Journal of Iraqi Al-Khwarizmi Society (JIKhS) Year 2018 pages: 168-177
Volume:2 Special issue of the first international scientific conference of Iraqi Al-Khawarizmi Society 28-29 March 2018

Numerical solution for second order Fuzzy Differential Equation by two methods

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Abstract: In this paper, two different methods were used the solve the 2nd fuzzy differential equation .there Picard's and rung-kutta methods the two methods were applied on MATLAB program to obtain comparative results .Both methods have shown numerical and graphical result which are close to the practical theorem (the exact solution) approximate solution with piratical theorem (the exact solution)

Keywords: Fuzzy differential equation, fuzzy numbers, Runge-Kutta method, Picard's iterative method.

I. Introduction

The fuzzy set was first introduced by Zadeh[1]. The theory of blurry groups is one of the important theories currently used in the processing of ambiguous information in mathematical models and is a powerful tool for modeling uncertainty. Therefore, for such mathematical modelling, using fuzzy differential equations are necessary. Important element of fuzzy differentialequation was fuzzy derivative(see[2]) . Many authors studied the solve of the fuzzy differentialequation by numerical method in the same and different method as [3],[4], [5],[7],[8],[9],[10].

In this paper, firstly, the fuzzy differential equation has been defined. Fuzzydifferential equations play important part in the biology, physics, and engineering as well as possible among other fields of science.

After that we used two methods to solve the 2nd fuzzydifferential equation, the first method is Runge_kutta method and discuss the Picard successive approximation method. Then, we explained the two methods by using an example and found the exact and approximate solutions by applying the matlab program. Eventually, the obtained results for the exact and approximation solutions were tabulated and discussed.

After that consider the numerical algorithm an approximation method for find power series solution fuzzy differential equation. $\acute{\dot{y}} = f(t, y, \acute{y})$.

II .Preliminaries

In this part, we show some definitions , and impotent notations preliminaries, which are used in this [2],[3],[11].

Definition2.1[11]:

A fuzzy numberis a map $u: R \longrightarrow [0,1]$ *which satisfies*

a.u *is upper semicontinuous*

b. $u(x) = 0$ *outside someinterval* $[c, d] \subset R$.

c.*There real numbers* a, b *such that* $c \le a \le b \le d$ *where*

1. $u(x)$ *is monotonic increasing on* $[c, a]$,

2. $u(x)$ is monotonic decreasing on $[b, d]$,

3. $u(x) = 1$, $a \le x \le b$

A set E is the family of fuzzy numbers and arbitrary fuzzy number is exemplify by an ordered pair of functions

Definition2.2. [11] :

The arbitrary fuzzy number inparametric form is represented by ordered pair functions

$$[u]_r = [\underline{u}(r), \overline{u}(r)] \quad ,0 \le r \le 1$$

That is satisfy the following:

1. $\underline{u}(r)$ a bounded left continuous nondecreasing function on[0,1].
2. $\overline{u}(r)$ abounded right continuous nonincreasing function on[0,1].
3. $\underline{u}(r) \le \overline{u}(r)$ for all $r \in [0,1]$.

A crisp number α is frugally represented by $\underline{u}(r) = \overline{u}(r) = \alpha, \ 0 \le r \le 1$. For arbitrary,

$$u = (\underline{u}(r), \overline{u}(r)), \quad v = (\underline{v}(r), \overline{v}(r))$$

and $k \in R$, we define equality, addition and multiplication by k as

a. u = v if and only if $\underline{u}(r) = \underline{v}(r), \overline{u}(r) = \overline{v}(r))$,

b. $u + v = (\underline{u}(r) + \underline{v}(r), \overline{u}(r) + \overline{v}(r))$,

c. $ku = \begin{cases} (k\underline{u}, k\overline{u}), & k \ge 0 \\ (k\overline{u}, k\underline{u}), & k < 0. \end{cases}$

Definition 2.3. [11]:

Let I is a real interval. A mapping $X: I \to E$ is fuzzy process and its r-level set is denoted by $[x(t)]_r = [x_1(t;r), x_2(t;r)],\ \ t \in I,\ r \in (0,1]$ thederivative $\dot{x}(t)$ of fuzzy process x is defined by $[\dot{x}(t)]_r = [\dot{x}_1(t;r), \dot{x}_2(t;r)],\ \ t \in I,\ r \in (0,1]$.

