Some Outcomes of αrps – Compact Space

Authors Names	**ABSTRACT**
Dunya Mohamed Hameed *Publication data:* 13 / 7 / 2026 *Keywords: αrps – closed sets, αrps – open sets, αrps – open the envelope, αrps– compact space*.	This essay is to submit a new sort of compact spaces termed as αrps– compact space. This article also discusses some findings of αrps– compact space.

1. Introduction

Compactness is an important and basic concept in general topology as well as other sophisticated mathematical disciplines.Several researchers[1,3,4,5,12] have investigated the essential p roperties of compact spaces in order to expand and study the concept of compact spaces . The ideas of [αrps - closed and αrps - open sets were first displayed by Dunya .M.H [6,7]. Article aims to present the notion of αrps - compact spaces and submit some of it is characteristics.

2. Preliminaries

This section proffers some definitions and outcomes that necessity in this research and in following (T. S) refers to a topological space throughout this article

Definition (2.1) :- Let K be a subset in T.S (X, ℓ) known as:

1. Semi – pre closed [2] if $\text{int}(\text{cl}(\text{int}(K)) \subseteq K$
2. α – closed [9] if $\text{cl}\left(\text{int}(\text{cl}(K))\right) \subseteq K$.
3. r– open [8] if K= (int (cl(K)) .

Definition (2-2), [2], [9]: In T.S (X, ℓ)

1. spcl(K) = ⋂ {Q : K ⊆ Q : ∀ Q is semi – pre closed in (X, ℓ)}
2. αcl (K) = ⋂ {Q : K □ Q : ∀ Q is α – closed in (X ,l)}

Definition (2-3): In T.S (X, ℓ) if K⊆ X , then K is termed as

1. rg – closed [10] if cl (K) ⊆ R , s.t K ⊆ R ᴣ R is r– open put within X.
2. rps – closed [11] if spcl(K) ⊆ R , s.t K ⊆ R ᴣ R is rg– open put within X.
3. αrps – closed [6] if αcl(K)⊆ R , s.t K ⊆ R ᴣ R is rps – open put within X.

The complement of above sets is knowns as open .

Remark (2-4),[6]: Each open set is αrps – open.

Definition (2- 5), [6],[7]: A function J: $(X, \ell) \rightarrow (\Upsilon, \mu)$ is:

1- αrps – continu. if $J^{-1}(K)$ is αrps – open set put within X for ∀ open K in ϒ.
2- Strongly αrps – continu. if $J^{-1}(K)$ is open set put within X for ∀ αrps– open K in ϒ.

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3- αrps – irresolute if $\mathrm{J}^{-1}(\mathrm{K})$ is αrps – open set put within X for ∀ αrps – open K in ϒ.

3. Some Outcomes of αrps – Compact Space

Following, we will begin define αrps – open the envelope

Definition (3.1) : A collection $\{\mathrm{W}_h : h \in \Lambda\}$ of αrps – open sets in T.S (X,ℓ) is known as αrps – open the envelope for K⊆X , if K ⊂ $\{\mathrm{W}_h : h \in \Lambda\}$ satisfy.

Now , we use this this cover to present of αrps – compact space

Definition (3.2) : A space (X,ℓ) is termed as αrps – compact space if each αrps – open the envelope of X has finite sub cover .

Remark (3.3):

It is elucidative that each αrps – compact space is compact , since if a space X is αrps – compact and $\{\mathrm{W}_h : h \in \Lambda\}$ be an open the envelope for X . By Remark(2.4), ∀ open set is αrps – open , implies that$\{\mathrm{W}_h : h \in \Lambda\}$ be αrps –open the envelope for X . from the postulate X is αrps – compact space .So we conclude $\{\mathrm{W}_h : h \in \Lambda\}$ has finite sub cover. Hence , X is compact space . But, the opposite of the prior this Remark is incorrect. Example (3.4) elucidates it.

Example (3.4):

Assume a set M be an infinite positive integer set and ℓ is indiscrete topology on M, hence elucidative that M is compact space but is not αrps – compact ,since $\{\{\eta\} : \eta \in \mathrm{M}\}$ is αrps – open the envelope of M with a no finite sub cover.

