptyset, X, \emptyset >$	$H^{n}_{26} =< \emptyset, \emptyset, X >$	$H^{n}_{27} =< \emptyset, \emptyset, \emptyset >$	$H^{n}_{28} =< \{a\}, \{b\}, \{c\} >$
$H^{n}_{29} =< \{b\}, \{c\}, \{a\} >$	$H^{n}_{30} =< \{a\}, \{c\}, \{b\} >$	$H^{n}_{31} =< \{b\}, \{a\}, \{c\}$	$H^{n}_{32} =< \{c\}, \{a\}, \{b\} >$
$H^{n}_{33} =< \{c\}, \{b\}, \{a\} >$	$H^{n}_{34} =< \{a\}, \{b\}, \emptyset >$	$H^{n}_{35} =< \{b\}, \{a\}, \emptyset >$	$H^{n}_{36} =< \{a\}, \{c\}, \emptyset >$
$H^{n}_{37} =< \{c\}, \{a\}, \emptyset >$	$H^{n}_{38} =< \emptyset, \{a\}, \{b\} >$	$H^{n}_{39} =< \emptyset, \{a\}, \{c\} >$	$H^{n}_{40} =< \emptyset, \{b\}, \{a\} >$
$H^{n}_{41} =< \emptyset, \{c\}, \{a\} >$	$H^{n}_{42} =< \emptyset, \{c\}, \{b\} >$	$H^{n}_{43} =< \{b\}, \{c\}, \emptyset >$	$H^{n}_{44} =< \{c\}, \{b\}, \emptyset >$
$H^{n}_{45} =< \{b, c\}, \{a\}, \emptyset >$	$H^{n}_{46} =< \{a, b\}, \{c\}, \emptyset >$	$H^{n}_{47} =< \{a, c\}, \{b\}, \emptyset >$	$H^{n}_{48} =< \{a\}, \{b, c\}, \emptyset >$
$H^{n}_{59} =< \{b\}, \{a, c\}, \emptyset >$	$H^{n}_{50} =< \{c\}, \{a, b\}, \emptyset >$	$H^{n}_{51} =< \emptyset, \{a, b\}, \{c\} >$	$H^{n}_{52} =< \emptyset, \{b\}, \{c\} >$
$H^{n}_{53} =< \emptyset, \{c\}, \{a, b\} >$	$H^{n}_{54} =< \emptyset, \{b\}, \{a, c\} >$	$H^{n}_{55} =< \emptyset, \{a\}, \{b, c\} >$	$H^{n}_{56} =< \emptyset, \{b, c\}, \{a\} >$
$H^{n}_{57} =< \emptyset, \{b\}, \{a, c\} >$	$H^{n}_{58} =< \emptyset, \{a\}, \emptyset >$	$H^{n}_{59} =< \emptyset, \{b\}, \emptyset >$	$H^{n}_{60} =< \emptyset, \{c\}, \emptyset >$
$H^{n}_{61} =< \emptyset, \{a, b\}, \emptyset >$	$H^{n}_{62} =< \emptyset, \{a, c\}, \emptyset >$	$H^{n}_{63} =< \emptyset, \{b, c\}, \emptyset >$	$H^{n}_{64} =< \emptyset, X, \emptyset >$

Theorem 7.4:

Every neutrosophic crisp pre – open $^{1}_{(1,2)}$ is a neutrosophic crisp pre – open $^{2}_{(1,2)}$.

Proof: The proof is similar to that of Theorem 6.4.

Remark 7.5:

The converse of Theorem 6.4 is not true, as illustrated by the following example:

Let Ҳ = $\{a, b\}$, and Ʈ = $\{\emptyset_1^{n}, Ҳ_1^{n}, Q^{n}, W^{n}\}$, $Q^{n} =< \emptyset, \emptyset, \{a\} >$, $W^{n} = < \{a\}, \emptyset, \emptyset >$ be $N_U Ç_ş$.
Therefore, (Ҳ, Ʈ) is a $N_U ÇƮ^1_{(1,2)}$–space. Hence, $D^{n} =< \emptyset, \{a\}, \emptyset >$ is classified as pre – open $^{2}_{(1,2)}$, but it does not qualify as pre – open $^{1}_{(1,2)}$.

Theorem 7.6:

Let $A^{n}{}_j$ be neutrosophic crisp pre – open $^{s}_{(1,2)}$sets, $s = 1,2, \forall\, j \in J$.Then $\cup_{2j} A^{n}{}_j$ is a neutrosophic crisp pre – open $^{s}_{(1,2)}$ set, $s = 1,2$.

Proof: The proof is similar to that of Theorem 6.6.