III. Second Order Fuzzy Differential Equation:

2nd fuzzy initial value differential equation is :

$$\begin{cases} \ddot{y} = f(t, y, \dot{y}) & t \in [t_0, T] \\ y(t_0) = y_0, \dot{y}(t_0) = y_0 \end{cases}$$

Such that y is afuzzy function of $t, f(t, y)$ is a fuzzy function of the scrips variable t and the fuzzy Variable y.

y_o the fuzzy derivativeof y and $y(t_o) = y_o$ is a triangular or a triangular shaped fuzzy number. We denote y = $[\underline{y}, \overline{y}]$. It denote that the r-level set of y(t) for $t \in [t_o, T]$ is

$$[\, y(t)]_r = [\underline{y}(t;r), \overline{y}(t;r)]$$

$$[\dot{y}(t)]_r = [\underline{\dot{y}}(t;r), \overline{\dot{y}}(t;r)]$$

Also:

$$[\, f(t, y(t))]_r = [\underline{f}(t, y(t); r), \overline{f}(t, y(t); r)]$$

We write:

$$f(t, y) = [\underline{f}(t, y), \overline{f}(t, y)]$$

We have:

$$\underline{\dot{y}}(t;r) = \underline{f}(t, y(t); r) = F[t, \underline{y}(t;r), \overline{y}(t;r)]$$

$$\overline{\dot{y}}(t;r) = \overline{f}(t, y(t); r) = G[t, \underline{y}(t;r), \overline{y}(t;r)]$$

Also we write

$$[\, y(t_o)]_r = [\underline{y}(t_o; r), \overline{y}(t_o; r)]$$

$$[y_o]_r = [(\underline{y}_o(r)), (\overline{y}_o(r))]$$

$$\underline{y}(t_o; r) = \underline{y}_o(r) \quad , \quad \overline{y}(t_o; r) = \overline{y_o}$$

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using the extension precept , have themembership function:

$$f(t,y(t))(s) = \sup\{y(t)(\tau) | s = f(t,\tau)\}, \quad s \in R$$

So Fuzzy number $f(t,y(t))$

$$[f(t,y(t))]_r = \left[\underline{f}(t,y(t);r), \overline{f}(t,y(t);r)\right], \quad r \in [0,1]$$

Where

$$\underline{f}(t,y(t);r) = \min\{f(t,u) | u \in [y(t)]_r\}$$
$$\overline{f}(t,y(t);r) = \max\{f(t,u) | u \in [y(t)]_r\}$$

IV. Fourth Order Runge-Kutta Method in fuzzy differentialequation

The form of 2[nd] fuzzy differential equation is

$$\begin{cases} \grave{\acute{y}} = f(t,y,\acute{y}) & t \in [t_o,T] \\ y(t_o) = y_o, \acute{y}(t_o) = \acute{y}_o \end{cases} \qquad (1)$$

From (1) can reduced two first order simultaneous fuzzy differential equation as

$$\begin{cases} \acute{y} = f(t,\acute{y},y) \\ \acute{y} = g(t,\acute{y},y) & t \in [t_o,T] \\ y(t_o) = y_o, \acute{y}(t_o) = y_o \end{cases}$$

The exact solution will be

$$[Y(t_n)]_r = [\underline{Y}(t_n;r), \overline{Y}(t_n;r)] \text{ ,}$$
$$[\acute{Y}(t_n)]_r = [\underline{\acute{Y}}(t_n;r), \overline{\acute{Y}}(t_n;r)]$$

The approximation solution is given by

$$[y(t_n)]_r = [\underline{y}(t_n;r), \overline{y}(t_n;r)]$$
$$[\acute{y}(t_n)]_r = [\underline{\grave{y}}(t_n;r), \overline{\acute{y}}(t_n;r)]$$

By using fourth order Runge-Kutta method, we have

$$[y(t_n)]_r = [\underline{y}(t_n;r), \overline{y}(t_n;r)$$

$$\underline{y}(t_{n+1};r) = \underline{y}(t_n;r) + \sum_{j=1}^{4} w_j\, k_{j,1}(t_n, y(t_n,r)), \quad \overline{y}(t_{n+1};r) = \overline{y}(t_n;r) + \sum_{j=1}^{4} w_j\, k_{j,2}(t_n, y(t_n,r))$$

$$\underline{\acute{y}}(t_{n+1};r) = \underline{\acute{y}}(t_n;r) + \sum_{j=1}^{4} w_j\, l_{j,1}(t_n, y(t_n,r)), \quad \overline{\acute{y}}(t_{n+1};r) = \overline{\acute{y}}(t_n;r) + \sum_{j=1}^{4} w_j\, l_{j,2}(t_n, y(t_n,r))$$

