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Intuitionistic fuzzy prime d-ideal of d-algebra

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ABSTRACT : In this paper we introduce the concept of intuitionistic fuzzy prime d-ideals of d-algebra , and we study several interesting properties and investigate some relation on intuitionistic fuzzy prime d-ideal and intuitionistic fuzzy prime BCK-ideal in d-algebra .

1. Introduction

BCK-algebra is a classe of abstract algebras introduced by Y. Imai and K. Iseki [5 ,12] . A d-algebra is a useful generalization of BCK-algebra was introduced by J. Negger and H. S. Kim [3]. J. Negger , Y. B. Jun and H. S. Kim [4] discussed ideal theory in d-algebra. After the introduction of intuitionistic fuzzy set by Atanassov in 1986 [6], there was a number of generalizations of this concept . This concept was generalizations for fuzzy set concept which was introduced by Zadeh in 1965 [7]. In [10] Y. B. Jun, J. Neggers and H. S. Kim apply the ideal theory in fuzzy d-ideals of d-algebras . Y. B. Jun , H. S. Kim and D.S. Yoo in [10] introduced the notion of intuitionistic fuzzy d-algebra. We introduce the notion of intuitionistic fuzzy d-ideals of d-algebra in [1] , and in this paper we introduce the concept of intuitionistic fuzzy d-ideals of d-algebra and some relation on intuitionistic fuzzy prime d-ideal and intuitionistic fuzzy prime BCK-ideal in d-algebra .

2. Background

Definition (2.1) : [3] A d-algebra is any non-empty set X with a binary operation $*$ and a constant 0 which satisfies that:

I. $a * a = 0$
II. $0 * a = 0$
III. If $a * b = b * a = 0$ then $a = b$

$\forall a, b \in X$. We will refer to $a * b$ by ab, and it is said to be commutative if $x(xy) = y(yx)$ for all $x, y \in X$, and $y(yx)$ is denoted by $(x \wedge y)$.Every set X in the following is a d-algebra

Definition (2.2) : **[4]** In a d-algebra X the set $\phi \neq A \subseteq X$is called a d-subalgebra of X if $ab \in A$ whenever $a, b \in A$. And if $\phi \neq A \subseteq X$, then A is called a BCK-ideal of X if it satisfies:
(D_0) $0 \in A$
(D_1) $ab \in A$ and $b \in A$ implies $a \in A$.

Definition (2.3) :[4] In A d-algebra $(X;*,0)$, the $\phi \neq A \subseteq X$. A is called a d-ideal if it satisfies:
(D_1) $ab \in A$ and $b \in A$ then $a \in A$.
(D_2) $a \in A$ and $b \in X$ then $ab \in A$, i. e. $AX \subseteq A$.

Definition (2.4): [7] A fuzzy set η in a non-empty set X is a map from X into [0,1] of the real numbers. A ***level subset*** of a fuzzy set η in X, is the set $\eta_t = \{a \in X, \eta(a) \geq t\}$, for all $t \in [0,1]$.

Definition (2.5): [8] A fuzzy set η is called a fuzzy d-subalgebra if it satisfies $\eta(ab) \geq \min\{\eta(a), \eta(b)\}$, for all $a, b \in X$. And it's called a fuzzy BCK-ideal if it satisfies that :
1) $\eta(0) \geq \eta(a)$, for all $a \in X$
2) $\eta(a) \geq \min\{\eta(ab), \eta(b)\}$, for all $a, b \in X$

Definition (2.6) : [11] Let η be a fuzzy set , then η is called a fuzzy d-ideal of X if it satisfies : $(\boldsymbol{Fd_1})$ $\eta(a) \geq min\{\eta(ab), \eta(b)\}$, and $(\boldsymbol{Fd_2})$ $\eta(ab) \geq \eta(a)$. for all $a , b \in X$.

Definition (2.7) [10] : If $r \in [0,1]$ and a fuzzy set υ in a non empty set of X, the set
$U(\upsilon, r) = \{a : \upsilon(a) \geq r\}$ is called an upper r-level cut of υ , and the set
$L(\upsilon(a), r) = \{a : \upsilon(a) \leq r\}$ is called a lower r-level cut of υ.