Definition (3.5): Let (X,ℓ) be a T.S and K ⊂ X , then K is termed as αrps – compact relative to X , if $\forall\{\mathrm{W}_h : h \in \Lambda\}$ of αrps –open subsets for X, s.t K $\subset \cup\{\mathrm{W}_h : h \in \Lambda\}$, ∃ finite subset Λ_0 of Λ s.t K $\subset \cup\{\mathrm{W}_h : h \in \Lambda_0\}$.

Definition (3.6) :

A sub set K of a space (X,ℓ) is termed as αrps – compact if T is αrps – compact as a subspace of X.

Theorem (3.7): Each αrps – closed subset of αrps – compact space (X,ℓ) is αrps – compact relative to X.

Proof:

Assume K ⊆ X is αrps – closed and $\{\mathrm{W}_h : h \in \Lambda\}$ be αrps – open the envelope of K this leads to K$\subseteq \bigcup_{h\in\Lambda} \mathrm{W}_h$, $\forall h \in \Lambda$, also since K^c is αrps – open , implies X $\subseteq \cup\{\mathrm{W}_h : h \in \Lambda_0\} \cup \mathrm{K}^c$, so we deduced $\cup\{\mathrm{W}_h : h \in \Lambda_0\} \cup \mathrm{K}^c$ is αrps – open the envelope of X. From the presumption a space X is αrps – compact , then ∃ h_1 , h_2 , $h_3, \cdots\cdots$, h_ω such that X= $(\bigcup_{j=1}^{\omega} \mathrm{W}_{h_j}) \cup \mathrm{K}^c$ is a finite sub cover of

X . But $Ꝁ \subseteq (\bigcup_{j=1}^{\omega} W_{h_j}) \cup Ꝁ^c$, where $Ꝁ \cap Ꝁ^c = \emptyset$ Implies that $Ꝁ \subseteq \bigcup_{j=1}^{\omega} W_{h_j}$, therefore $\{W_{h_1}, W_{h_2}, \cdots\cdots, W_{h_j}\}$ is a finite sub cover of Ꝁ , hence Ꝁ is αrps – compact.

Theorem(3.8): Each Finite subset Ꝁ in T.S (X, ℓ) is αrps – compact.

Proof:

Assume $Ꝁ = \{k_1, k_2, \cdots\cdots, k_\sigma\}$be a finite subset of (X, ℓ) and $\{W_h : h \in \Lambda\}$ be αrps – open the envelope of Ꝁ , thus $Ꝁ \subseteq \bigcup_{h\in\Lambda} W_h$, this implies $k_1 \in W_{h_1}$, $k_2 \in W_{h_2}$, $\cdots\cdots$, $k_\sigma \in W_{h_\sigma}$ Hence, $Ꝁ \subseteq \bigcup_{j=1}^{\sigma} W_{h_j}$ which is finite sub cover of Ꝁ , It is a result that Ꝁ is αrps – compact.

Theorem(3.9): In T.S (X, ℓ) the intersection of αrps – compact subset with αrps – closed subset is αrps – compact.

Proof:

If postulate that Ꝁ and P are subsets in T.S (X, ℓ), where Ꝁ is αrps – compact and P is αrps – closed put within X, let $\{W_h : h \in \Lambda\}$ be αrps – open the envelope of $Ꝁ \cap P$, this leads to $Ꝁ \cap P \subseteq \bigcup_{h\in\Lambda} W_h$, since P^c is αrps – open put within X , so we conclude that $Ꝁ \subseteq \bigcup_{h\in\Lambda} W_h \cup P^c$, thus $\{\{W_h\}, P^c\}$ is αrps – open the envelope of Ꝁ, by assumption Ꝁ is αrps – compact , this implies Ꝁ has finite sub cover for Ꝁ , this lead $Ꝁ \subseteq W_{h_1} \cup\in W_{h_2} \cup \cdots\cdots \cup \in W_{h_k} \cup P^c$ .So $Ꝁ \cap P \subseteq W_{h_1} \cup W_{h_2} \cup \cdots\cdots \cup \in W_{h_k}$. Hence $Ꝁ \cap P$ is αrps – compact.

Theorem(3.10): If Ꝁ and P are αrps – compact put within X then the union of $Ꝁ \cup P$ is also αrps – compact in (X, ℓ).