Where $k_{j,١}, k_{j,٢}$ could be defined as follow:

$$k_{1,1}(t_n, y(t_n;r)) = \min\, \{y(t_n, u, v) | u \in \left(\underline{y}(t_n;r), \overline{y}(t_n;r)\right), v \in \left(\underline{\acute{y}}(t_n;r), \overline{\acute{y}}(t_n;r)\right)\}$$

$$k_{1,2}(t_n, y(t_n;r)) = \max\, \{y(t_n, u, v) | u \in \left(\underline{y}(t_n;r), \overline{y}(t_n;r)\right), v \in \left(\underline{\acute{y}}(t_n;r), \overline{\acute{y}}(t_n;r)\right)\}$$

$$k_{2,1}(t_n, y(t_n;r)) = \min\, \{y(t_n + \tfrac{h}{2}, u) | u \in \left(p_{1,1}(t_n; y(t_n,r), \grave{y}(t_n,r)), p_{1,2}(t_n; y(t_n,r), \grave{y}(t_n,r))\right),$$
$$v \in \left(q_{1,1}(t_n; y(t_n,r), \grave{y}(t_n,r)), q_{1,2}(t_n; y(t_n,r), \grave{y}(t_n,r))\right)\}$$

$$k_{2,1}(t_n, y(t_n;r)) = \max\, \{y(t_n + \tfrac{h}{2}, u, v) | u \in \left(p_{1,1}(t_n; y(t_n,r), \grave{y}(t_n,r)), p_{1,2}(t_n; y(t_n,r), \grave{y}(t_n,r))\right), v \in \left(q_{1,1}(t_n; y(t_n,r), \overline{y}(t_n;r)), q_{1,2}(t_n; y(t_n,r), \overline{y}(t_n;r))\right)\}$$

$$k_{3,1}(t_n, y(t_n;r)) = \min\, \{y(t_n + \tfrac{h}{2}, u, v) | u \in \left(p_{2,1}(t_n; y(t_n,r), \grave{y}(t_n,r)), p_{2,2}(t_n: y(t_n,r), \grave{y}(t_n,r))\right), v \in \left(q_{2,1}(t_n; y(t_n,r)\overline{,y}(t_n;r)), q_{2,2}(t_n; y(t_n,r)\overline{,y}(t_n;r))\right)\}$$

$$k_{3,1}(t_n, y(t_n;r)) = \max\, \{y(t_n + \tfrac{h}{2}, u, v) | u \in \left(p_{2,1}(t_n; y(t_n,r), \grave{y}(t_n,r)), p_{2,2}(t_n; y(t_n,r), \grave{y}(t_n,r))\right), v \in \left(q_{2,1}(t_n; y(t_n,r), \overline{y}(t_n;r)), q_{2,2}(t_n; y(t_n,r), \overline{y}(t_n;r))\right)\}$$

$$k_{4,1}(t_n, y(t_n;r)) = \min\, \{y(t_n + \tfrac{h}{2}, u, v) | u \in \left(p_{3,1}(t_n; y(t_n,r), \grave{y}(t_n,r)), p_{3,2}(t_n; y(t_n,r), \grave{y}(t_n,r))\right), v \in \left(q_{3,1}(t_n; y(t_n,r), \overline{y}(t_n;r)), q_{3,2}(t_n; y(t_n,r), \overline{y}(t_n;r))\right)\}$$

$$k_{4,1}(t_n, y(t_n;r)) = \max\, \{y(t_n + \tfrac{h}{2}, u, v) | u \in \left(p_{3,1}(t_n; y(t_n,r), \grave{y}(t_n,r)), p_{3,2}(t_n; y(t_n,r), \grave{y}(t_n,r))\right), v \in \left(q_{3,1}(t_n; y(t_n,r), \overline{y}(t_n;r)), q_{3,2}(t_n; y(t_n,r), \overline{y}(t_n;r))\right)\}$$

