



Chaos In Soft Topological Spaces Based On Soft Point

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Abstract: In this work, new space is introduced called soft chaos space which is denoted by $S\tilde{C}$ -space. Some properties on this space are proved. Also, we prove for a soft continuous mapping on a soft topological space $(\tilde{X}_A, \tilde{\tau}, A)$, that there is relation between soft transitivity of \tilde{f} and soft density of soft orbit of \tilde{f} , and soft transitivity of \tilde{f} has a soft density of soft periodicity.

Keywords: soft dense set, soft isolated point, soft nowhere dense set, soft chaotic mapping, soft chaos space.

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1. Introduction:

The soft set was introduced by Molodtsov D. [1] in 1999 to handle the indeterminate information. In 2002, Maji K. et al they introduced the definition of processes on soft sets and their properties of soft sets, see [2], [3], [4] and [5]. After that Sujoy D. et al [10], specified the definition of soft set which is called soft point is a generalization of a crisp point. Soft point is applied to a soft topological structures.

We observe it is main properties and equivalent characterizations, see [6], [7], [8], [11], [13], [14], [15], [16], [19] and [22]. Weijin R. in 2012, was introduced new concept of the soft topological space called the soft second countable, [21]. Also In 2012, sujoy D. [9], introduced soft real sets and soft real numbers and their properties. In 2012, Sujoy D. and Smanta S. [10], were introduced the concept of soft metric spaces. In 2014, the concept of soft isolated points of a soft sets, soft dense sets and soft nowhere dense was first introduced by Subhashinin J. et al [16].

Al-aamery N and Al-Swidi L. in 2016, [18], were introduced a new type of soft topological spaces called soft second category. Now chaos is the science of the unexpected, it is very important in our lives. Many scientists have studied chaos and give different definitions of chaos such as Vinod K., [20]. Finally, in this work, we study some of the chaotic properties such as soft orbit, soft periodicity, and soft sensitivity.

2. Background Material:

We introduced some elementary concept which we need in our work. In this section, we give basic definitions in soft topological spaces and some necessary relationship and speculation. Also, we will introduced necessary notations introduced in soft topological space such as soft second countable and soft second category. Finally, we introduced some results about soft isolated points of a soft set, soft dense set and soft nowhere dense set.



Definition (2.1) [1]:

Let X be a universe set and E be a set of parameters , $P(X)$ the power set of X and $A \subseteq E$. A pair (F, A) is called soft set over X with respect to A and F is a mapping given by:

$$F: A \rightarrow P(X) \text{ with } (F, A) = \{F(e) \in P(X): e \in A\}.$$

(i) $F(e)$, for all $e \in A$ may be arbitrary set, may be empty set.

(ii) The soft set can be represented by two ways:

- $F_A = (F, A) = \{ F(e) \in P(X), \text{ for all } e \in A \}$.

- By ordered pairs:

$$F_A = \{ (e, F(e)) : \text{for all } e \in A, F(e) \in P(X) \}.$$

Definition (2.2) [10]:

A soft (F, A) over a universe X is called a soft point and its denoted by:

$$x^e = \{(e, F(e))\}, \text{ if exactly one } e \in A, F(e) = \{x\} \text{ for some } x \in X \text{ and } F(e') = \emptyset \text{ for all } e' \in A \setminus \{e\}.$$

Definition (2.3) [16]:

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a *STS* and F_A be a soft set over X . Then:

(i) A soft point x^e of a soft set F_A is called a soft limit point of F_A if and only if for every soft nbhd U_A of x^e , then $F_A \tilde{\cap} U_A \tilde{\setminus} \{x^e\} \neq \tilde{\emptyset}_A$. The soft set of all soft limit points of a soft set F_A is called soft derived set of F_A , and denoted by F_A' . It easy x^e is not soft limit point of a soft set F_A , if there is a soft nbhd U_A of x^e with $F_A \tilde{\cap} U_A \tilde{\setminus} \{x^e\} = \tilde{\emptyset}_A$.

(ii) A soft point x^e of F_A is called soft isolated point (for soft *s. i. p.*) of F_A , if there is a soft open set U_A of x^e such that $F_A \tilde{\cap} U_A \tilde{\setminus} \{x^e\} = \tilde{\emptyset}_A$.

(iii) A soft set F_A is called soft dense set (for short $S\tilde{D}$) if and only if $\tilde{cl}(F_A) = \tilde{X}_A$.

(iv) A soft set F_A is called $S\tilde{D}$ in itself if and only if it has no *s. i. p.*

(v) A soft set F_A is called soft nowhere dense set (for short $S\tilde{N}\tilde{D}$), if and only if $\tilde{int}(\tilde{cl}(F_A)) = \tilde{\emptyset}_A$.

Definition (2.4):

A *STS* $(\tilde{X}_A, \tilde{\tau}, A)$ is called:

(i) Soft second countable, if $(\tilde{X}_A, \tilde{\tau}, A)$ has a countable soft base, [21].

(ii) Soft second category, if for any soft set F_A of $(\tilde{X}_A, \tilde{\tau}, A)$ cannot be represented as a countable union of $S\tilde{N}\tilde{D}$, [18].

3. Soft Transitivity And Soft Sensitivity:

In this section we quote some definitions, theorems and remarks in *STS*'s such as soft orbit, soft periodic, soft transitive, soft sensitive and their relation with soft density of a soft set and soft isolated points of a soft sets.



