Journal of Iraqi Al-Khwarizmi Society (JIKhS) Year 2018 pages: 178-187

Volume:2 Special issue of the first international scientific conference of Iraqi Al-Khawarizmi Society 28-29 March 2018

Comparing Some Estimators of the Parameter of Basic Gompertz distribution

Huda A. Rasheed[1] Wafaa S. Hasanain[2]

Nadia J. Al-Obedy's[3]

Mustansiriyah University, Collage of Science, Department of Mathematics

[1] hudamath@uomustansiriyah.edu.iq
[2] w.s.hasanain@uomustansiriyah.edu.iq
[3]abualhassan76@uomustansiriyah.edu.iq

Abstract

In this paper, some estimators for the unknown shape parameter of Basic Gompertz distribution have been obtained, such as Maximum likelihood estimator and Bayesian estimators under Generalized weighted loss function, using Gamma prior. All these estimators have been compared empirically using Mont-Carlo simulation by employing the mean squared errors (MSE's). Finally, the discussion is provided to illustrate the results that summarized in tables.

Keywords: Gompertz distribution; Maximum likelihood estimator; Bayes estimator; Generalized weighted loss function.

1. Introduction

The Basic Gompertz distribution has been introduced by Benjamin Gompertz in (1779-1856). It is often applied to describe the actuarial, biological and demographic sciences, and also it has a simple connections to the standard uniform distribution and exponential distribution by means of the distribution and quintile functions. The probability density function of the Basic Gompertz distribution is defined as follows

$$f(x;\theta) = \theta \exp[x + \theta(1 - e^x)] \quad ; \quad x \geq 0 \ , \quad \theta > 0 \tag{1}$$

Where θ is a shape parameter .

The corresponding cumulative distribution function is given by

$$F(x) = 1 - \exp[\theta(1 - e^x)] \quad ; \quad x \geq 0 \tag{2}$$

The Reliability or survival function is

$$R(t) = \overline{F(t)} = \exp[\theta(1 - e^t)] \quad ; \quad t \geq 0$$

2. Some estimators of the shape parameter (θ)

2.1. Maximum likelihood estimator

Let $x_1, x_2, \dots, x_n$ be a random sample of size n from the Basic Gompertz distribution defined by (1) , the likelihood function for the sample observation will be as follows [6] , [7]:

$$L(x_1, x_2, \dots, x_n; \theta) = \pi_{i=1}^{n} f(x_i; \theta)$$

$$= \theta^n \exp\left[\sum_{i=1}^{n} x_i + \theta \sum_{i=1}^{n}(1 - e^{x_i})\right] \quad (3)$$

The natural log-likelihood function will be

$$\ln L(x_i; \theta) = n \ln\theta + \sum_{i=1}^{n} x_i + \theta \sum_{i=1}^{n}(1 - e^{x_i}) \quad (4)$$

By differentiating partially $\ln L(x_i; \theta)$ given by (4), with respect to θ and equate to zero, the MLE of θ become

$$\hat{\theta}_{MLE} = \frac{-n}{\sum_{i=1}^{n}(1 - e^{x_i})}$$

$$= \frac{-n}{T}, \text{ where } T = \sum_{i=1}^{n}(1 - e^{x_i}) \quad (5)$$

2.2. Bayes estimator

In this paper, we assumed that, prior distribution of the shape parameter θ is Gamma distribution which defined as follows [3]:

$$g(\theta) = \frac{\alpha^\beta}{\Gamma(\beta)} \theta^{\beta-1} e^{-\alpha\theta} \qquad \alpha, \beta > 0 \quad ; \quad \theta > 0 \quad (6)$$

