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The Use of Topological Homeomorphism in Architectural Design

Authors Names	**ABSTRACT**
Hadeel Husham Kadhim *Publication date: 30 / 5 /2026* *Keywords:* Homeomorphism topology , Geometric Flexibility .	Topology , a branch of mathematics called elastic geometry , because it is studies properties without focusing on length or angles the geometric properties of shapes that are preserved of shapes that are preserved when subjected to stretching , contraction , and twisting without tearing or closing holes .

1. Introduction

Topology is involved in understanding the structure of the DNA molecule specifically in studying its structural properties . Topology also utilizes , architecture , and sculpture , where artists employ concepts such as single- faced surfaces.

The concept of Topological space was introduced in 1970 by S. Willard. S. A. Morse [2] presented homeomorphism of topological space . M.P. BendsØe . and O. Sigmund are studied in [1] Topology Optimization (Theory , Methods and Applications

In 1736 (Leonhard Euler) : published the first topological paper (Konigsberg Bridges) . In 1847 (Johan Lesting) used word " Topology " for the first time . Late 19th-early 20th century : The science developed significantly with Cantors set theory , then T_i- space when i= $0,\frac{1}{2}$, 1,2 ,3 ,4 (1914) , and the modern definition by Koratowski in 1922 .

2. Preliminaries

This section introduces definition of topological space and topological homeomorphisms .All these concepts are found in references [2,3]

Definition(2.1)[3] If is X$\neq \emptyset$. Then the collection τ of subsets of X is called topology for X if τ satisfies the following axioms

i- X and $\emptyset \in \tau$
ii- If A_1 and A_1 are any two sets in τ then $A_1 \cap A_1$ subset in τ
iii- If $\{ A_\gamma : \gamma \in \Delta\} \in \tau$ then $\cup\{ A_\gamma : \gamma \in \Delta\} \in \tau$.
If τ topology on X then (X, τ) is called topological space . the member of τ is called open sets the complement of open is called closed set .

Definition(2.2.1)[2]A function $f:(X,\tau) \to (Y,\tau')$ is called homeomorphism if it is bijective , continuous and open .

When the above conditions are met thus X and Y have the same topological properties .

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Examples and antonyms

Example(2.2.1) A metal spring , if you pull or compress it changes its length (geometric properties) but its topological properties don't changes such as .

Connectedness :The spring is a single , continuous process(which is a topological homeomorphism)cannot transform a continuous object into separate .

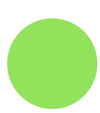
– Neighborhoods : points that were "neighbors" to each other on the wire before being pulled remain neighbors afterward . two adjacent points cannot be separated without being "torn" and this is forbidden in symmetry

Figures(1) and (2) an example of the concept in mathematics , specifically how some properties remain constant of the object changes

Figer(1)

Figer(2)

Example(2.2.2) The circle , square and rectangle are topological homeomorphisms through the concept of " complete elasticity"
If you have a very flexible rubber ring . If you leave it as it is , it's a circle . If you pull it from four equal points , it will turn into a square . If you stretch two opposite sides of a square more than others , it becomes a rectangle
Why is one formulation considered topological ?
- Connectedness : In all three cases you didn't need to cut the elastic or glue parts together .
– Neighborhoods : Any point on the circle has "neighbors" that remain its neighbors after it's transformed into a square (the fabric doesn't tear)
That is the square and the rectangle are circles with angles . Topology does not distinguish between angles ;it assumes you can return to the initial state without interruption .
The figer(1) ,(2) and (3) provide a further example of the concept of homeomorphism in topology , and confirm that these shapes are considered topologically equivalent .



Figer(1)

Figer(2)

Figer(3)

Example(2.2.3) The latter **" D "** and the latter **" B "** (the first contains one ring and the second contains two rings , one cannot be converted to the other without cutting or gluing)

3. Using Homeomorphism in architectural design

In architecture , homeomorphism topological space is applied by focusing on "spatial relationships " rather than rigid geometric shapes . The topological architect is concerned with "How do we get from point A to point B ? and not with "is the angle 90 degrees or curved?"

3.1. Leveraging the experience of the spring in architectural design

A spring is essentially a " straight line "wound around a cylinder . In architecture this principle is used to create continuous ramps instead of traditional stairs

3.1.1. Architectural application : Replacing long horizontal corridors with compressed spiral paths . The engineer does not changes the "type "of path, but rather its "geometry" to save space , while maintaining a smooth transition from point (A) to point(B)without interruption .

3.1.2. Transforming a " Cylinder" into a " Twisted Tower "

A tower with a spring-like shape **(like the Cayan Tower in Dubai)** is essentially a cylinder that has undergone topological torsion.

This homeomorphism allows the engineer to distribute loads helically. Instead of the force being transmitted only vertically , it is distributed topologically along the twisting path, increasing resistance to wind and earthquakes.

3.1.3. Structural flexibility (continuous deformation):

Homeomorphism topology allows a building to " breath ."Spring- like structures can expand and contract (like a mechanical spring) to cope with thermal changes without the structure collapsing .because the topology ensures that the parts remain connected regardless of changes in the external shape .

3.2. Local homeomorphism

It is used in architecture when we want to deal with the complex surfaces by breaking them down into simple and familiar parts (such as a flat plane).

How do architects use local topological homeomorphism in the design of buildings and bridges ?

3.2.1. Complex surface Tessellation : Buildings with organic shapes (such as the museum of the Future in Dubai or the works of Zaha Hadid) are complex topological forms .

A fully curved surface is divided into small pieces (such as triangles or squares). Each small piece is locally homeomorphism to a simple " Euclidean plane" This allows for the fabrication and assembly of flat glass or metal panels to form a complex overall curvature .

3.2.2. Stress Analysis in Bridges : In suspension or helical bridges , the forces and stress directions are constantly changing along the span.

Engineers use local symmetry to simplify the study of a specific "point" on the bridge . Topologically , a small area around any point is treated as part of a straight line or flat surface , allowing for precise load calculations before generalization to the overall curved shape

3.2.3.Maps and Architectural Representation: An engineer needs to translate a three-dimensional "mass" of a building into two-dimensional " plans"

A plan (map) is a local representation of the building's surface . this ensures that the relationships between rooms and columns remain accurate when transferred from paper to the building , preserving overhangs and connections without distortion that would compromise functionality.

4. Conclusions

Topological homeomorphism demonstrate their importance in architectural design by enabling flexible formal transformations of forms while preserving the fundamental structure of each form . The application of the spring emerges as a practical tool that allows for the systematic and consistent transformation of architectural forms, reflecting the ability of mathematics and digital technologies to support modern architectural creativity.

References

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