Definition (3.1):

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a STS. Then:

A **soft orbit** (for short $S\tilde{O}$) of a soft point x^e under the soft mapping $\tilde{f} : (\tilde{X}_A, \tilde{\tau}, A) \rightarrow (\tilde{X}_A, \tilde{\tau}, A)$, is denoted by $\tilde{O}_{\tilde{f}}(x^e)$ and defined as:

$$\tilde{O}_{\tilde{f}}(x^e) = \{x^e, \tilde{f}^1(x^e), \tilde{f}^2(x^e), \dots, \tilde{f}^n(x^e)\};$$

where $\tilde{f}^1(x^e) = \tilde{f}(x^e)$, $\tilde{f}^2(x^e) = \tilde{f}(x^e) \circ \tilde{f}(x^e)$, etc. More generally, if n is any integer, then n is n -th iterate of x^e for \tilde{f} .

The collection of all $S\tilde{O}$'s of all soft points x^e under \tilde{f} is denoted by $S\tilde{O}_{\tilde{f}}(x^e)$.

Definition (3.2):

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a STS. Then:

For a soft point x^e is called a **soft periodic point**

(for short $S\tilde{P}$) of a soft mapping

$\tilde{f} : (\tilde{X}_A, \tilde{\tau}, A) \rightarrow (\tilde{X}_A, \tilde{\tau}, A)$, and it is denoted by $\tilde{P}_{\tilde{f}}(x^e)$, if $\tilde{f}^n(x^e) = x^e$ for some $n \in \mathbb{Z}_+$.

In this case x^e may be called n -soft iterates (the smallest positive integer n satisfy: $\tilde{f}^n(x^e) = x^e$

called soft period of \tilde{f}). A soft periodic set is denoted by $S\tilde{P}_{\tilde{f}}(x^e)$.

Remark (3.3):

It is clear that $\tilde{P}_{\tilde{f}}(x^e)$ and $\tilde{O}_{\tilde{f}}(x^e)$ are independent.

Example (3.4):

Let \mathbb{R} be a real line and let $A = \{e\}$. Then $\tilde{\tau} = \{F_A : F(e) = [a, b] ; a < b\}$ is ST on \mathbb{R} . A soft mapping $\tilde{f} : (\tilde{\mathbb{R}}_A, \tilde{\tau}, A) \rightarrow (\tilde{\mathbb{R}}_A, \tilde{\tau}, A)$ such that $\tilde{f}(r^e) = -r^e$ for all $r \in \mathbb{R}$. Then r^e is a $S\tilde{P}$ point of a soft period 2.

i.e. $\tilde{P}_{\tilde{f}}(r^e) = \{r^e\}$, but $\tilde{O}_{\tilde{f}}(r^e) = \{r^e, -r^e, r^e\}$.

Definition (3.5):

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a STS. Then:

(i) A soft mapping $\tilde{f} : (\tilde{X}_A, \tilde{\tau}, A) \rightarrow (\tilde{X}_A, \tilde{\tau}, A)$ is called a **soft transitive** (for short $S\tilde{T}$) on $(\tilde{X}_A, \tilde{\tau}, A)$, if for any non-null soft open sets U_A and V_A in $(\tilde{X}_A, \tilde{\tau}, A)$, there is $n \in \mathbb{Z}_+$ such that: $\tilde{f}^n(U_A) \tilde{\cap} V_A \neq \tilde{\emptyset}_A$.

(ii) Let $\tilde{f} : (\tilde{X}_A, \tilde{\tau}, A) \rightarrow (\tilde{X}_A, \tilde{\tau}, A)$ be a soft continuous mapping. Then \tilde{f} is called a **soft sensitive** ($S\tilde{S}$) at x^e , if for any soft open set U_A containing x^e , there is a soft open set V_A and $n \in \mathbb{Z}_+$ such that $\tilde{f}^n(x^e) \tilde{\in} V_A$, $\tilde{f}^n(y^e) \tilde{\notin} \tilde{cl}(V_A)$ and $y^e \tilde{\in} U_A$.

Also, \tilde{f} be a $S\tilde{S}$ on a soft compact set F_A , if $\tilde{f}|_{F_A}$ is a $S\tilde{S}$ at every soft point of F_A .



Example (3.6):

(i) Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft co-finite topological space (where A is a finite set of parameters) and $\tilde{f}: (\tilde{X}_A, \tilde{\tau}, A) \rightsquigarrow (\tilde{X}_A, \tilde{\tau}, A)$ be an identity soft mapping. Then \tilde{f} is $S\tilde{T}$.

(ii) Let $(\tilde{\mathbb{R}}_A, \tilde{\tau}, A)$ be a STS in example (3.4) and $\tilde{f}: (\tilde{\mathbb{R}}_A, \tilde{\tau}, A) \rightsquigarrow (\tilde{\mathbb{R}}_A, \tilde{\tau}, A)$ be a soft constant mapping. Then \tilde{f} is not $S\tilde{T}$.

Example (3.7):

(i) Let $X = \{n : n \in \mathbb{Z}^+\}$ be a universe set with the set $A = \{\eta, \eta'\}$ of a parameters, $(\tilde{X}_A, \tilde{\tau}, A)$ is a STS with $\tilde{\tau} = \{\tilde{\emptyset}_A, \tilde{X}_A, F_A, G_A\}$, where

$$F_A = \{(\eta, \{n, n \in \mathbb{Z}_e^+\}), (\eta', \emptyset)\};$$

$$G_A = \{(\eta, \{n, n \in \mathbb{Z}_o^+\}), (\eta', \emptyset)\}.$$

Then a soft mapping $\tilde{f}: (\tilde{X}_A, \tilde{\tau}, A) \rightsquigarrow (\tilde{X}_A, \tilde{\tau}, A)$ is

defined by:

$$\tilde{f}(1^n) = 2^n; \tilde{f}((n)^n) = (n)^n, \text{ for all } n > 1 \text{ and } \tilde{f}((n)^{n'}) = (n)^{n'} \text{ for all } n \in \mathbb{Z}^+, \text{ is } S\tilde{S} \text{ at } 1^n.$$