Remarks 7.7:

- If A^{n} and B^{n} are two pre – open $^{1}_{(1,2)}$ sets, then $A^{n} \cap_1 B^{n}$ is not necessarily pre–open $^{1}_{(1,2)}$, as shown in Example 7.3. In particular, if $H_2^{n} =< \{b\}, \emptyset, \{a\} >$, and $H_8^{n} =< \emptyset, \emptyset, \{b\} >$, then $H_2^{n} \cap_1 H_8^{n} =< \emptyset, \emptyset, \{a, b\} >$ is not pre–open $^{1}_{(1,2)}$.
- If A^{n} and B^{n} are two pre– open $^{2}_{(1,2)}$ sets, then $A^{n} \cap_1 B^{n}$ is not necessarily pre–open $^{2}_{(1,2)}$, as shown in Example 7.3. In particular, if $H_1^{n} =< \{a\}, \emptyset, \{b\} >$, and $H_2^{n} = < \{b\}, \emptyset, \{a\} >$, then $H_1^{n} \cap_1 H_2^{n} =< \emptyset, \emptyset, \{a, b\} >$ is not pre–open $^{2}_{(1,2)}$.

Theorem 7.8:

If (Ҳ, Ʈ) is a $N_U ÇƮ^2_{(1,2)}$–space. Then every neutrosophic crisp open set is a neutrosophic crisp pre– – open $^{s}_{(1,2)}$, $s = 1,2$.

Proof : Let $Ɓ^{n} =< Ɓ_1, \emptyset, Ɓ_1{}^{c} >$ be $N_U ÇO - set$.
So, $Ɓ^{n} = < Ɓ_1, \emptyset, Ɓ_1{}^{c} > \subseteq_i N_U ÇÇL_{(1,2)}(Ɓ^{n}), i = 1,2$.
Therefore, $Ɓ^{n} = N_U ÇLnt_{(1,2)}(Ɓ^{n}) \subseteq_i N_U ÇLnt_{(1,2)}\left(N_U ÇÇL_{(1,2)}(Ɓ^{n})\right), i = 1,2$.

Hence, $Ɓ^{n} \subseteq_i N_U ÇLnt_{(1,2)}(N_U ÇÇL_{(1,2)}(Ɓ^{n})). i = 1,2$.

Remarks 7.9:

- Theorem 7.8 does not hold in the $N_U ÇƮ^1_{(1,2)}$–space , as illustrated by the following example:
  Let Ҳ = $\{a, b, c\}$, and Ʈ = $\{\emptyset_1^{n}, Ҳ_1^{n}, A^{n}, B^{n}, C^{n}, D^{n}\}$, $A^{n} =< \emptyset, \emptyset, \{a, c\} >$, $B^{n} = < \{a\}, \emptyset, \emptyset >$, $C^{n} =< \{a\}, \emptyset, \{b\} >$, and $D^{n} =< \{a\}, \{c\}, \{b\} >$. be $N_U Ç_ş$.
  Therefore, (Ҳ, Ʈ) is a $N_U ÇƮ^1_{(1,2)}$–space. Hence, $D^{n} =< \{a\}, \{c\}, \{b\} >$ is classified as $N_U ÇO - set$, but it does not qualify as pre – open $^{s}_{(1,2)}$, $s = 1,2$.

- Theorem 7.8 does not hold in the $\mathrm{N_U ÇT^3_{(1,2)}}$–space , as illustrated by the following example:
  Let $X = \{a, b, c\}$, and $T = \{\emptyset_1^{n}, X_1^{n}, A^{n}, B^{n}, C, D^{n}, E^{n}\}$, $A^{n} =< X, \emptyset, \{a, c\} >$, $B^{n} =< \{b\}, \emptyset, X >$, $C^{n} =< \{a\}, \emptyset, X >$, $D^{n} =< \{a\}, \{b\}, X >$, $E^{n} =< \{a, b\}, \emptyset, X >$, be a $\mathrm{N_U Ç_s}$.
  Therefore, (X, T) is $a\ \mathrm{N_U ÇT^3_{(1,2)}}$–space. Hence, $D^{n} =< \{a\}, \{b\}, X >$ is classified as $\mathrm{N_U ÇO} - set$, but it does not qualify as pre – open $^{s}_{(1,2)}, s = 1,2$.

Theorem 7.10:

Every neutrosophic crisp open set is a neutrosophic crisp pre – open $^{2}_{(1,2)}$.

