Journal of Iraqi Al-Khwarizmi (JIKh) Volume: 10 Issue: 2 Year: 2026 pages: 320-328

Extraction Lie Algebra and Extraction Lie Derivation

Authors Names

[1]*Adnan A. Oudah,*
[2]*Mohammed H. Hamza,*
Publication date: 28/8/2026
Keyword: Extraction, Extraction Lie algebra, Extraction Lie derivation, direct sum, derivation.

ABSTRACT

In this research paper, we discussed a new Lie algebra generated from the composition of tow direct sum, such that the result is not contained in their direct sum. We called it the Extraction Lie algebra. Then we restricted ourselves to the special class $sl_2(g)$. We defined the same definition of the derivation Lie algebra and the adjoin, along with some properties, such as $Der(L_{Ext}) \subseteq Der(L_k)$. And the result obtained from the derived elements always yields the zero result, as we have clarified in this paper, from the components of the Extraction Lie algebra.

1. Introduction

In these papers, the resulting space is initially defined as the space whose components are direct sums, such that each resulting space or component is distinct from the two components that generated it, according to the following definition. Which denoted by v_{Ext} or L_{Ext}.

A Extraction Lie algebra of type A over a field K of characteristic dose not equal 2 direct sum vector space $v = \oplus_{\alpha\in\Delta}$ over K together with a bilinear map $(a,b) \to [a,b]$ of $v_h \times v_j$ into v_k which satisfies the following conditions:

$$[v_h, v_j] = v_{Ext} \subseteq v_k$$

Such that $v = \oplus v_{\alpha\in\Delta} = v_h \oplus v_j \oplus \dots \oplus v_k \oplus \dots$

In [20] if nilpotent Lie algebra L has dimension $(c+1)$ and class (c), then it has maximal class.This means the algebra is as "long" as possible relative to its dimension, making its structure more intricate than what might be inferred just by examining interconnected Lie algebras of maximal class (p) −groups with. Put differently:

$$L = L_1 \oplus L_2 \oplus \dots \oplus L_c$$

In [7] it was proven in Part 4 of the research that the constrained Lie algebra of the direct sum possesses the property characteristic ≠**2** in general. In [18] The relationship between Lie algebra semi-direct sums and integrals couplings for continuous soliton equations was discussed, and a practical method was presented for generation of integrals of motion by semi direct sums of Lie Algebras in [16,17] some methods for construction of integrals couplings, such as using perturbations, are proposed, in [13,15] extending spectral problems and in [6,24]. creating new loop algebras. In the last few years, several researchers studied the topic of biderivations of many Lie (super) algebras by individual cases, see [12,4,21]. In fact, there is no single way for obtaining all the derivations of certain families of Lie (super) algebras.

In [3] It follows that, for the semi simple direct sum, the derived sub algebras of its components are trivial. An approach to integrals couplings, defined in [16, 14], was recently developed based on semi-direct sums of Lie algebra [18, 5]. Examples of continuous in [18] and discrete integral coupling [19] reveal intriguing mathematical structures underlying integrals equations. This theory aids in the

[1]*General Directorate of Al-Muthanna Education, Iraq. E-mail address: adnann8@gmail.com*
[2]*General Directorate of Al-Muthanna Education, Iraq. E-mail address: muhammadhassan@ijsu.edu.iq*

algebraic classification of integrals equations, as every A Lie algebra can be broken down into a semi−direct sum of a semi−simple Lie algebra and a solvable Lie algebra [9, 5]. Derivation algebras (and super algebras) play a significant role in the study of algebraic structures. The most comprehensive results on generalized derivation algebras of Lie algebras and their sub algebras were provided by Leger and Luk [1]. For a Lie algebra (L), the relationship between the structure of (L) and its derivation algebra Der(L) has been extensively studied by various authors in [2, 11]. Specifically, they defined Lie algebras that meet specific requirements and explained the structure of generalized derivation algebras. Lie super algebras, which , which are Z_2-graded Lie algebras, were introduced by Kac [10]. These findings are fascinating from a purely mathematical perspective. Later, Zhang and Zhang [23] extended the results of Leger and Luk [18] to the framework of Lie super algebras. Additionally, Wang investigated the structure of super derivations of Lie super algebras in [22].

In these papers, Part 2 defines the Extraction Lie algebra with components as direct sum. It specifically addresses the derivative of Extraction Lie algebra within L_K and explores the relationship between the derivative of Extraction Lie algebra and the compound containing L_K. In Part 3 further discusses the equality between them.

2. Extraction Lie algebra

in this section, we employed Extraction Lie algebra for direct sum Lie algebra. As demonstrated in definition 2.1 and example 2.2 below.

Definition 2.1 .A Extraction Lie algebra L_{Ext} is a Lie algebra L which are decayed into a direct sum of sub Lie algebra such that.

$$L = \oplus_{i=1}^{p} L_i$$

where the Lie bracket

$$[L_h , L_j] \subseteq L_k$$

is compatible with the gradation. It satisfies Extraction Lie algebra antisymmetric and the Extraction Lie algebra Jacobi identity, extending standard Lie algebra structures.

Refer to Extraction Lie algebra by L_{Ext} then

$$L_{Ext} = [L_h, L_j] \subseteq L_k$$

Every element of $L_{Ext} \subseteq L_k$ generated by tow element one from L_h and L_j the other from by definition Lie algebra

$$\forall\, x \in L_{Ext} , \exists\, x_h \in L_h \; and \; x_j \in L_j \; : x = [x_h, x_j]$$

Structure 2.2 A Lie algebra is Extraction Lie algebra if it’s a direct sum of subspaces L_h, L_j such that the bracket of an element in L_h and an element in L_j Lies in L_k .