(ii) Let $X = \{x, y, z\}$ be a universe set with the set $A = \{e, e'\}$ of a parameters such that $(\tilde{X}_A, \tilde{\tau}, A)$ is a STS, where $\tilde{\tau} = \{\tilde{\emptyset}_A, \tilde{X}_A, F_A, G_A\}$ such that

$$F_A = \{(e, x), (e', \emptyset)\}, G_A = \{(e, \{y, z\}), (e', X)\}.$$

Then a soft continuous mapping $\tilde{f}: (\tilde{X}_A, \tilde{\tau}, A) \rightsquigarrow (\tilde{X}_A, \tilde{\tau}, A)$, which is defined by:

$$\tilde{f}(x^e) = y^{e'}; \tilde{f}(y^e) = x^{e'}; \tilde{f}(z^e) = z^{e'}; \tilde{f}(x^{e'}) = x^{e'}; \tilde{f}(y^{e'}) = z^{e'} \text{ and } \tilde{f}(z^{e'}) = y^{e'} \text{ is not } S\tilde{S}.$$

Remark (3.8):

(i) On $(\tilde{X}_A, \tilde{\tau}_{ind}, A)$ and $(\tilde{X}_A, \tilde{\tau}_d, A)$, there are no $S\tilde{S}$ mappings because in $(\tilde{X}_A, \tilde{\tau}_{ind}, A)$ we can't find a soft open set V_A satisfying (3.5.ii) and for $(\tilde{X}_A, \tilde{\tau}_{ind}, A)$ can't hold when $U_A = \{x^e\}$.

(ii) A soft constant mappings are not $S\tilde{S}$, since $\tilde{f}^n(y^e) \notin \tilde{cl} V_A$ is not holds.

(iii) In general in any soft compact STS $(\tilde{X}_A, \tilde{\tau}, A)$, if $\{x^e\}$ is soft open, then a soft continuous mapping $\tilde{f}: (\tilde{X}_A, \tilde{\tau}, A) \rightsquigarrow (\tilde{X}_A, \tilde{\tau}, A)$ is not $S\tilde{S}$ at x^e .

4. Relationship between definitions:

Remark (4.1):

In general the two definitions of $S\tilde{T}$ and $S\tilde{D}$ of a $S\tilde{O}$ are not equivalent.



Example (4.2):

Let $X = \{0\} \cup \{\frac{1}{n} : n \in \mathbb{Z}_+\}$ with a soft metric topology $\tilde{\tau}_d$. Then A soft mapping

$\tilde{f} : (\tilde{X}_A, \tilde{\tau}_d, A) \rightarrow (\tilde{X}_A, \tilde{\tau}_d, A)$, defined by:

$$\tilde{f}(0^e) = 0^e \text{ and } \tilde{f}((\frac{1}{n})^e) = (\frac{1}{n+1})^e \text{ for all } n \in \mathbb{Z}_+, \text{ is not } S\tilde{T}.$$

Since we choose $U_A = \{(\frac{1}{n})^e\}$ and $V_A = \{1^e\}$ are soft open sets, then $\tilde{f}^n(U_A) \cap V_A = \emptyset_A$ for all $n \in \mathbb{Z}_+$. But 1^e is (the only) soft point such that $S\tilde{O}_{\tilde{f}}(1^e) = \{1^e, (\frac{1}{2})^e, (\frac{1}{3})^e, \dots\}$, and $\tilde{cl}(S\tilde{O}_{\tilde{f}}(1^e)) = \tilde{X}_A$. Thus \tilde{f} has a $S\tilde{D}$ of $S\tilde{O}$.

Notation (4.3):

Let F_A be a soft set of a STS $(\tilde{X}_A, \tilde{\tau}, A)$. Then:

(i) $S\tilde{D}\tilde{O}_{\tilde{f}}(F_A) = \{x^e \in F_A : \tilde{cl}_{F_A}(S\tilde{O}_{\tilde{f}}(x^e)) = F_A\}.$

(ii) $S\tilde{D}\tilde{P}_{\tilde{f}}(F_A) = \{x^e \in F_A : \tilde{cl}_{F_A}(S\tilde{P}_{\tilde{f}}(x^e)) = F_A\}.$

Definition (4.4):

Let $(\tilde{X}_A, \tilde{\tau}, A)$ be a STS and $\tilde{f} : (\tilde{X}_A, \tilde{\tau}, A) \rightarrow (\tilde{X}_A, \tilde{\tau}, A)$ be a soft mapping.

A soft set F_A of $(\tilde{X}_A, \tilde{\tau}, A)$ is said to be \tilde{f} -soft invariant under $(\tilde{X}_A, \tilde{\tau}, A)$, if $\tilde{f}^n(F_A) \subseteq F_A$, for all $n \in \mathbb{Z}_+$.

Theorem (4.5):

Let F_A be a soft subset of a STS $(\tilde{X}_A, \tilde{\tau}, A)$ and $\tilde{f} : (F_A, \tilde{\tau}_{F_A}, A) \rightarrow (F_A, \tilde{\tau}_{F_A}, A)$ be a soft continuous mapping, if F_A has no s. i. p and $S\tilde{D}\tilde{O}_{\tilde{f}}(F_A) \neq \emptyset_A$. Then $\tilde{cl}_{F_A}(S\tilde{D}\tilde{O}_{\tilde{f}}(F_A)) = F_A$.