$$l_{1,1}(t_n, y(t_n;r)) = \min\, \{g(t_n, u, v) | u \in \left(\underline{y}(t_n;r), \overline{y}(t_n;r)\right), v \in \left(\underline{\acute{y}}(t_n;r), \overline{\acute{y}}(t_n;r)\right)\}$$

$$l_{1,2}(t_n, y(t_n;r)) = \max\, \{g(t_n, u, v) | u \in \left(\underline{y}(t_n;r), \overline{y}(t_n;r)\right), v \in \left(\underline{\acute{y}}(t_n;r), \overline{\acute{y}}(t_n;r)\right)\}$$

$$l_{2,1}(t_n, y(t_n;r)) = \min \{g(t_n + \frac{h}{2}, u) | u \in \left(p_{1,1}(t_n; y(t_n,r), \dot{y}(t_n,r)), p_{1,2}(t_n; y(t_n,r), \dot{y}(t_n,r))\right), v \in \left(q_{1,1}(t_n; y(t_n,r), \dot{y}(t_n,r),), q_{1,2}(t_n; y(t_n,r), \dot{y}(t_n,r))\right)\}$$

$$l_{2,1}(t_n, y(t_n;r)) = \max \{g(t_n + \frac{h}{2}, u, v) | u \in \left(p_{1,1}(t_n; y(t_n,r), \dot{y}(t_n,r)), p_{1,2}(t_n; y(t_n,r), \dot{y}(t_n,r))\right), v \in \left(q_{1,1}(t_n; y(t_n,r), \dot{y}(t_n,r)), q_{1,2}(t_n; y(t_n,r), \dot{y}(t_n,r))\right)\}$$

$$l_{3,1}(t_n, y(t_n;r)) = \min \{g(t_n + \frac{h}{2}, u, v) | u \in \left(p_{2,1}(t_n; y(t_n,r), \dot{y}(t_n,r)), p_{2,2}(t_n; y(t_n,r), \dot{y}(t_n,r))\right), v \in \left(q_{2,1}(t_n; y(t_n,r), \dot{y}(t_n,r)), q_{2,2}(t_n; y(t_n,r), \dot{y}(t_n,r))\right)\}$$

$$l_{3,1}(t_n, y(t_n;r)) = \max \{g(t_n + \frac{h}{2}, u, v) | u \in \left(p_{2,1}(t_n; y(t_n,r), \dot{y}(t_n,r)), p_{2,2}(t_n; y(t_n,r), \dot{y}(t_n,r))\right), v \in \left(q_{2,1}(t_n; y(t_n,r), \dot{y}(t_n,r)), q_{2,2}(t_n; y(t_n,r), \dot{y}(t_n,r))\right)\}$$

$$l_{4,1}(t_n, y(t_n;r)) = \min \{g(t_n + \frac{h}{2}, u, v) | u \in \left(p_{3,1}(t_n; y(t_n,r), \dot{y}(t_n,r)), p_{3,2}(t_n; y(t_n,r), \dot{y}(t_n,r))\right), v \in \left(q_{3,1}(t_n; y(t_n,r), \dot{y}(t_n,r)), q_{3,2}(t_n; y(t_n,r), \dot{y}(t_n,r))\right)\}$$

$$l_{4,1}(t_n, y(t_n;r)) = \max \{y(t_n + \frac{h}{2}, u, v) | u \in \left(p_{3,1}(t_n; y(t_n,r), \dot{y}(t_n,r)), p_{3,2}(t_n; y(t_n,r), \dot{y}(t_n,r))\right), v \in v \in \left(q_{3,1}(t_n; y(t_n,r), \dot{y}(t_n,r)), q_{3,2}(t_n; y(t_n,r), \dot{y}(t_n,r))\right)\}$$

Where:

$$p_{1,1}(t_n; y(t_n,r)) = \underline{y}(t_n,r) + \frac{h}{2} k_{1,1}(t_n, y(t_n;r), \dot{y}(t_n,r))$$

$$p_{1,2}(t_n; y(t_n,r)) = \overline{y}(t_n,r) + \frac{h}{2} k_{1,2}(t_n, y(t_n;r)\dot{y}(t_n,r))$$

$$p_{2,1}(t_n; y(t_n,r)) = \underline{y}(t_n,r) + \frac{h}{2} k_{2,1}(t_n, y(t_n;r), \dot{y}(t_n,r))$$

$$p_{2,2}(t_n; y(t_n,r)) = \overline{y}(t_n,r) + \frac{h}{2} k_{2,2}(t_n, y(t_n;r), \dot{y}(t_n,r))$$