Proof:

Let $B^{n} =< B_1, B_2, B_3 >$ be a $\mathrm{N_U ÇO} - set$ and let $\{\mathcal{F}^{n}_j = <\mathcal{F}_{1j}, \mathcal{F}_{2j}, \mathcal{F}_{3j}>: j \in J\}$ be the family of $\mathrm{N_U ÇÇ} - set$ such that $B_1 \subseteq \mathcal{F}_{3j}{}^c\ \forall j$, $B_2 \subseteq \mathcal{F}_{2j}{}^c \forall j$, $B_3{}^c \subseteq \mathcal{F}_{1j} \forall j$.

Thus, $\mathrm{N_U ÇÇL}^{\breve{\alpha}}_{(1,2)}(B^{n}) \bigcap_{1j} \mathcal{F}^{n}_j =<\cap_j \mathcal{F}_{j1}, \cap_j \mathcal{F}_{j2}, \cup_j \mathcal{F}_{j3}>$.

Now let and $\{\mathfrak{S}^{n}_j =< \mathfrak{S}_{1j}, \mathfrak{S}_{2j}, \mathfrak{S}_{3j} >: j \in J\}$ be the family of $\mathrm{N_U ÇO} - set$ such that

$\mathfrak{S}_{1j}{}^c \supseteq \cup_j \mathcal{F}_{3j} \forall j$, $\mathfrak{S}_{2j}{}^c \supseteq \cap_j \mathcal{F}_{3j} \forall j$, $\mathfrak{S}_{3j}{}^c \subseteq \cap_j \mathcal{F}_{j1} \forall j$.

Therefore, $\mathrm{N_U ÇLnt}^{\breve{\alpha}}_{(1,2)}(\mathrm{N_U ÇÇL}_{(1,2)}(B^{n})) = \bigcup_{2j} \mathfrak{S}^{n}_j =<\cup \mathfrak{S}_{1j}, \cap \mathfrak{S}_{2j}, \cap \mathfrak{S}_{3j}>$.

Since, $\mathfrak{S}_{1j}{}^c \supseteq \cup_j \mathcal{F}_{3j}\ \forall j$, and $B_1 \subseteq \mathcal{F}_{3j}{}^c\ \forall j$. Thus, $\mathcal{F}_{3j} \subseteq B_1{}^c\ \forall j$. So, $\cup_j \mathcal{F}_{3j} \subseteq B_1{}^c$.

Since, $B_1{}^c \in \{\mathfrak{S}_{1j}{}^c\}$. Therefore, $(\cap_j \mathfrak{S}_{1j})^c \subseteq B_1{}^c$. Hence, $B_1 \subseteq \cup_j \mathfrak{S}_{1j}$. ...(1)

Since $\mathfrak{S}_{2j}{}^c \supseteq \cap_j \mathcal{F}_{2j}\ \forall j$, and $B_2 \subseteq \mathcal{F}_{2j}{}^c\ \forall j$. Thus, $\mathcal{F}_{2j} \subseteq B_2{}^c$. So, $\cap_j \mathcal{F}_{2j} \subseteq B_2{}^c$

Since $B_2{}^c \in \{\mathfrak{S}_{2j}{}^c\}$. Therefore, $(\cup_j \mathfrak{S}_{2j})^c \supseteq B_2{}^c$. Hence, $B_2 \supseteq \cap_j \mathfrak{S}_{2j}$. ...(2)

Since $\mathfrak{S}_{3j}{}^c \subseteq \cap_j \mathcal{F}_{1j} \forall j$ and $B_3{}^c \subseteq \mathcal{F}_{1j} \forall j$. Thus, $B_3{}^c \subseteq \cap_j \mathcal{F}_{1j}$.

Since $B_3{}^c \in \{\mathfrak{S}_{3j}{}^c\}$. Therefore, $(\cup_j \mathfrak{S}_{3j})^c \supseteq B_3{}^c$. Hence, $B_3 \supseteq \cap_j \mathfrak{S}_{3j}$. ...(3)

From (1), (2) and (3), we get $B^{n} \subseteq_2 \mathrm{N_U ÇLnt}_{(1,2)}(\mathrm{N_U ÇÇL}_{(1,2)}(B^{n}))$.

8. Conclusions:

In this study, four neutrosophic crisp topological spaces were constructed through new definitions of the closure of a neutrosophic crisp set as well as new definitions of the interior of a neutrosophic crisp set. Moreover, all classical topological theorems, results, and relations that hold in these four neutrosophic crisp topological spaces were proved, and illustrative examples were provided for those that do not hold.

In future work, these four neutrosophic crisp topological spaces may be employed to study other general topological concepts, such as separation axioms, compact spaces, and Lindelöf spaces. In addition, further investigations of these six spaces may be incorporated into future scientific research (see [7–11]) within the framework of neutrosophic crisp topological spaces, which may lead to the development of new theorems, novel results, and innovative concepts.

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