The posterior probability density function of the shape parameter θ can be expressed as form[4]:

$$H(\theta|\underline{x}) = \frac{L(x_1, x_2, \dots, x_n; \theta)\, g(\theta)}{\int_{\forall\theta} L(x_1, x_2, \dots, x_n; \theta)\, g(\theta) d\theta}$$

$$= \frac{\pi_{i=1}^{n} f(x_i; \theta)\, g(\theta)}{\int_{\forall\theta} L(x_1, x_2, \dots, x_n; \theta)\, g(\theta) d\theta} \quad (7)$$

Now, combining (3) with (6) in (7), yields:

$$H(\theta|\underline{x}) = \frac{\theta^{n+\beta-1} e^{-\theta[\alpha - \sum_{i=1}^{n}(1 - e^{x_i})]}}{\int_0^\infty \theta^{n+\beta-1} e^{-\theta[\alpha - \sum_{i=1}^{n}(1 - e^{x_i})]} d\theta}$$

After Simplification ,we get (8)

That is, the posterior p.d.f. of the parameter θ is obviously Gamma distribution. On the other hand

$\theta|\underline{x} \sim \text{Gamma}(n + \beta, \alpha - T)$, with:

$E(\theta|\underline{x}) = \frac{n+\beta}{\alpha-T}$, $Var(\theta|\underline{x}\sim\theta) = \frac{n+\beta}{(\alpha-T)^2}$

2.2.1 Bayesian estimators under Generalized weighted loss function

AL-Nasser and Saleh[9] (2009) suggested the Generalized square error loss function as follows

$$L(\hat{\theta},\theta) = \left(\sum_{j=0}^{k} b_j\theta^j\right)(\hat{\theta}-\theta)^2 \quad ; \quad k = 0,1,\dots ; \quad b_j = (j = 0,\dots,k) \text{ are constants}$$

Rasheed (2015) [8] proposed a a modulation of the the Generalized square error loss function and named it Generalized weighted loss function which is defined as follows

$$L(\hat{\theta},\theta) = \frac{\left(\sum_{j=0}^{k} b_j\theta^j\right)(\hat{\theta}-\theta)^2}{\theta^c} \quad , \quad \theta > 0 \quad ; \quad c = 1, 2, 3, \dots \text{ are constants} \qquad (9)$$

Then the Risk function under Generalized weighted loss function denoted by $R(\hat{\theta},\theta)$ is:

$$R(\hat{\theta},\theta) = E[L(\hat{\theta},\theta)]$$

$$= \int_0^\infty \frac{1}{\theta^c}\left(\sum_{j=0}^{k} b_j\theta^j\right)(\hat{\theta}-\theta)^2 H(\theta|\underline{x})d\theta$$

$$= \int_0^\infty \frac{1}{\theta^c}(b_0 + b_1\theta + \cdots + b_k\theta^k)(\hat{\theta}^2 - 2\hat{\theta}\theta + \theta^2)H(\theta|\underline{x})d\theta$$

$$= b_0\hat{\theta}^2 E\left(\frac{1}{\theta^c}|\underline{x}\right) - 2b_0\hat{\theta}E\left(\frac{1}{\theta^{c-1}}|\underline{x}\right) + b_0 E\left(\frac{1}{\theta^{c-2}}|\underline{x}\right) + b_1\hat{\theta}^2 E\left(\frac{1}{\theta^{c-1}}|\underline{x}\right) - 2b_1\hat{\theta}E\left(\frac{1}{\theta^{c-2}}|\underline{x}\right) + b_1 E\left(\frac{1}{\theta^{c-3}}|\underline{x}\right) + \cdots + b_K\hat{\theta}^2 E\left(\frac{1}{\theta^{c-K}}|\underline{x}\right) - 2b_K\hat{\theta}E\left(\frac{1}{\theta^{c-(K+1)}}|\underline{x}\right) + b_K E\left(\frac{1}{\theta^{c-(K+2)}}|\underline{x}\right)$$