Definition (2.8) [6] : An IFS " intuitionistic fuzzy set " A in a set X is an object having the form $A = \{< a, \alpha_A(a), \beta_A(a) >: a \in X\}$, such that $\alpha_A: X \to [0,1]$ and $\beta_A: X \to [0,1]$ denoted the degree of membership (namely $\alpha_A(a)$) and the degree of non membership (namely $\beta_A(a)$) for any elements $a \in X$ to the set A, and $0 \leq \alpha_A(a) + \beta_A(a) \leq 1$, for all $a \in X$.

To simplicity, we shall use $A =< \alpha_A, \beta_A >$ instead of $A = \{< a, \alpha_A(a), \beta_A(a) >: a \in X\}$.

Definition (2.9) [9] : If A is an IFS from non empty set X, we define

(i) $\square A = \{< a, \alpha_A(a): a \in X >\} = \{< a, \alpha_A(a), 1 - \alpha_A(a) : a \in X >\} = \{< a, \alpha_A(a), \overline{\alpha_A}(a) >\}$

(ii) $\Diamond A = \{< a, 1 - \beta_A(a) >: a \in X\} = \{< a, 1 - \beta_A(a), \beta_A(a): a \in X\} = \{< a, \overline{\beta_A}(a), \beta_A(a) >\}$

Definition (2.10) [10] : Let X be a d-algebra . An IFS $A =< \alpha_A, \beta_A >$ in X is called an intuitionistic fuzzy d-algebra if satisfies $\alpha_A(ab) \geq \min\{\alpha_A(a), \alpha_A(b)\}$ and $\beta_A(ab) \leq \max\{\beta_A(a), \beta_A(b)\}$, for all $a, b \in X$.

Proposition(2.11) [10] : Every IFS d-algebra $A =< \alpha_A, \beta_A >$ of X satisfies $\alpha_A(0) \geq \alpha_A(a)$ and $\beta_A(0) \leq \beta_A(a)$ $\forall a \in X$.

Definition (2.12) [10]: An IFS $A =< \alpha_A, \beta_A >$ in X *is called* an intuitionistic fuzzy BCK-ideal of X if it satisfies that:

(i) $\alpha_A(0) \geq \alpha_A(a)$, $\beta_A(0) \leq \beta_A(a)$

(ii) $\alpha_A(a) \geq \min\{\alpha_A(ab), \alpha_A(b)\}$

(iii) $\beta_A(a) \leq \max\{\beta_A(ab), \beta_A(b)\}$, for all $a, b \in X$

Definition(2.13) : **[1]** An intuitionistic fuzzy d-ideal of X " shortly $IFd - ideal$ " is the IFS $A =< \alpha_A, \beta_A >$ in X with the following inequalities :

(IFd_1) $\alpha_A(a) \geq \min\{\alpha_A(ab), \alpha_A(b)\}$

(IFd_2) $\beta_A(a) \leq \max\{\beta_A(ab), \beta_A(b)\}$

(IFd_3) $\alpha_A(ab) \geq \alpha_A(a)$

(IFd_4) $\beta_A(ab) \leq \beta_A(a)$, for all $a, b \in X$

Lemma (2.14) : **[1]** An IFS $A =< \alpha_A, \beta_A >$ is an $IFd - ideal$ of X if and only if α_A and $\overline{\beta_A}$ are a fuzzy d-ideals .

Theorem (2.15) : **[1]** Let $A =< \alpha_A, \beta_A >$ be an IFS in X. Then $A =< \alpha_A, \beta_A >$ is an $IFd - ideal$ of X if and only if $\square A = \{< \alpha_A, \overline{\alpha_A} >\}$ and $\Diamond A = \{< \overline{\beta_A}, \beta_A >\}$ are $IFd - ideal$ of X.

Theorem (2.16) : **[1]** " An IFS $A =< \alpha_A, \beta_A >$ is an $IFd - ideal$ of X if and only if for all $s, t \in [0,1]$ the sets $U(\alpha_A, t)$ and $L(\beta_A, s)$ are either empty or d-ideal of X. "

Theorem (2.17) : **[1]** " If an IFS $A =< \alpha_A, \beta_A >$ is an $IFd - ideal$ of X, then the sets $X_\alpha = \{x \in X: \alpha_A(x) = \alpha_A(0)\}$ and $X_\beta = \{x \in X: \beta_A(x) = \beta_A(0)\}$ are d-ideal of X. "

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3. Intuitionistic fuzzy prime d-ideal

Definition(3.1) : In a commutative d-algebra X , a d-ideal I is said to be prime if $a \wedge b \in I$ implies $a \in I$ or $b \in I$ **,** for all $a, b \in X$.