Proof:

Since $S\tilde{D}\tilde{O}_{\tilde{f}}(F_A) \neq \emptyset_A$, there is $x^e \in S\tilde{D}\tilde{O}_{\tilde{f}}(F_A)$ with $\tilde{cl}_{F_A}(S\tilde{O}_{\tilde{f}}(x^e)) = F_A$. Since F_A has no s. i. p, then:

$S\tilde{O}_{\tilde{f}}(\tilde{f}(x^e)) = \{S\tilde{O}_{\tilde{f}}(x^e) \setminus \{x^e\}\}$ is a $S\tilde{D}$ set in F_A (because $S\tilde{D}\tilde{O}_{\tilde{f}}(F_A)$ is \tilde{f} -soft invariant under \tilde{f}). So, $\tilde{f}(x^e) \in S\tilde{D}\tilde{O}_{\tilde{f}}(F_A)$. If $x^e \in S\tilde{D}\tilde{O}_{\tilde{f}}(F_A)$, then $S\tilde{O}_{\tilde{f}}(x^e) \subseteq S\tilde{D}\tilde{O}_{\tilde{f}}(F_A)$. But $\tilde{cl}_{F_A}(S\tilde{O}_{\tilde{f}}(x^e)) = F_A$, implies $\tilde{cl}_{F_A}(S\tilde{D}\tilde{O}_{\tilde{f}}(F_A)) = F_A$.

Theorem (4.6):

Let F_A be a soft subset of a STS $(\tilde{X}_A, \tilde{\tau}, A)$ and $\tilde{f} : (F_A, \tilde{\tau}_{F_A}, A) \rightarrow (F_A, \tilde{\tau}_{F_A}, A)$ be a soft continuous mapping and F_A has no s. i. p. If $\tilde{cl}_{F_A}(S\tilde{D}\tilde{O}_{\tilde{f}}(F_A)) = F_A$, implies that \tilde{f} is $S\tilde{T}$ on F_A .

Proof:

Let $x^e \in S\tilde{D}\tilde{O}_{\tilde{f}}(F_A)$ such that $\tilde{cl}_{F_A}(S\tilde{O}_{\tilde{f}}(x^e)) = F_A$ and F_A has no s. i. p. Let U_A and V_A be two non-null soft open sets in $(F_A, \tilde{\tau}_{F_A}, A)$.

There is $n \in \mathbb{Z}_+$ such that $\tilde{f}^n(x^e) \in U_A$. The soft set $V_A \setminus \{x^e, \tilde{f}(x^e), \dots, \tilde{f}^n(x^e)\}$ is non-null soft open set, (because F_A has no s. i. p). Thus there is $m \in \mathbb{Z}_+$ such that:



$$\tilde{f}^m(x^e) \tilde{\in} V_A \tilde{\setminus} \{x^e, \tilde{f}(x^e), \dots, \tilde{f}^n(x^e)\}.$$

It follows that $m > n$,

$$\tilde{f}^m(x^e) = \tilde{f}^{m-n}(\tilde{f}^n(x^e)) \tilde{\in} \tilde{f}^{m-n}(U_A) \tilde{\cap} V_A,$$

thus $\tilde{f}^{m-n}(U_A) \tilde{\cap} V_A \neq \tilde{\emptyset}_A$. We deduce that \tilde{f} is $S\tilde{T}$.

Theorem (4.7):

Let F_A be a soft subset of a $STS (\tilde{X}_A, \tilde{\tau}, A)$ and $\tilde{f}: (F_A, \tilde{\tau}_{F_A}, A) \rightsquigarrow (F_A, \tilde{\tau}_{F_A}, A)$ be a soft continuous mapping, if F_A be a soft second countable and soft second category. Then a $S\tilde{T}$ of \tilde{f} on F_A , implies that $\tilde{c}l_{F_A}(S\tilde{O}_{\tilde{f}}(x^e)) = F_A$.

Proof:

Suppose that $\tilde{c}l_{F_A}(S\tilde{O}_{\tilde{f}}(x^e)) \neq F_A$, $\{(U_A)_n : n \in \mathbb{N}\}$ be a countable soft base of $(F_A, \tilde{\tau}_{F_A}, A)$.

For each $x^e \tilde{\in} F_A$, there is a soft open nbhd $(U_A)_n$ of x^e in a collection $\{(U_A)_n : n \in \mathbb{N}\}$ such that $\tilde{f}^k(x^e) \tilde{\notin} (U_A)_n$ for all $k \geq 0$. Since \tilde{f} be a soft continuous mapping, then $\tilde{f}^{-k}((U_A)_n)$ is soft open nbhd of x^e in $(F_A, \tilde{\tau}_{F_A}, A)$ for all $k \geq 0$. But $\tilde{\bigcup}_{k=0}^{\infty} \tilde{f}^{-k}((U_A)_n)$ is a soft open set and $\tilde{\bigcup}_{k=0}^{\infty} \tilde{f}^{-k}((U_A)_n) \tilde{\cap} V_A \neq \tilde{\emptyset}_A$, for all soft open set V_A in $(F_A, \tilde{\tau}_{F_A}, A)$, (because \tilde{f} be a $S\tilde{T}$). If we let $(G_A)_n$ be a soft complement of $\tilde{\bigcup}_{k=0}^{\infty} \tilde{f}^{-k}((U_A)_n)$, then $(G_A)_n$ contains x^e which is soft closed and soft nowhere dense (see [16]).

However $F_A = \tilde{\bigcup}_{x^e \tilde{\in} F_A} (G_A)_n$ is a soft countable union, this is a contradiction with the fact that $(F_A, \tilde{\tau}_{F_A}, A)$ be a soft second category. Hence $\tilde{c}l_{F_A}(S\tilde{O}_{\tilde{f}}(x^e)) = F_A$.