By taking the partial derivative for $R(\hat{\theta},\theta)$ with respect to $\hat{\theta}$ and setting it equal to zero yields

$$\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}} = 2b_0\hat{\theta}E\left(\frac{1}{\theta^c}|\underline{x}\right) - 2b_0 E\left(\frac{1}{\theta^{c-1}}|\underline{x}\right) + 2b_1\hat{\theta}E\left(\frac{1}{\theta^{c-1}}|\underline{x}\right) - 2b_1 E\left(\frac{1}{\theta^{c-2}}|\underline{x}\right) + \cdots + 2b_K\hat{\theta}E\left(\frac{1}{\theta^{c-K}}|\underline{x}\right) - 2b_K E\left(\frac{1}{\theta^{c-(K+1)}}|\underline{x}\right) = 0$$

Hence, Bayes estimator using Generalized weighted loss function, denoted by $\hat{\theta}$ can be written as

$$\hat{\theta} = \frac{b_0 E\left(\frac{1}{\theta^{c-1}}|\underline{x}\right) + b_1 E\left(\frac{1}{\theta^{c-2}}|\underline{x}\right) + \cdots + b_K E\left(\frac{1}{\theta^{c-(k+1)}}|\underline{x}\right)}{b_0 E\left(\frac{1}{\theta^{c}}|\underline{x}\right) + b_1 E\left(\frac{1}{\theta^{c-1}}|\underline{x}\right) + \cdots + b_K E\left(\frac{1}{\theta^{c-k}}|\underline{x}\right)} \qquad (10)$$

Now, we will discuss some special cases of the Generalized weighted loss function:

Putting k = 0 and c = 0, leads to the Square error loss function which is defined as follows

$$L(\hat{\theta}, \theta) = (\theta - \hat{\theta})^2$$

When k = 0 and c = 1, yields the Weighted square error loss function which is as follows:

$$L(\hat{\theta}, \theta) = \frac{(\hat{\theta}, \theta)^2}{\theta}$$

If k = 0 and c = 2, leads to the Quadratic loss function, which is written as

$$L(\hat{\theta}, \theta) = \left(\frac{\theta - \hat{\theta}}{\theta}\right)^2$$

To obtain Bayes estimators for θ by using the Generalized weighted loss function, according to the posterior density function (8), we derived $E(\theta^m|\underline{x})$, $E\left(\frac{1}{\theta^m}|\underline{x}\right)$ as follows

$$E(\theta^m|\underline{x}) = \int_0^\infty \theta^m \, H(\theta|\underline{\mathbf{x}}) d\theta$$

$$= \int_0^\infty \theta^m \frac{(\alpha - T)^{n+\beta} \, \theta^{n+\beta-1} \, e^{-\theta(\alpha-T)}}{\Gamma(n+\beta)} d\theta$$

$$= \frac{\Gamma(n+m+\beta)}{\Gamma(n+\beta)(\alpha-T)^m} \quad , \; m = 1,2,\ldots \qquad (11)$$

And $E\left(\frac{1}{\theta^m}|\underline{x}\right) = \int_0^\infty \frac{1}{\theta^m} H(\theta|\underline{\mathbf{x}}) d\theta$

$$= \frac{(\alpha-T)^m \, \Gamma(n+\beta-m)}{\Gamma(n+\beta)} \quad , \quad m = 1,2,\ldots \qquad (12)$$

Which can be substituted into equation (10) to obtain $\hat{\theta}$ for different values of k and c.

In this paper, we use k = 0,1,2 and c = 0,1, 2, based on Gamma distribution prior.