Proof:

If assuming $\{W_h : h \in \Lambda\}$ is αrps – open the envelope of $Ꝁ \cup P$, this implies $Ꝁ \cup P \subseteq \bigcup_{h\in\Lambda} W_h$ So, $Ꝁ \subseteq \bigcup_{h\in\Lambda} W_h$, $P \subseteq \bigcup_{h\in\Lambda} W_h$. From postulate Ꝁ and P are αrps – compact , hence ∃ finite sub cover say $Ꝁ \subseteq W_{h_1} \cup\in W_{h_2} \cup \cdots\cdots \cup \in W_{h_k}$, $P \subseteq W_{h_{k+1}} \cup\in W_{h_{k+2}} \cup \cdots\cdots \cup \in W_{h_{k+m}}$, this implies $Ꝁ \cup P \subseteq \bigcup_{j=1}^{k+m} W_{h_j}$. Hence , $Ꝁ \cup P$ is αrps – compact.

Theorem(3.11): Let a space (X, ℓ) be αrps – compact, and if a function J: $(X, \ell) \rightarrow (\Upsilon, \mu)$ is onto and

- **(1)** αrps – continu. , then ϒ is also compact .
- **(2)** αrps – irresolute , then ϒ is αrps – compact .
- **(3)** strongly αrps – continu. , then ϒ is αrps – compact.

Proof

(1): Consider open the envelope $\{W_h : h \in \Lambda\}$ for ϒ. From the assumption a function J is αrps – continu. , this leads to $\{J^{-1}((W_h)) : h \in \Lambda\}$ is αrps – open the envelope of X , from presumption X is αrps – compact , so X has finite sub cover .This means ∃ $h_1, h_2, \cdots\cdots, h_k$, s.t $X = \cup\{J^{-1}(W_{h_\sigma}) : \sigma = 1,$

2, ⋯⋯ ,k},but J is onto this implies ϒ =J(X)= ∪{ J(J^{-1}((W$_{h_\sigma}$)) : σ= 1, 2, ⋯⋯ ,k} hence ϒ =∪{W$_{h_\sigma}$: σ= 1, 2, ⋯⋯ ,k} , therefore ϒ is compact.

(2): Consider αrps – open the envelope {W$_h$: $h \in \Lambda$} for ϒ. Since J is αrps – irresolute, we conclude { J^{-1}(W$_h$) : $h \in \Lambda$} is αrps – open the envelope of X , from presumption X is αrps – compact, so X has finite sub cover This means ∃ h_1 , h_2, ⋯⋯, h_k , s.t X= ∪{ J^{-1}(W$_{h_\sigma}$) : σ= 1, 2, ⋯⋯ ,k},but J is onto this implies ϒ =J(X)= ∪{ J(J^{-1}((W$_{h_\sigma}$)) : σ= 1, 2, ⋯⋯ ,k} hence ϒ =∪{W$_{h_\sigma}$: σ= 1, 2, ⋯⋯ ,k} , therefore ϒ is αrps – compact.

Likewise, point (3) proves.

Corollary (3.12): Let a space (X , ℓ) be αrps – compact, and if a function J: (X , ℓ) → (ϒ, μ) is onto and

(1) αrps – irresolute, then Y is compact.
(2) strongly αrps – continu. , then Y is compact**.**

Proof:

It is intelligibly that by Th.(3.11) point(2) and (3) and Remark(3.3) .

Theorem(3.13): If a function J: (X , ℓ) → (ϒ, μ) is αrps – irresolute and a subset Ꝁ is αrps – compact relative to X , then J(Ꝁ) is αrps – compact relative to ϒ.

Proof:

Consider {W$_h$: $h \in \Lambda$} be a collection of αrps – open subset for ϒ , s.t J(Ꝁ) ⊆ ∪{W$_h$: $h \in \Lambda$} , hence Ꝁ⊆ ∪{J^{-1}(W$_h$) : $h \in \Lambda$} .So, ∀ h ∈ Λ . From presumption Ꝁ is αrps – compact relative to X. Then ∃ finite subset Λ_0 of Λ s.t Ꝁ⊆ ∪{J^{-1}(W$_h$) : $h \in \Lambda_0$},this leads to J(Ꝁ) ⊆ ∪ {(W$_h$): $h \in \Lambda_0$} . So we infer that J(Ꝁ) is αrps – compact relative to ϒ.

4. Conclusions

This research led to the identification of novel type of compact space termed as αrps – compact also we presentation of several findings to discussed characteristics of these spaces.

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