$$p_{3,1}(t_n; y(t_n,r)) = \underline{y}(t_n,r) + \frac{h}{2} k_{3,1}(t_n, y(t_n;r), \dot{y}(t_n,r))$$

$$p_{3,2}(t_n; y(t_n,r)) = \overline{y}(t_n,r) + \frac{h}{2} k_{3,2}(t_n, y(t_n;r), \dot{y}(t_n,r))$$

$$q_{1,1}(t_n; y(t_n,r)) = \underline{\acute{y}}(t_n,r) + \frac{h}{2} l_{1,1}(t_n, y(t_n;r), \dot{y}(t_n,r))$$

$$q_{1,2}(t_n; y(t_n,r)) = \overline{\acute{y}}(t_n,r) + \frac{h}{2} l_{1,2}(t_n, y(t_n;r), \dot{y}(t_n,r))$$

$$q_{2,1}(t_n; y(t_n,r)) = \underline{\acute{y}}(t_n,r) + \frac{h}{2} l_{2,1}(t_n, y(t_n;r), \dot{y}(t_n,r))$$

$$q_{2,2}(t_n; y(t_n,r)) = \overline{\acute{y}}(t_n,r) + \frac{h}{2} l_{2,2}(t_n, y(t_n;r), \dot{y}(t_n,r))$$

$$q_{3,1}(t_n; y(t_n,r)) = \underline{\acute{y}}(t_n,r) + \frac{h}{2} l_{3,1}(t_n, y(t_n;r), \dot{y}(t_n,r))$$

$q_{٣,٢}\left(t_n;y(t_n,r)\right)=\overline{\acute{y}}(t_n,r)+\frac{h}{٢}l_{٣,٢}\left(t_n,y(t_n;r),\acute{y}(t_n,r)\right)$

Now using the initial condition, we compute:

$$\underline{y}(t_{n+1};r)=\underline{y}(t_n;r)+\frac{h}{6}(k_{1,1}\left(t_n,y(t_n;r)\right)+2k_{2,1}\left(t_n,y(t_n;r)\right)+2\,k_{3,1}\left(t_n,y(t_n;r)\right)+k_{4,1}\left(t_n,y(t_n;r)\right)$$

$$\overline{y}(t_{n+1};r)=\overline{y}(t_n;r)+\frac{h}{6}(k_{1,2}\left(t_n,y(t_n;r)\right)+2k_{2,2}\left(t_n,y(t_n;r)\right)+2\,k_{3,2}\left(t_n,y(t_n;r)\right)+k_{4,2}\left(t_n,y(t_n;r)\right)$$

$$\underline{\acute{y}}(t_{n+1};r)=\underline{\acute{y}}(t_n;r)+\frac{h}{6}(l_{1,1}\left(t_n,y(t_n;r)\right)+2l_{2,1}\left(t_n,y(t_n;r)\right)+2\,l_{3,1}\left(t_n,y(t_n;r)\right)+l_{4,1}\left(t_n,y(t_n;r)\right)$$

$$\overline{\acute{y}}(t_{n+1};r)=\overline{\acute{y}}(t_n;r)+\frac{h}{6}(k_{1,2}\left(t_n,y(t_n;r)\right)+2k_{2,2}\left(t_n,y(t_n;r)\right)+2\,k_{3,2}\left(t_n,y(t_n;r)\right)+k_{4,2}\left(t_n,y(t_n;r)\right)$$

The solution at t_n $,٠\leq n\leq N$ $and\; a=t_o\leq t_١\leq t_٢\leq\cdots\leq t_n=b$, and $h=\frac{b-a}{N}=t_{n+١}-t_n,$

Example (1.1):

Let afuzzy initial value problem

$\acute{\acute{y}}-٤\acute{y}+٤y=٠$, $t\in[٠,١]$

$y(0)=(2+r,4-r)$

$\acute{y}(0)=(5+r,7-r)$

Solve:

Can be reduced the 2^{nd} fuzzy differential equation to two first order fuzzydifferential

$y_١=y,\quad \acute{y}_١=\acute{y}=y_٢,\; \acute{y}_٢=\acute{\acute{y}}=٤\acute{y}-٤y$

Then $y_٢=\acute{y},=f,\; \acute{\acute{y}}=g=٤y_٢-٤y_١$

The exact solation is as follow

$\underline{y}(t)=(٢+r)e^{٢t}+(١-r)te^{٢t}$

$\underline{y}(t)=(٤-r)e^{٢t}+(r-١)te^{٢t}$

Numerical results

We used MATLAB software in all the calculations which were done in this section.