Now, let k = 0, yields

$$\hat{\theta} = \frac{E\left(\frac{1}{\theta^{c-1}} \,|\, \underline{x}\right)}{E\left(\frac{1}{\theta^{c}} \,|\, \underline{x}\right)}$$

Put c = 0, gives

$$\hat{\theta}_1 = E(\theta) = \frac{(n+\beta)}{(\alpha-t)}$$

By letting c =1, gives

$$\hat{\theta}_2 = [E\ (\tfrac{1}{\theta})\ ]^{-1} = \frac{(n+\beta-1)}{(\alpha-T)} \quad (13)$$

Let c =2 , we get,

$$\hat{\theta}_3 = \frac{E\left(\frac{1}{\theta}\right)}{E\left(\frac{1}{\theta^2}\right)} = \frac{(n+\beta-2)}{(\alpha-T)} \quad (14)$$

If k = 1, yields,

$$\hat{\theta} = \frac{b_0 E\left(\frac{1}{\theta^{C-1}} \mid \underline{x}\right) + b_1 E\left(\frac{1}{\theta^{C-2}} \mid \underline{x}\right)}{b_0 E\left(\frac{1}{\theta^{C}} \mid \underline{x}\right) + b_1 E\left(\frac{1}{\theta^{C-1}} \mid \underline{x}\right)}$$

Putting c = 0, gives,

$$\hat{\theta}_4 = \frac{b_0 E(\theta) + b_1 E(\theta^2)}{b_0 + b_1 E(\theta)} = \frac{b_0 \frac{(n+\beta)}{(\alpha-T)} + b_1 \frac{(n+\beta)(n+\beta+1)}{(\alpha-T)^2}}{b_0 + b_1 (\frac{n+\beta}{\alpha-T})} \quad (15)$$

Let c =1, then we have

$$\hat{\theta}_5 = \frac{b_0 + b_1 E(\theta)}{b_0 E\left(\frac{1}{\theta}\right) + b_1} = \frac{b_0 + b_1 (\frac{n+\beta}{\alpha-T})}{b_0 (\frac{\alpha-T}{n+\beta-1}) + b_1} \quad (16)$$

By letting c =2, gives,

$$\hat{\theta}_6 = \frac{b_0 E\left(\frac{1}{\theta}\right) + b_1}{b_0 E\left(\frac{1}{\theta^2}\right) + b_1 E\left(\frac{1}{\theta}\right)} = \frac{b_0 (\frac{\alpha-T}{n+\beta-1}) + b_1}{b_0 \frac{(\alpha-T)^2}{(n+\beta-1)(n+\beta-2)} + b_1 (\frac{\alpha-T}{n+\beta-1})} \quad (17)$$

Now, let k = 2

Put c = 0, yields

$$\hat{\theta}_7 = \frac{b_0 E(\theta) + b_1 E(\theta^2) + b_2 E(\theta^3)}{b_0 + b_1 E(\theta) + b_2 E(\theta^2)} = \frac{b_0 \frac{(n+\beta)}{(\alpha-T)} + b_1 \frac{(n+\beta)(n+\beta+1)}{(\alpha-T)^2} + b_2 \frac{(n+\beta)(n+\beta+1)(n+\beta+2)}{(\alpha-T)^3}}{b_0 + b_1 \left(\frac{n+\beta}{\alpha-T}\right) + b_2 \frac{(n+\beta)(n+\beta+1)}{(\alpha-T)^2}} \quad (18)$$

When c =1, then we have

$$\hat{\theta}_8 = \frac{b_0 + b_1 E(\theta) + b_2 E(\theta^2)}{b_0 E\left(\frac{1}{\theta}\right) + b_1 + b_2 E(\theta)} = \frac{b_0 + b_1 \frac{(n+\beta)}{(\alpha-T)} + b_2 \frac{(n+\beta)(n+\beta+1)}{(\alpha-T)^2}}{b_0 (\frac{\alpha-T}{n+\beta-1}) + b_1 + b_2 \left(\frac{n+\beta}{\alpha-T}\right)} \quad (19)$$