Theorem (4.8):

Let F_A be a soft subset of a $STS (\tilde{X}_A, \tilde{\tau}, A)$ and $\tilde{f}: (F_A, \tilde{\tau}_{F_A}, A) \rightsquigarrow (F_A, \tilde{\tau}_{F_A}, A)$ be a soft continuous mapping, if \tilde{f} be a $S\tilde{S}$ on F_A . Then F_A has no *s. i. p.*

Proof:

Suppose that F_A has x^e as a *s. i. p.*, then $F_A \tilde{\cap} U_A \tilde{\setminus} \{x^e\} = \tilde{\emptyset}_A$, for all soft open set U_A in $(F_A, \tilde{\tau}_{F_A}, A)$ containing x^e . Since x^e be an arbitrary, this means that \tilde{f} is not $S\tilde{S}$ on F_A , contradiction.

Theorem (4.9):

Let F_A be a soft closed set has no *s. i. p.* of a soft $STS (\tilde{X}_A, \tilde{\tau}, A)$, which is soft T_3 -space and let $\tilde{f}: (F_A, \tilde{\tau}_{F_A}, A) \rightsquigarrow (F_A, \tilde{\tau}_{F_A}, A)$ is soft continuous mapping such that \tilde{f}^n is non-soft constant for some $n \geq 1$ on $F_A \tilde{\cap} G_A \neq \tilde{\emptyset}_A$, for every soft open set G_A in $(\tilde{X}_A, \tilde{\tau}, A)$. Then \tilde{f} is $S\tilde{S}$.



Proof:

Let $x^e \in F_A$, since F_A be a soft closed has no *s.i.p.* Then $F_A \cap U_A \neq \emptyset_A$, for every soft open set U_A in $(\tilde{X}_A, \tilde{\tau}, A)$ containing x^e , so there is $y^e \in F_A \cap U_A$ with $\tilde{f}^n(x^e) \neq \tilde{f}^n(y^e)$ for some n (otherwise \tilde{f}^n becomes soft constant on $F_A \cap U_A$). Since $(\tilde{X}_A, \tilde{\tau}, A)$ be soft T_3 -space, then there is a soft open set V_A such that $\tilde{f}^n(x^e) \in V_A$ and $\tilde{f}^n(y^e) \notin \tilde{cl}(V_A)$, thus \tilde{f} is $S\tilde{S}$ on F_A .

Corollary (4.10):

Let $(\tilde{X}_A, \tilde{\tau}_A, A)$ be a soft compact (soft metric space for short *SMS*) and F_A be a soft closed set without *s.i.p.* Then for each soft continuous mapping $\tilde{f} : (F_A, \tilde{\tau}_{F_A}, A) \rightarrow (F_A, \tilde{\tau}_{F_A}, A)$, which is non-soft constant on every soft open set is $S\tilde{S}$.

Proof: Clear.

Now, we put conditions to have a $S\tilde{T}$ and $S\tilde{D}\tilde{O}_{\tilde{f}}(F_A)$ are equivalent.

Theorem (4.11):

Let F_A be a soft compact subset in a soft Hausdorff space $(\tilde{X}_A, \tilde{\tau}, A)$. Then a soft continuous mapping $\tilde{f} : (F_A, \tilde{\tau}_{F_A}, A) \rightarrow (F_A, \tilde{\tau}_{F_A}, A)$ is $S\tilde{T}$ if and only if $\tilde{cl}_{F_A}(S\tilde{O}_{\tilde{f}}(x^e)) = F_A$, for some $x^e \in F_A$.

Proof:

Since $(F_A, \tilde{\tau}_{F_A}, A)$ be a soft Hausdorff, then F_A has no *s.i.p.* Since F_A be a soft second countable and soft second category. This complete the proof.

5. Soft Chaos Space:

In this section, we introduce chaos in soft topological spaces, which is called (soft chaos) and we quote some definitions, theorems and remarks about this concepts. Our definition of soft chaotic (soft mapping) on a soft compact soft topological spaces or a restriction on a soft compact set of a soft topological spaces. There are many definitions of soft chaotic mapping and relation between them are given.

Definition (5.1):

Let F_A be a soft compact set in *STS* $(\tilde{X}_A, \tilde{\tau}, A)$ and $\tilde{f} : (F_A, \tilde{\tau}_{F_A}, A) \rightarrow (F_A, \tilde{\tau}_{F_A}, A)$ be a soft continuous mapping. Then \tilde{f} is called **soft chaotic** (for short $S\tilde{C}$) on F_A if:

- (i) \tilde{f} is $S\tilde{T}$ on F_A .
- (ii) $\tilde{cl}_{F_A}(S\tilde{P}_{\tilde{f}}(x^e)) = F_A$.
- (iii) \tilde{f} is $S\tilde{S}$ on F_A .



Example (5.2):

(i) Let $X = \{n : n \in \mathbb{Z}^+\}$ be a universe set with the set $A = \{\eta, \eta'\}$ of a parameters and $(\tilde{X}_A, \tilde{\tau}, A)$ is a STS with $\tilde{\tau} = \{\tilde{\emptyset}_A, \tilde{X}_A, F_A\}$ and $F_A = \{(\eta, \{n, n \in \mathbb{Z}_0^+\}), (\eta', \emptyset)\}$.

A soft mapping $\tilde{f}: (\tilde{X}_A, \tilde{\tau}, A) \rightsquigarrow (\tilde{X}_A, \tilde{\tau}, A)$ which is defined by: $\tilde{f}(n^\eta) = (n^e)^2$; for all $e \in A$, is $S\tilde{C}$. on \tilde{X}_A .