Example (3.2) : Let $X = \{0, \text{ɱ}, \text{ɳ}, \acute{r}, \acute{s}\}$ be a d-algebra with the following table

*	0	ɱ	ɳ	ŕ	ś
0	0	0	0	0	0
ɱ	ɱ	0	ɱ	0	ɱ
ɳ	ɳ	ɳ	0	ŕ	0
ŕ	ŕ	ŕ	ɳ	0	ɳ
ś	ŕ	ŕ	ɱ	ɱ	0

We can take $I = \{0, \text{ɱ}\}$, which is a prime d-ideal

Definition(3.3) : An $IFd - ideal$ $A =< \alpha_A, \beta_A >$ of X is an intuitionistic fuzzy prime d-ideal " shortly $IFPd - ideal$ " in X if the IFS in X with the following inequalities :

$(IFPd_1)$ $\alpha_A(a \wedge b) \leq \max\{\alpha_A(a), \alpha_A(b)\}$

$(IFPd_2)$ $\beta_A(a \wedge b) \geq \min\{\beta_A(a), \beta_A(b)\}$, for all $a, b \in X$

Example (3.4) : Let $X = \{0, p, q\}$ with the following table

*	0	p	q
0	0	0	0
p	q	0	q
q	p	p	0

Note that if $\alpha_A(a) = \begin{cases} 0.9 & if a = 0 \\ 0.01 & if a = p, q \end{cases}$, $\beta_A(a) = \begin{cases} 0.1 & if a = 0 \\ 0.5 & if a = p, q \end{cases}$,

so $A =< \alpha_A, \beta_A >$ *is* $IFd - ideal$ *by* ***[1]***, *and it is clear that* $A =< \alpha_A, \beta_A >$ is an $IFPd - ideal$.

Remark (3.5) : It is easy to show that every $IFPd - ideal$ is an $IFP - BCK - ideal$.

The converse of this remark cannot be true in general , and the next example showing that

Example (3.6) : Let $X = \{0, \text{ɱ}, \text{ɳ}, \rho, \}$ with the following table

*	0	ɱ	ɳ	ρ
0	0	0	0	0
ɱ	ɱ	0	0	0
ɳ	ɳ	ɱ	0	0
ρ	ρ	ɳ	ɱ	0

Let $A =< \alpha_A, \beta_A >$ be an IFS in X , *define* $\alpha_A(0) = 1$, $\alpha_A(m) = 0.9$, $\alpha_A(n) = 0.5$, $\alpha_A(p) = 0$

$\beta_A(0) = 0$, $\beta_A(m) = 0.1$, $\beta_A(n) = 0.5$, $\beta_A(p) = 1$, *Then* $A =< \alpha_A, \beta_A >$ in X *is* an $IFP - BCK - ideal$ of X *but it is not* an $IFPd - ideal$ (in fact it is not $IFd - ideal$) ,
where $\alpha_A(n) = 0.5 \ \geq \min\{\alpha_A(nm), \alpha_A(m)\} = 0.9$.

Theorem (3.7) : If $\{A_i \ , i \in \Lambda\}$ is an arbitrary family of $IFPd - ideal$, then $\cap A_i$ is an $IFPd - ideal$, when
$\cap A_i = \{< a, \wedge \alpha_{A_i}(a) \ , \vee \beta_{A_i}(a) \ | a \in X\}$

Proof : Since $\alpha_A(a \wedge b) \leq \max\{\alpha_A(a), \alpha_A(b)\}$ and $\beta_A(a \wedge b) \geq \min\{\beta_A(a), \beta_A(b)\} \ \forall a, b \in X$. Now for all $i \in \Lambda$
$\wedge \alpha_{A_i}(a \wedge b) \leq \wedge\{\max\{\alpha_{A_i}(a), \alpha_{A_i}(b)\}\} \leq \{\max\{\wedge \alpha_{A_i}(a), \wedge \alpha_{A_i}(b)\}\}$, and

$\vee \beta_{A_i}(a \wedge b) \geq \vee\{\min\{\beta_{A_i}(a), \beta_{A_i}(b)\}\} \geq \{\min\{\vee \beta_{A_i}(a) \ , \vee \beta_{A_i}(b)\}\}$

Hence $\cap A_i = \{< a, \wedge \alpha_{A_i}(a) \ , \vee \beta_{A_i}(a) \ | a \in X\}$ is an $IFPd - ideal$.