Put c =2, yields

$$\hat{\theta}_9 = \frac{b_0\, E\left(\frac{1}{\theta}\right) + b_1 + b_2 E(\theta)}{b_0\, E\left(\frac{1}{\theta^2}\right) + b_1\, E\left(\frac{1}{\theta}\right) + b_2} = \frac{b_0\left(\frac{\alpha - T}{n+\beta-1}\right) + b_1 + b_2 \frac{(n+\beta)}{(\alpha - T)}}{b_0 \frac{(\alpha - T)^2}{(n+\beta-1)\,(n+\beta-2)} + b_1\left(\frac{\alpha - T}{n+\beta-1}\right) + b_2} \tag{20}$$

3. Simulation study

In this section, a Monte Carlo simulation has been done to compare the performance of the different estimators of the unknown shape parameter for Basic Gompertz distribution θ. The process (R) have been repeated 2500 times with different sample sizes n = 15, 50, and100 to represent small, moderate and large sample size.

The values of shape parameter θ are chosen as θ = 0.5, 1 and 3 to propose the different shape values of density function. Bayes estimates have been generated considering two different values for the parameters of Gamma prior, α and β as $\alpha = 0.7$ and $1.5,\ \beta = 1 \ and\ 1.5$ taking b_0= -5000, b_1 = 10 and b_2= 10. All estimators that derived in the previous section are evaluated based on their mean squared errors (MSE's), where,

$$MSE(\theta) = \frac{\sum_{i=0}^{R}\left(\hat{\theta}_i - \theta\right)^2}{R} \qquad ; \qquad i = 1, 2, 3, \dots, R$$

The expected values of θ are tabulated in tables (1-3). The Tables (4-6) represent mean squared errors (MSE's) for all estimates for different cases.

4. Discussion and Conclusion

The results in tables 4 and 5 which are represent MSE values for different estimates when θ = 0.5, 1 respectively, show that, the Bayes estimates using generalized weighted loss function $\hat{\theta}_9$ (when k = 2 and c = 2) is the best estimate in comparison with the other estimates for all sample sizes, while the best estimates for θ when θ = 3 is $\hat{\theta}_7$ (when k = 2 and c = 0) for all sample sizes.

Finally, it is obvious that, MSE's for Bayes estimators using Gamma prior, are decreasing when the value of the scale parameter of Gamma prior (β) is close to the unknown shape parameter θ of Basic Gompertz distribution.

Table 1: The expected values of different estimators for unknown shape parameter of Basic Gompertz distribution when θ = 0.5

n	α	β	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_4$	$\hat{\theta}_5$	$\hat{\theta}_6$	$\hat{\theta}_7$	$\hat{\theta}_8$	$\hat{\theta}_9$	ML
15	0.7	0.5	0.536348	0.501746	0.467142	0.536308	0.501708	0.467108	0.536256	0.501662	0.467067	0.532893
		1.5	0.570952	0.536348	0.501746	0.570909	0.536308	0.501708	0.570850	0.536256	0.501662	
	1.5	0.5	0.520945	0.487336	0.453727	0.520908	0.487302	0.453694	0.520861	0.487260	0.453658	
		1.5	0.554556	0.520945	0.487336	0.554515	0.520908	0.487302	0.554461	0.520861	0.487260	
50	0.7	0.5	0.512261	0.502116	0.491973	0.512250	0.502106	0.491963	0.512238	0.502095	0.491952	0.510896
		1.5	0.522404	0.512261	0.502116	0.522394	0.512250	0.502106	0.522382	0.512238	0.502095	
	1.5	0.5	0.508047	0.497987	0.487928	0.508038	0.497977	0.487917	0.508026	0.497967	0.487906	
		1.5	0.518108	0.508047	0.497987	0.518098	0.508038	0.497977	0.518086	0.508026	0.497967	
100	0.7	0.5	0.505579	0.500548	0.495518	0.505574	0.500543	0.495513	0.505569	0.500538	0.495508	0.504858
		1.5	0.510609	0.505579	0.500548	0.510604	0.505574	0.500543	0.510599	0.505569	0.500538	
	1.5	0.5	0.503532	0.498521	0.493512	0.503527	0.498516	0.493506	0.503522	0.498511	0.493501	
		1.5	0.508542	0.503532	0.498521	0.508537	0.503527	0.498516	0.508531	0.503522	0.498511	