R	Y lower	Upper	$\tilde{Y}$ lower	Upper
0	3.097279125469690	5.467304090157996	6.857961009099550	9.345805422155820
0.1	3.219392040370772	5.347394508590248	7.015806563085697	9.249946534836342
0.2	3.341504955271854	5.227484927022498	7.173652117071844	9.154087647516860
0.3	3.463617870172936	5.107575345454750	7.331497671057989	9.058228760197380
0.4	3.585730785074018	4.987665763887001	7.489343225044137	8.962369872877899
0.5	3.707843699975100	4.867772848985919	7.647188779030285	8.866644318891753
0.6	3.829956614876182	4.747899934084837	7.805034333016431	8.771078764905607
0.7	3.951189529777264	4.628027019183755	7.943679887002579	8.675513210919458
0.8	4.071762444678345	4.508154104282674	8.067925440988724	8.579947656933314
0.9	4.192335359579428	4.388281189381592	8.192170994974871	8.484382102947166
1	4.312908274480510	4.568408274480509	8.316416548961020	8.388816548961020

Table1: represent approximate solution to Rung-Kutta method

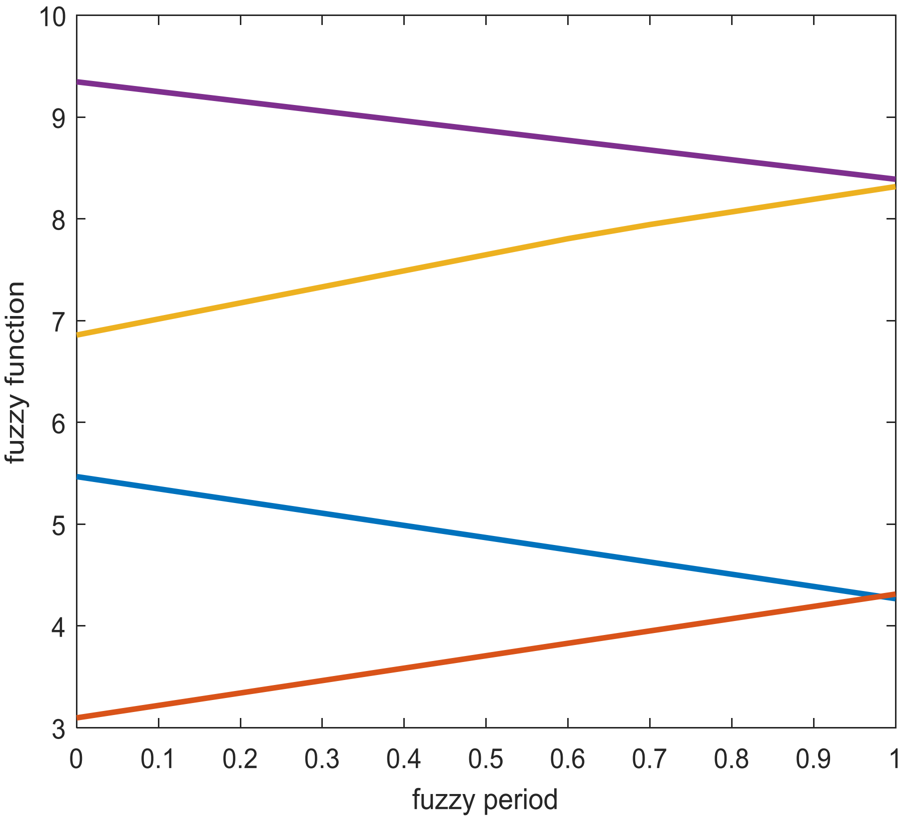

Figure 1: represent Rung-kutta method

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Volume:2 Special issue of the first international scientific conference of Iraqi Al-Khawarizmi Society 28-29 March 2018