(ii) Let $X = \mathbb{Z}$ be a universe set, $A = \{e, e'\}$ be a parameter set and $(\tilde{X}_A, \tilde{\tau}, A)$ be a soft compact STS, where $\tilde{\tau} = \{F_A \cong \tilde{X}_A : 0^e \cong F_A\} \cup \{\tilde{\emptyset}_A\}$. Then the identity soft mapping $\tilde{f}: (\tilde{X}_A, \tilde{\tau}, A) \rightsquigarrow (\tilde{X}_A, \tilde{\tau}, A)$ is not $S\tilde{C}$.

Theorem (5.3):

Let F_A be a soft compact set of STS $(\tilde{X}_A, \tilde{\tau}, A)$ and $\tilde{f}: (F_A, \tilde{\tau}_{F_A}, A) \rightsquigarrow (F_A, \tilde{\tau}_{F_A}, A)$ be a soft continuous mapping. Then \tilde{f} is $S\tilde{C}$ on F_A if and only if:

(i) $\tilde{cl}_{F_A}(S\tilde{O}_{\tilde{f}}(x^e)) = F_A$, for some $x^e \cong F_A$.

(ii) $\tilde{cl}_{F_A}(S\tilde{P}_{\tilde{f}}(x^e)) = F_A$.

(iii) \tilde{f} is $S\tilde{S}$ on F_A .

Proof: By using theorem (4.11) and definition (5.1)

Notation (5.4):

Let F_A be a soft subset of a STS $(\tilde{X}_A, \tilde{\tau}, A)$. Then:

(i) $S\tilde{S}(F_A) = \{\tilde{f}: \tilde{f} \text{ is } S\tilde{S} \text{ on } F_A\}$.

(ii) $S\tilde{C}(F_A) = \{\tilde{f}: \tilde{f} \text{ is } S\tilde{C} \text{ on a soft compact set } F_A\}$.

(iii) $S\tilde{C}C(X_A) =$

$\{F_A: F_A \text{ soft compact set ; } S\tilde{C}(F_A) \neq \tilde{\emptyset}_A\}$.

Remark (5.5):

It is clear that $S\tilde{C}(F_A) \cong S\tilde{S}(F_A)$. The other inclusion need not to be true in general.

Let $X = \mathbb{R}$ be a universal set, $A = \{\eta\}$ be a set of parameter and $(\tilde{X}_A, \tilde{\tau}, A)$ be a STS in example (3.7.i). It is clear that \tilde{X}_A is a soft compact set, a soft mapping $\tilde{f}: (\tilde{X}_A, \tilde{\tau}, A) \rightsquigarrow (\tilde{X}_A, \tilde{\tau}, A)$, is defined by $\tilde{f}(r^e) = (r^e)^2$. Thus $\tilde{f} \in S\tilde{S}(\tilde{X}_A)$. But $\tilde{f} \notin S\tilde{C}(F_A)$, since \tilde{f} is $S\tilde{T}$ on \tilde{X}_A .

Definition (5.6):

A STS $(\tilde{X}_A, \tilde{\tau}, A)$ is called **soft chaotic** (for short $S\tilde{C}$ -space), if there is a $S\tilde{C}$ -mapping on a soft compact set F_A of $(\tilde{X}_A, \tilde{\tau}, A)$. i.e. $S\tilde{C}C(F_A) \neq \tilde{\emptyset}_A$. If $(\tilde{X}_A, \tilde{\tau}, A)$ be a $S\tilde{C}$ -space, then the soft elements of $S\tilde{C}C(\tilde{X}_A)$ are called $S\tilde{C}$ -sets.

Example (5.7):

(i) A soft topological space in example (5.2.i), is $S\tilde{C}$ -space.



(ii) A STS $(\tilde{X}_A, \tilde{\tau}_d, A)$ is not $S\tilde{C}$ -space, where $\tilde{\tau}_d$ is a soft discrete topology on \tilde{X}_A .

Remark (5.8):

Let F_A be a soft compact set in a STS $(\tilde{X}_A, \tilde{\tau}, A)$, and $\tilde{f} : (F_A, \tilde{\tau}_{F_A}, A) \rightsquigarrow (F_A, \tilde{\tau}_{F_A}, A)$ be a soft continuous mapping, if (i) and (ii) of a notation (4.3) are satisfied. Then \tilde{f} need not to be $S\tilde{S}$ (\tilde{f} need not to be a $S\tilde{C}$), as the following example shows:

Example (5.9):

Let $X = \{x, y\}$, $A = \{e\}$ and $(\tilde{X}_A, \tilde{\tau}, A)$ be a STS with $\tilde{\tau} = \{\emptyset_A, \tilde{X}_A, F_A\}$ and $F_A = \{x^e\}$.

It is clear that $(\tilde{X}_A, \tilde{\tau}, A)$ is a soft compact space. given $\tilde{f} : (\tilde{X}_A, \tilde{\tau}, A) \rightsquigarrow (\tilde{X}_A, \tilde{\tau}, A)$ be a soft identity mapping on $(\tilde{X}_A, \tilde{\tau}, A)$. Then $\tilde{cl}(S\tilde{O}_{\tilde{f}}(x^e)) = \tilde{X}_A$ and $\tilde{cl}(S\tilde{P}_{\tilde{f}}(x^e)) = \tilde{X}_A$, but \tilde{f} is not $S\tilde{S}$, because $F_A = \{x^e\}$ is soft open.

Theorem (5.10):

If each $(\tilde{X}_{i_A}, \tilde{\tau}_i, A)$, $i = 1, 2, \dots, n$ be a soft T_3 -space and $S\tilde{C}$ -space, then $(\prod_{i=1}^n \tilde{X}_{i_A}, \tilde{\tau}, A)$ is $S\tilde{C}$ -space and $\tilde{\tau}$ is a soft product topology on $\prod_{i=1}^n \tilde{X}_{i_A}$.