Table 2: The expected values of different estimators for unknown shape parameter of Basic Gompertz distribution when θ = 1

n	α	β	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_4$	$\hat{\theta}_5$	$\hat{\theta}_6$	$\hat{\theta}_7$	$\hat{\theta}_8$	$\hat{\theta}_9$	ML
15	0.7	0.5	1.045638	0.978178	0.910716	1.045487	0.978035	0.910586	1.045102	0.977697	0.910290	1.065785
		1.5	1.113097	1.045638	0.978178	1.112937	1.045487	0.978035	1.112502	1.045102	0.977697	
	1.5	0.5	0.988940	0.925138	0.861335	0.988807	0.925012	0.861219	0.988487	0.924733	0.860976	
		1.5	1.052743	0.988940	0.925138	1.052601	0.988807	0.925012	1.052241	0.988487	0.924733	
50	0.7	0.5	1.017141	0.996999	0.976858	1.017099	0.996958	0.976818	1.017008	0.996871	0.976734	1.021792
		1.5	1.037283	1.017141	0.996999	1.037240	1.017099	0.996958	1.037146	1.017008	0.996871	
	1.5	0.5	1.000674	0.980858	0.961043	1.000634	0.980819	0.961005	1.000546	0.980736	0.960926	
		1.5	1.020489	1.000674	0.980858	1.020448	1.000634	0.980819	1.020358	1.000546	0.980736	
100	0.7	0.5	1.007573	0.997548	0.987524	1.007553	0.997528	0.987503	1.007510	0.997486	0.987461	1.009717
		1.5	1.017601	1.007573	0.997548	1.017580	1.007553	0.997528	1.017535	1.007510	0.997486	
	1.5	0.5	0.999477	0.989534	0.979587	0.999458	0.989512	0.979566	0.999415	0.989470	0.979527	
		1.5	1.009421	0.999477	0.989534	1.009401	0.999458	0.989512	1.009358	0.999415	0.989470	

Table 3: The expected values of different estimators for unknown shape parameter of Basic Gompertz distribution when θ = 3

n	α	β	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_4$	$\hat{\theta}_5$	$\hat{\theta}_6$	$\hat{\theta}_7$	$\hat{\theta}_8$	$\hat{\theta}_9$	ML
15	0.7	0.5	2.851550	2.667578	2.483607	2.850436	2.666540	2.482636	2.842691	2.659757	2.476756	3.197359
		1.5	3.035517	2.851550	2.667578	3.034332	2.850436	2.666540	3.025556	2.842691	2.659757	
	1.5	0.5	2.471174	2.311743	2.152316	2.470353	2.310974	2.151597	2.465570	2.306784	2.147956	
		1.5	2.630604	2.471174	2.311743	2.629731	2.470353	2.310974	2.624319	2.465570	2.306784	
50	0.7	0.5	2.966031	2.907300	2.848562	2.965671	2.906948	2.848221	2.963362	2.904727	2.846088	3.065383
		1.5	3.024764	2.966031	2.907300	3.024400	2.965671	2.906948	3.021994	2.963362	2.904727	
	1.5	0.5	2.830449	2.774403	2.718354	2.830126	2.774087	2.718046	2.828139	2.772171	2.716207	
		1.5	2.886498	2.830449	2.774403	2.886171	2.830126	2.774087	2.884095	2.828139	2.772171	
100	0.7	0.5	2.980470	2.950813	2.921154	2.980291	2.950638	2.920986	2.979165	2.949532	2.919898	3.029158
		1.5	3.010128	2.980470	2.950813	3.009943	2.980291	2.950638	3.008793	2.979165	2.949532	
	1.5	0.5	2.910748	2.881785	2.852824	2.910578	2.881617	2.852658	2.909529	2.880590	2.851654	
		1.5	2.851550	2.667578	2.483607	2.850436	2.666540	2.482636	2.842691	2.659757	2.476756	