Lemma (3.8) : An IFS $A =< \alpha_A, \beta_A >$ is an $IFPd - ideal$ of X if and only if α_A and $\overline{\beta_A}$ are a fuzzy prime d-ideals .

proof : Suppose that $A =< \alpha_A, \beta_A >$ an $IFPd - ideal$ of X. Since $A =< \alpha_A, \beta_A >$ is an $IFd - ideal$ (by Theorem (3.5)), so (by Lemma (2.14)) α_A and $\overline{\beta_A}$ are a fuzzy d-ideals. Now for all $a, b \in X$ we get $\alpha_A(a \wedge b) \leq \max\{\alpha_A(a), \alpha_A(b)\}$ and
$\beta_A(a \wedge b) \geq \min\{\beta_A(a), \beta_A(b)\}$,

$$\text{Now } \overline{\beta_A}(a \wedge b) = 1 - \beta_A(a \wedge b) \geq 1 - \min\{\beta_A(a), \beta_A(b)\}$$
$$= \max\{1 - \beta_A(a), 1 - \beta_A(b)\}$$
$$= \max\{\overline{\beta_A}(a), \overline{\beta_A}(b)\}$$

Hence α_A and $\overline{\beta_A}$ are a fuzzy prime d-ideals . The converse is clear.

Theorem (3.9) : Let $A =< \alpha_A, \beta_A >$ be an $IFPd - ideal$ of X , then $\Box A = \{< \alpha_A, \overline{\alpha_A} >\}$ and $\Diamond A = \{< \overline{\beta_A}, \beta_A >\}$ are $IFPd - ideals$ of X.

proof : Since $A =< \alpha_A, \beta_A >$ is an $IFPd - ideal$, we get that $\Box A = \{< \alpha_A, \overline{\alpha_A} >\}$ and
$\Diamond A = \{< \overline{\beta_A}, \beta_A >\}$ are $IFd - ideals$ (by Theorem (2.15)) . So for all $a, b \in X$ we get
$\alpha_A(a \wedge b) \leq \max\{\alpha_A(a), \alpha_A(b)\}$, then $1 - \alpha_A(a \wedge b) \geq 1 - \max\{\alpha_A(a), \alpha_A(b)\}$

$$\geq \min\{1 - \alpha_A(a), 1 - \alpha_A(b)\}$$
$$\geq \min\{\overline{\alpha_A}(a), \overline{\alpha_A}(b)\}$$

Hence $\Box A = \{< \alpha_A, \overline{\alpha_A} >\}$ is an $IFPd - ideal$ of X .

And $\beta_A(a \wedge b) \geq \min\{\beta_A(a), \beta_A(b)\}$,
Now $\overline{\beta_A}(a \wedge b) = 1 - \beta_A(a \wedge b) \leq 1 - \min\{\beta_A(a), \beta_A(b)\} \ \leq \max\{1 - \beta_A(a), 1 - \beta_A(b)\}$

$$\leq \max\{\overline{\beta_A}(a), \overline{\beta_A}(b)\}$$

Hence $\Diamond A = \{< \overline{\beta_A}, \beta_A >\}$ is an $IFPd - ideal$ of X.

Theorem (3.10) : $A =< \alpha_A, \beta_A >$ is an $IFPd - ideal$ if and only if for all $s, t \in [0,1]$ the sets $U(\alpha_A, t)$ and $L(\beta_A, s)$ are prim d-ideals.

proof : Let $A =< \alpha_A, \beta_A >$ *be an* $IFPd - ideal$ of X, so it is an $IFd - ideal$, and let $U(\alpha_A, t), L(\beta_A, s)$ non empty set for any $s, t \in [0,1]$. So (by Theorem (2.16)), $U(\alpha_A, t), L(\beta_A, s)$ are d-ideal . Now let $a, b \in X$ such that $a \wedge b \in U(\alpha_A, t)$ this implies $\alpha_A(a \wedge b) \geq t$, and $\alpha_A(b) \geq t$ then $\alpha_A(a \wedge b) \leq \max\{\alpha_A(a), \alpha_A(b)\}$, and hence $\max\{\alpha_A(a), \alpha_A(b)\} \geq \alpha_A(a \wedge b) \geq t$. Then $\alpha_A(a) \geq t$ or $\alpha_A(b) \geq t$ so that $a \in U(\alpha_A, t)$ or $b \in U(\alpha_A, t)$. Hence $U(\alpha_A, t)$ is a prime d-ideal in X. In the same way we can find that $L(\beta_A, s)$ is a prime d-ideal in X.