Proof:

Since each $(\tilde{X}_{i_A}, \tilde{\tau}_i, A)$, $i = 1, 2, \dots, n$ be a soft T_3 -space, then $(\prod_{i=1}^n \tilde{X}_{i_A}, \tilde{\tau}, A)$ is also.

Given that each $(\tilde{X}_{i_A}, \tilde{\tau}_i, A)$, $i = 1, 2, \dots, n$ be a $S\tilde{C}$ -space. So there is $(F_A)_i$ be a soft compact set of $(\tilde{X}_{i_A}, \tilde{\tau}_i, A)$, $i = 1, 2, \dots, n$ and $\tilde{f}_i \in S\tilde{C}((F_A)_i)$. Let $F_A = \prod_{i=1}^n (F_A)_i$. Then F_A is soft compact set, since each $(F_A)_i$ has no *s.i.p*, then F_A it also has no *s.i.p*.

Define a soft mapping $\tilde{f} : (F_A, \tilde{\tau}_{F_A}, A) \rightsquigarrow (F_A, \tilde{\tau}_{F_A}, A)$ by:

$$\tilde{f}(x^e) = \tilde{f}(x_1^e, x_2^e, \dots, x_n^e) = (\tilde{f}_1(x_1^e), \tilde{f}_2(x_2^e), \dots, \tilde{f}_n(x_n^e))$$

, where $x^e = (x_1^e, x_2^e, \dots, x_n^e)$.

It is clear that $\tilde{f} : (F_A, \tilde{\tau}_{F_A}, A) \rightsquigarrow (F_A, \tilde{\tau}_{F_A}, A)$ is a soft continuous mapping.

Now, to show that $\tilde{f} \in S\tilde{C}(F_A)$. i.e. We have to show that:

(i) $\tilde{cl}(S\tilde{O}_{\tilde{f}}(x^e) = F_A)$ for some $x^e \in F_A$,

(ii) $\tilde{cl}(S\tilde{P}_{\tilde{f}}(x^e) = F_A$,

(iii) $\tilde{f} \in S\tilde{S}(F_A)$.

First we prove (i). Since $\tilde{f}_i \in S\tilde{C}((F_A)_i)$, there is $x_i^e \in (F_A)_i$ such that $\tilde{cl}(S\tilde{O}_{\tilde{f}_i}(x_i^e) = (F_A)_i$, for $i = 1, 2, \dots, n$. Let $x^e = (x_1^e, x_2^e, \dots, x_n^e)$. Then we have:

$$\tilde{cl}(S\tilde{O}_{\tilde{f}_1}(x_1^e) \times \tilde{cl}(S\tilde{O}_{\tilde{f}_2}(x_2^e) \times \dots \times \tilde{cl}(S\tilde{O}_{\tilde{f}_n}(x_n^e) = (F_A)_1 \times (F_A)_2 \times \dots \times (F_A)_n = F_A.$$

This implies that:



$$\tilde{cl}(S\tilde{O}_{\tilde{f}_1}(x_1^e) \times \tilde{cl}(S\tilde{O}_{\tilde{f}_2}(x_2^e) \times \dots \times \tilde{cl}(S\tilde{O}_{\tilde{f}_n}(x_n^e) = \tilde{cl}(S\tilde{O}_{\tilde{f}_1}(x_1^e) \times S\tilde{O}_{\tilde{f}_2}(x_2^e) \times \dots \times S\tilde{O}_{\tilde{f}_n}(x_n^e)).$$

$$\text{But } \tilde{cl}(S\tilde{O}_{\tilde{f}_1}(x_1^e) \times S\tilde{O}_{\tilde{f}_2}(x_2^e) \times \dots \times S\tilde{O}_{\tilde{f}_n}(x_n^e)) = \tilde{cl}(S\tilde{O}_{\tilde{f}}((x_1^e, x_2^e, \dots, x_n^e))) = \tilde{cl}(S\tilde{O}_{\tilde{f}}(x^e)) = F_A.$$

(ii) By similar argument we have:

$$\tilde{cl}(S\tilde{P}_{\tilde{f}}(x^e) = F_A \text{ and (iii) } \tilde{f} \in S\tilde{S}(F_A). \text{ Hence } \tilde{f} \in S\tilde{C}(F_A), \text{ so } (\prod_{i=1}^n \tilde{X}_{i_A}, \tilde{S}, A) \text{ is } S\tilde{C}\text{-space.}$$

Remark (5.11):

A $S\tilde{C}$ -space need not to be a soft hereditary property in general, as the following example shows:

Let $F_A = \{2^n\}$, be a soft compact subset of a STS $(\tilde{X}_A, \tilde{\tau}, A)$ in example (5.2.i).

Clearly $(\tilde{X}_A, \tilde{\tau}, A)$ is a $S\tilde{C}$ -space, but F_A with soft relative topology of $\tilde{\tau}$ is not $S\tilde{C}$ -subspace, see remark (3.8.iii).

Theorem (5.12):

A $S\tilde{C}$ -space is soft topologically property.

Proof:

Let $(\tilde{X}_A, \tilde{\tau}_1, A), (\tilde{Y}_A, \tilde{\tau}_2, A)$ be two STSs

and $\tilde{h} : (\tilde{X}_A, \tilde{\tau}_1, A) \rightarrow (\tilde{Y}_A, \tilde{\tau}_2, A)$ is a soft homeomorphism with $(\tilde{X}_A, \tilde{\tau}_1, A)$ is a $S\tilde{C}$ -space. We want to prove that $(\tilde{Y}_A, \tilde{\tau}_2, A)$ is $S\tilde{C}$ -space. Since $(\tilde{X}_A, \tilde{\tau}_1, A)$ be a $S\tilde{C}$ -space, then there is a soft compact set F_A in $(\tilde{X}_A, \tilde{\tau}_1, A)$.

i.e. $S\tilde{C}(\tilde{X}_A) \neq \tilde{\emptyset}_A$. Let $\tilde{f} \in S\tilde{C}(F_A)$.