Table 4: The MSE values of different estimators for unknown shape parameter of Basic Gompertz distribution when θ = 0.5

n	α	β	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_4$	$\hat{\theta}_5$	$\hat{\theta}_6$	$\hat{\theta}_7$	$\hat{\theta}_8$	$\hat{\theta}_9$	ML
15	0.7	0.5	0.021588	0.017740	0.016454	0.021579	0.017734	0.016451	0.021561	0.017722	0.016445	0.022364
		1.5	0.028001	0.021588	0.017740	0.027988	0.021579	0.017734	0.027962	0.021561	0.017722	
	1.5	0.5	0.018313	0.015802	0.015700	0.018306	0.015799	0.015699	0.018292	0.015790	0.015695	
		1.5	0.023231	0.018313	0.015802	0.023220	0.018306	0.015799	0.023201	0.018292	0.015790	
50	0.7	0.5	0.005915	0.005543	0.005381	0.005914	0.005542	0.005381	0.005913	0.005542	0.005381	0.005941
		1.5	0.006497	0.005915	0.005543	0.006496	0.005914	0.005542	0.006495	0.005913	0.005542	
	1.5	0.5	0.005638	0.005359	0.005286	0.005637	0.005358	0.005286	0.005636	0.005358	0.005286	
		1.5	0.006124	0.005638	0.005359	0.006123	0.005637	0.005358	0.006122	0.005636	0.005358	
100	0.7	0.5	0.002633	0.002551	0.002519	0.002633	0.002550	0.002519	0.002633	0.002550	0.002519	0.002637
		1.5	0.002766	0.002633	0.002551	0.002766	0.002633	0.002550	0.002766	0.002633	0.002550	
	1.5	0.5	0.002572	0.002511	0.002501	0.002572	0.002511	0.002501	0.002572	0.002511	0.002501	
		1.5	0.002684	0.002572	0.002511	0.002684	0.002572	0.002511	0.002683	0.002572	0.002511	

Table 5: The MSE values of different estimators for unknown shape parameter of Basic Gompertz distribution when θ = 1

n	α	β	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_4$	$\hat{\theta}_5$	$\hat{\theta}_6$	$\hat{\theta}_7$	$\hat{\theta}_8$	$\hat{\theta}_9$	ML
15	0.7	0.5	0.074690	0.064017	0.063051	0.074631	0.063984	0.063040	0.074409	0.063845	0.062968	
		1.5	0.095070	0.074690	0.064017	0.094982	0.074631	0.063984	0.094659	0.074409	0.063845	0.089457
	1.5	0.5	0.057349	0.055685	0.062640	0.057319	0.055675	0.062647	0.057202	0.055615	0.062631	
		1.5	0.067631	0.057349	0.055685	0.067579	0.057319	0.055675	0.067391	0.057202	0.055615	
50	0.7	0.5	0.022681	0.021518	0.021184	0.022676	0.021515	0.021183	0.022660	0.021503	0.021176	
		1.5	0.024672	0.022681	0.021518	0.024665	0.022676	0.021515	0.024645	0.022660	0.021503	0.023762
	1.5	0.5	0.020944	0.020489	0.020835	0.020941	0.020487	0.020835	0.020929	0.020480	0.020831	
		1.5	0.022201	0.020944	0.020489	0.022196	0.020941	0.020487	0.022180	0.020929	0.020480	
100	0.7	0.5	0.010317	0.010062	0.010011	0.010316	0.010062	0.010010	0.010312	0.010059	0.010009	
		1.5	0.010774	0.010317	0.010062	0.010773	0.010316	0.010062	0.010768	0.010312	0.010059	0.010547
	1.5	0.5	0.009932	0.009844	0.009957	0.009931	0.009844	0.009957	0.009928	0.009842	0.009956	
		1.5	0.010219	0.009932	0.009844	0.010218	0.009931	0.009844	0.010214	0.009928	0.009842	