V. Fourth Order Picard's iterative Method in fuzzy differential equation

By using fourth order by Picard's iterative Method of second order fuzzy differential equation

$$\underline{y_n}(t_{n+1};r) = \underline{y}(t_n;r) + \int_{t_o}^{t} \underline{f}(r,s,y_{n-1}(r,s))ds$$

$$\overline{y_n}(t_{n+1};r) = \overline{y}(t_n;r) + \int_{t_o}^{t} \overline{f}(r,s,y_{n-1}(r,s))ds$$

$$\underline{\acute{y}_n}(t_{n+1};r) = \underline{\acute{y}}(t_n;r) + \int_{t_o}^{t} \underline{g}(r,s,y_{n-1}(r,s))ds$$

$$\overline{\acute{y}_n}(t_{n+1};r) = \overline{\acute{y}}(t_n;r) + \int_{t_o}^{t} \underline{g}(r,s,y_{n-1}(r,s))ds$$

By using the example (1.1) in this method where $\text{٠} \le r \le \text{١}$, $t_o=0$, n=1,2,3,4 and the two function f, g is

$y_{\text{١}} = y,$ $\acute{y}_{\text{١}} = \acute{y} = y_{\text{٢}},$ $\acute{y}_{\text{٢}} = \acute{\acute{y}} = \text{٤}\acute{y} - \text{٤}y$

Then $y_{\text{٢}} = \acute{y}$, $= f,$ $\acute{\acute{y}} = g = \text{٤}y_{\text{٢}} - \text{٤}y_{\text{١}}$

$$\underline{y_1}(t_{n+1};r) = \underline{y}(t_n;r) + \int_0^t \underline{f}(r,s,y_{n-1}(r,s))ds$$

$$\underline{y_2}(t_{n+1};r) = \underline{y}(t_n;r) + \int_0^t \underline{y_1}(t_{n+1};r)ds$$

$$\underline{y_3}(t_{n+1};r) = \underline{y}(t_n;r) + \int_0^t \underline{y_2}(t_{n+1};r)ds$$

$$\underline{y_4}(t_{n+1};r) = \underline{y}(t_n;r) + \int_0^t \underline{y_3}(t_{n+1};r)ds$$

$$\overline{y_1}(t_{n+1};r) = \overline{y}(t_n;r) + \int_0^t \overline{f}(r,s,y_{n-1}(r,s))ds$$

$$\overline{y_2}(t_{n+1};r) = \overline{y}(t_n;r) + \int_0^t \overline{y_1}(t_{n+1};r)$$

$$\overline{y_3}(t_{n+1};r) = \overline{y}(t_n;r) + \int_0^t \overline{y_2}(t_{n+1};r)$$

$$\overline{y_4}(t_{n+1};r) = \overline{y}(t_n;r) + \int_0^t \overline{y_3}(t_{n+1};r)$$

$$\underline{\acute{y}_1}(t_{n+1};r) = \underline{\acute{y}}(t_n;r) + \int_0^t \underline{g}(r,s,y_{n-1}(r,s))ds$$

$$\underline{\acute{y}_2}(t_{n+1};r) = \underline{\acute{y}}(t_n;r) + \int_0^t \underline{\acute{y}_1}(t_{n+1};r)ds$$

we obtan

$$\underline{\acute{y}_3}(t_{n+1};r) = \underline{\acute{y}}(t_n;r) + \int_0^t \underline{\acute{y}_2}(t_{n+1};r)ds$$

$$\underline{\acute{y}_4}(t_{n+1};r) = \underline{\acute{y}}(t_n;r) + \int_0^t \underline{\acute{y}_3}(t_{n+1};r)ds$$

$$\overline{\acute{y}_1}(t_{n+1};r) = \overline{\acute{y}}(t_n;r) + \int_{t_o}^t \underline{g}(r,s,y_{n-1}(r,s))ds$$

$$\overline{\acute{y}_2}(t_{n+1};r) = \overline{\acute{y}}(t_n;r) + \int_0^t \overline{\acute{y}_1}(t_{n+1};r)ds$$

$$\overline{\acute{y}_3}(t_{n+1};r) = \overline{\acute{y}}(t_n;r) + \int_0^t \overline{\acute{y}_2}(t_{n+1};r)ds$$

$$\overline{\acute{y}_4}(t_{n+1};r) = \overline{\acute{y}}(t_n;r) + \int_0^t \overline{\acute{y}_3}(t_{n+1};r)ds$$