Put $\tilde{g} = \tilde{h} \circ \tilde{f} \circ \tilde{h}^{-1}$. Then $\tilde{g} : \tilde{h}(F_A) \rightarrow \tilde{h}(F_A)$ is a soft continuous. We prove that $\tilde{g} \in S\tilde{C}(\tilde{h}(F_A))$, so that $\tilde{h}(F_A) \in S\tilde{C}(\tilde{Y}_A)$. We have to prove that:

(i) $\tilde{cl}(S\tilde{O}_{\tilde{g}}(y^e) = \tilde{h}(F_A)$ for some $y^e \in \tilde{h}(F_A)$.

(ii) $\tilde{cl}(S\tilde{P}_{\tilde{g}}(y^e) = \tilde{h}(F_A)$ for any $y^e \in \tilde{h}(F_A)$.

(iii) $\tilde{g} \in S\tilde{S}(\tilde{h}(F_A))$.

To prove (i):

Since $F_A \in S\tilde{C}(\tilde{X}_A)$, then there is $x^e \in F_A$ such that $\tilde{cl}(S\tilde{O}_{\tilde{f}}(x^e) = F_A$.

Suppose $\tilde{cl}(S\tilde{O}_{\tilde{g}}(y^e) \neq \tilde{h}(F_A)$, for all $y^e \in \tilde{h}(F_A)$. i.e. $\tilde{cl}(S\tilde{O}_{\tilde{g}}(\tilde{h}(x^e)) \neq \tilde{h}(F_A)$ for all $x^e \in F_A$. i.e. There is a soft open set V_A in $\tilde{h}(F_A)$ such that $S\tilde{O}_{\tilde{g}}(\tilde{h}(x^e)) \cap V_A = \tilde{\emptyset}_A$.

$$\Rightarrow \tilde{g}^n(\tilde{h}(x^e)) \notin V_A \text{ for all } n \in \mathbb{Z}_+.$$

$$\Rightarrow (\tilde{h} \circ \tilde{f} \circ \tilde{h}^{-1})^n(\tilde{h}(x^e)) \notin V_A \text{ for all } n \in \mathbb{Z}_+.$$

$$\Rightarrow \tilde{h} \circ \tilde{f}^n \circ \tilde{h}^{-1}(\tilde{h}(x^e)) \notin V_A \text{ for all } n \in \mathbb{Z}_+.$$

$$\Rightarrow \tilde{h} \circ \tilde{f}^n(x^e) \notin V_A \text{ for all } n \in \mathbb{Z}_+.$$



$\Rightarrow \tilde{f}^n(x^e) \not\subseteq \tilde{h}^{-1}(V_A)$ for all $n \in \mathbb{Z}_+$, which is a contradiction. Thus $\tilde{cl}(S\tilde{O}_{\tilde{g}}(y^e)) = \tilde{h}(F_A)$ for some $y^e \in \tilde{h}(F_A)$.

Proof of (ii) is similar.

To prove (iii).

Let $y_1^e \in \tilde{h}(F_A)$. Then $y_1^e = \tilde{h}(x_1^e)$, for some $x_1^e \in F_A$. Let V_A be a soft open nbhd of y_1^e . Then $\tilde{h}^{-1}(V_A)$ is soft open in F_A and soft nbhd of x_1^e . Since $\tilde{f} \in S\tilde{S}(F_A)$, then there is $x_2^e \in \tilde{h}^{-1}(V_A) \cap V_A$, $n \in \mathbb{Z}$ and soft open set W_A such that $\tilde{f}^n(x_1^e) \in W_A$ and $\tilde{f}^n(x_2^e) \in \tilde{cl}(W_A)$.

Since $\tilde{f}^n(x_1^e) \in W_A$, $(\tilde{h} \circ \tilde{f}^n)(x_1^e) \in \tilde{h}(W_A)$.

$\Rightarrow (\tilde{h} \circ \tilde{f} \circ \tilde{h}^{-1})^n(\tilde{h}(x_1^e)) \in \tilde{h}(W_A)$.

$\Rightarrow \tilde{g}^n(y_1^e) \in \tilde{h}(W_A)$.

In the same way $\tilde{g}^n(y_2^e) \in \tilde{cl}(\tilde{h}(W_A))$. So $\tilde{g} \in S\tilde{C}(\tilde{h}(F_A))$ and so $\tilde{h}(F_A) \in S\tilde{C}C(Y_A)$. Thus $(\tilde{Y}_A, \tilde{\tau}_2, A)$ is $S\tilde{C}$ -space

المستخلص:

أن الهدف الرئيس من هذا العمل هو دراسة السلوك الفوضوي للدوال الواهنة المعرفة على الفضاءات التوبولوجية الواهنة وذلك بالاعتماد على النقطة الواهنة (soft point). كذلك تم دراسة تحول وسرعة تأثر (حساسية) الدوال المستمرة الواهنة وإيجاد العلاقة بينهما وصولاً إلى إعطاء تعريف الدوال الفوضوية الواهنة والتي تعتبر التعريف الأساس في هذا البحث. وفي الجزء الأخير من هذا البحث قدمنا نوع جديد من الفضاءات التوبولوجية الواهنة التي اسمناها الفضاءات التوبولوجية الفوضوية الواهنة والتي تعتبر من النمط المتقطع من حيث التعامل مع مجموعة الأعداد الصحيحة.

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