Table 6: The MSE values of different estimators for unknown shape parameter of Basic Gompertz distribution when θ = 3

n	α	β	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_4$	$\hat{\theta}_5$	$\hat{\theta}_6$	$\hat{\theta}_7$	$\hat{\theta}_8$	$\hat{\theta}_9$	ML
15	0.7	0.5	0.457675	0.491745	0.597131	0.457288	0.491809	0.597588	0.451626	0.489811	0.598418	
		1.5	0.494923	0.457675	0.491745	0.494026	0.457288	0.491809	0.483768	0.451626	0.489811	
	1.5	0.5	0.519786	0.683843	0.900734	0.520328	0.684613	0.901698	0.522419	0.687952	0.905907	
		1.5	0.408565	0.519786	0.683843	0.408840	0.520328	0.684613	0.409261	0.522419	0.687952	0.805111
50	0.7	0.5	0.180598	0.181001	0.188445	0.180533	0.180982	0.188467	0.179810	0.180564	0.188333	
		1.5	0.187234	0.180598	0.181001	0.187124	0.180533	0.180982	0.186069	0.179810	0.180564	
	1.5	0.5	0.177070	0.193401	0.216131	0.177111	0.193478	0.216243	0.177134	0.193727	0.216700	
		1.5	0.167138	0.177070	0.193401	0.167141	0.177111	0.193478	0.166919	0.177134	0.193727	0.213860
100	0.7	0.5	0.087627	0.087938	0.090024	0.087613	0.087934	0.090032	0.087455	0.087847	0.090012	
		1.5	0.089094	0.087627	0.087938	0.089068	0.087613	0.087934	0.088836	0.087455	0.087847	
	1.5	0.5	0.087283	0.091721	0.097853	0.087295	0.091743	0.097884	0.087308	0.091817	0.098016	
		1.5	0.084539	0.087283	0.091721	0.084540	0.087295	0.091743	0.084489	0.087308	0.091817	0.094925

References

1- Abu-Zinadah, H. H., Six method of estimations for the shape parameter of exponentiated Gompertz distribution. Applied Mathematical Sciences, 8(88), 4349-4359, 2014.

2- El-Gohary, A., Alshamrani, A., Al-Otaibi, A., The generalized Gompertz distribution", Applied Mathematical Modeling, 37 (1), 13–24, 2013.

3- Griffin, J. E. and Brown, P. J., Inference with normal-gamma prior distributions in regression problems, Bayesian Analysis, 5(1), 171-188, 2010.

4- Jaheen, Z. F., A Bayesian analysis of record statistics from the Gompertz model, Applied Mathematics and Computation, 145(2-3), 307-320, 2003

5- John H. and Emil J., The Gompertz distribution and its application, Journal Article , 48 (314), 15-28, 1992.

6- Lenart, A., The moments of the Gompertz distribution and maximum likelihood estimation of its parameters, Scandinavian Actuarial Journal, (3), 255-277, 2014.

7- M. L. Garg, B. R. Rao, and C. K. Redmond, Maximum likelihood estimation of the parameters of the Gompertz survival function, Journal of the Royal Statistical Society C, 19, 152–159, 1970.

8- Rasheed, H. A. and Khalifa, Z. N., Semi-Minimax Estimators of Maxwell Distribution under New Loss Function, Mathematics and Statistics Journal, 2 (3), 16-22, 2016.

9- Saleh, M.A and AL-Nasser, H. A, The use of a generalized loss function of the method for estimating the Bayes scale parameter function and reliability of the Weibull distribution. Iraqi Journal of Statistical Science, 16, 1-14, 2009.