Conversely , suppose that for all $s, t \in [0,1]$, the set $U(\alpha_A, t)$ and $L(\beta_A, s)$ are prime d-ideals. Then $U(\alpha_A, t)$ and $L(\beta_A, s)$ are d-ideals of X. Since $A =< \alpha_A, \beta_A > IFd - ideal$ by (theorem 2.16). If $A =< \alpha_A, \beta_A >$ is not $IFPd - ideal$ of X , then there exist $a, b \in X$ such that $\alpha_A(a \wedge b) > \max\{\alpha_A(a), \alpha_A(b)\}$. Let $t = \frac{1}{2}[(\alpha_A(a \wedge b) + \max\{\alpha_A(a), \alpha_A(b)\}]$, this implies $(\alpha_A(a \wedge b) > t > \max\{\alpha_A(a), \alpha_A(b)\}$. we get that $a \wedge b \in U(\alpha_A, t)$, but $a \notin U(\alpha_A, t)$ and $b \notin U(\alpha_A, t)$ which is a contradiction . Hence $A =< \alpha_A, \beta_A >$ is an $IFPd - ideal$ of X .

Theorem (3.11) : If an IFS $A =< \alpha_A, \beta_A >$ is an $IFPd - ideal$, then the sets $X_\alpha = \{a \in X : \alpha_A(a) = \alpha_A(0)\}$ and $X_\beta = \{a \in X : \beta_A(a) = \beta_A(0)\}$ are prime d-ideals.

proof : Let $a \wedge b \in X_\alpha$, so $\alpha_A(a \wedge b) = \alpha_A(0)$. Since $A =< \alpha_A, \beta_A >$ is an $IFPd - ideal$, then $\alpha_A(a \wedge b) \leq \max\{\alpha_A(a), \alpha_A(b)\}$, but $\alpha_A(0) \geq \alpha_A(a)$, so $\alpha_A(0) = \max\{\alpha_A(a), \alpha_A(b)\}$. Now either $\alpha_A(a) = \alpha_A(0)$ and this implies $a \in X_\alpha$, or $\alpha_A(b) = \alpha_A(0)$ and hence $b \in X_\alpha$. Thus X_α is a prime d-ideal .

Now let $a \wedge b \in X_\beta$. Then $\beta_A(a \wedge b) = \beta_A(0)$, so $\beta_A(a \wedge b) = \beta_A(0) \geq \min\{\beta_A(a), \beta_A(b)\}$ (since $A =< \alpha_A, \beta_A >$ is an $IFPd - ideal$), but $\beta_A(0) \leq \beta_A(a)$, so either $\beta_A(a) = \beta_A(0)$ then $a \in X_\beta$, or $\beta_A(b) = \beta_A(0)$ then $b \in X_\beta$ .Thus X_β is a prime d-ideal .

The proof of the next tow propositions is clear .

Proposition (3.12) : Let $A =< \alpha_A, \beta_A >$ be an $IFPd - ideal$, then the sets $P_1 = \{a \in X : \alpha_A(a) = 0\}$ and $P_2 = \{a \in X : \beta_A(a) = 0\}$ either empty or prime d-ideals.

Proposition (3.13) : Let $A =< \alpha_A, \beta_A >$ be an $IFPd - ideal$, then the sets $I_1 = \{a \in X : \alpha_A(a) = 1\}$ and $I_2 = \{a \in X : \beta_A(a) = 1\}$ are either empty or prime d-ideals.

الخلاصة : قدمنا في هذا البحث مفهوم مثالي d الأولي الضبابي البديهي وتطرقنا إلى العديد من الخصائص المهمة حول هذا المفهوم وبعض ارتباطاته بالإضافة إلى دراسة بعض العلاقات على مثالي d البديهي الأولي ومقارنته بمثالي BCK البديهي الأولي في جبر d .

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