Numerical results

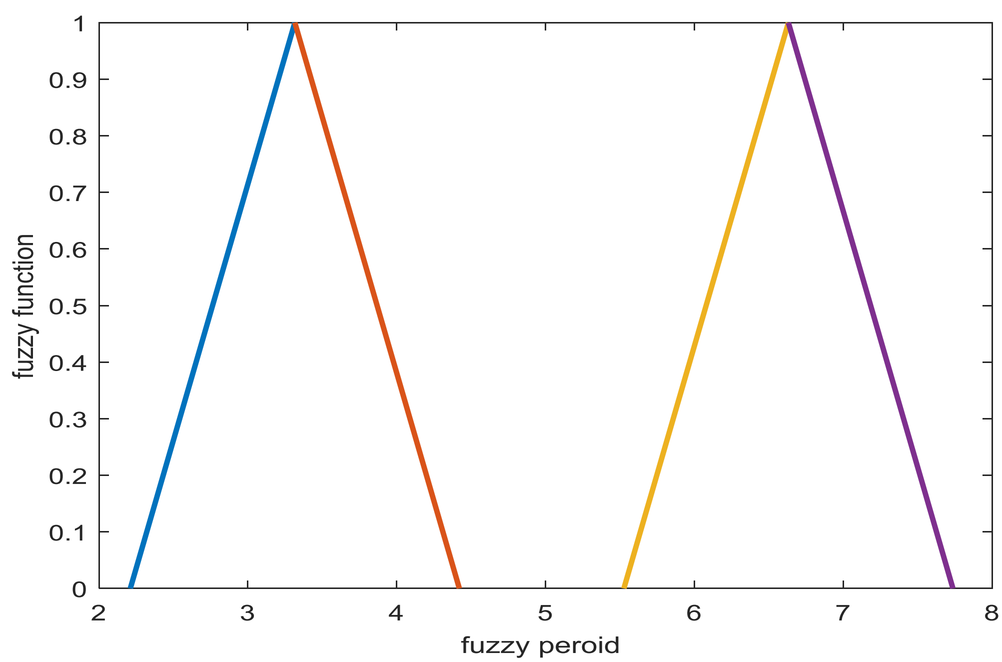
We used MATLAB software in all the calculations, which were done in this section

R	**$Y_{n\,LOW}$**	**$Y_{n\,UPP}$**	**$\tilde{Y}_{n\,LOW}$**	**$\tilde{Y}_{n\,UPP}$**
0	**2.2104**	**4.4207**	**5.5259**	**7.7370**
.1	**2.3209**	**4.3102**	**5.6262**	**7.6264**
.2	**2.4314**	**4.1997**	**5.7265**	**7.5158**
.3	**2.5419**	**4.0891**	**5.8268**	**7.4052**
.4	**2.6524**	**3.9786**	**5.9271**	**7.2946**
.5	**2.7629**	**3.8681**	**6.0274**	**7.1840**
.6	**2.8735**	**3.7576**	**6.1277**	**7.0734**
.7	**2.9840**	**3.6471**	**6.2281**	**6.9628**
.8	**3.0945**	**3.5365**	**6.3283**	**6.8522**
.9	**3.2050**	**3.4260**	**6.4286**	**6.7416**
1	**3.3155**	**3.3155**	**6.5289**	**6.6310**

Table 2

r	$Y_{n\,LOW}$	$Y_{n\,UPP}$	$\tilde{Y}_{n\,LOW}$	$\tilde{Y}_{n\,UPP}$
0	2.2104	4.4207	5.5259	7.7363
0.1	2.3209	4.3102	5.6364	7.6257
0.2	2.4314	4.1997	5.7470	7.5152
0.3	2.5419	4.0891	5.8575	7.4047
0.4	2.6524	3.9786	5.9680	7.2942
0.5	2.7629	3.8681	6.0785	7.1836
0.6	2.8735	3.7576	6.1890	7.0731
0.7	2.9840	3.6471	6.2995	6.9626
0.8	3.0945	3.5366	6.4100	6.8521
0.9	3.2050	3.4260	6.5205	6.7415
1	3.3155	3.3155	6.6311	6.6310

Table 3



Conclusion:

Form the observation of the approximate and practical result, we observed efficiency of using both methods .However, Picard iteration method was more accurate and less ration of error between the approximate and theoretical solution .This is clear as explained by table 3, where the theoretical solution is compared with table 2 obviously, Picard iteration method has shown very simple differences between the result.

Fuzzy differential equations serve as mathematical models for many exciting real-world problems, not only in science and technology but also in such diverse fields as population models ,civil engineering, etc

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