2.3 example 2.3 we can find something Extraction Lie algebra through the previous examples as found in [8] as we will.

be a vector space

$$L = h \oplus j \oplus k$$

such that we have a direct sum decomposition into E , F , H

Let us take algebra $sl_2(R)$

$$k = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad , \quad h = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad , \quad j = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

with the bracket relations

$$[k,h] = -2h \quad , \quad [k,j] = 2j \quad , \quad [h,j] = k$$

We can obtain a larger using spans as following

$span(k) = g_1 = \{\begin{pmatrix} a & 0 \\ 0 & -b \end{pmatrix} : a,b \in R\}$

$span(h) = g_2 = \{\begin{pmatrix} 0 & c \\ 0 & 0 \end{pmatrix} : \quad c \in R\}$

$span(j) = g_3 = \{\begin{pmatrix} 0 & 0 \\ d & 0 \end{pmatrix} : \quad d \in R\}$

Where

$$[g_1, g_2] = -2g_2 \quad , \quad [g_1, g_3] = 2g_3 \quad , \quad [g_2, g_3] = g_1$$

From the above definition Extraction Lie algebra. We find that , $[g_2, g_3] = g_1$ satisfies the condition of Extraction Lie algebra.

3. Extraction Lie derivation.

Derivation 3.1. A Extraction Lie algebra of over K then $Der(L_{Ext})$.

$$D[L_h, L_j] = [D(L_h), L_j] + [L_h, D(L_j)]$$

$$Der(L_{Ext}) \subseteq Der(L_k)$$

Corollary 3.2: if $x, y \in L_{Ext} \subseteq L$ and $D \in Der(L_{Ext})$ by define derivation then

$$D[x,y] = [D(x), y] + [x, D(y)]$$

Sense $x, y \in L_{Ext}$ then there exists $x_h, y_h \in L_h$ and $x_j, y_j \in L_j$ such that

$$x = [x_h, x_j] \ , \ y = [y_h, y_j]$$

Then

$$D[x,y] = \left[D[x_h, x_j], [y_h, y_j]\right] + \left[[x_h, x_j] \ , D[y_h, y_j]\right]$$

$$= \left[[D(x_h), x_j] + [x_h, D(x_j)], [y_h, y_j]\right] + \left[[x_h, x_j] \ , [D(y_h), y_j] + [y_h, D(y_j)]\right]$$

$$\subseteq Der(L)$$

We have obtained $D(L_{Ext}) \subseteq Der(L)$

(1) Note**:** In some cases , a match occurs between $Der(L_{Ext})$ and $Der(L_k)$

If $Der(L_k)$ simple derivation then $Der(L_k) = Der(L_{Ext})$

Definition 3.3: a map $ad_{Ext}: L_{Ext} \to L_{Ext}$ defined by $y \to [x, y]$ is side to be lanner derivation.

$$ad_{Ext}y \to [x, y]$$

Let $x, y \in L_{Ext}$

Proposition 3.4: is concerned which the assumptions to impose on the differential systems. We first introduce the notion of a regular differential system (Def derivation) : A differential system D on a manifold L is called regular if $D_j = D$ or $D_i = D$ such that $D_i \in Der(L_i)$, $D_j \in Der(L_j)$ such that $D_{Ext} = [D_i, D_j] + D_k$, then $D_{Ext} = D_k$.

If $D_j = D$

$$D_{Ext} = [D_i, D_j] + D_k = D_i \circ D_j - D_j \circ D_i + D_k$$

$$= D_i(D_j) - D_j(D_i) + D_k$$

Sense $D_j = D$ then.

$$= D_i(D) - D(D_i) + D_k$$

$$= D_i - D_i + D_k = D_k$$

We get

$$D_{Ext} = D_k$$

Then

$$Der(L_k) = Der(L_{Ext})$$

We can generalize to the rest for all $i \text{ and } j$.

Proposition 3.5: A homomorphism of Extraction Lie algebra. It always has a zero result.

Let $x_h, y_h \in L_h$ and $x_j, y_j \in L_j$ sines $D \in Der(L) \text{ and } L_h, L_j \subseteq L$ then .and Let $x, y \in L_{Exs}$ such that.

$x = [x_h, x_j]$, $y = [y_h, y_j]$ by Corollary (3.2) then

$D(x) = D[x_h, x_j] = [D(x_h), x_j] + [x_h, D(x_j)]$; $x_h \in L_h$ and $x_j \in L_j$ (1)

$D(y) = D[y_h, y_j] = [D(y_h), y_j] + [y_h, D(y_j)]$; $y_h \in L_h$ and $y_j \in L_j$ (2)

Then

By define homomorphism of Extraction Lie algebra

$$D[x, y] = [D(x), d(y)] = D(x)D(y) - D(y)D(x)$$

Sines $x = [x_h, x_j]$, $y = [y_h, y_j]$ then

$$D[x_h, x_j]D[y_h, y_j] - D[y_h, y_j]D[x_h, x_j]$$

By (1) and (2) we get

$$\left([D(x_h),x_j]+[x_h,D(x_j)]\right)\left([D(y_h),y_j]+[y_h,D(y_j)]\right)$$
$$-\left([D(y_h),y_j]+[y_h,D(y_j)]\right)\left([D(x_h),x_j]+[x_h,D(x_j)]\right)$$
$$=[D(x_h),x_j][D(y_h),y_j]+[x_h,D(x_j)][D(y_h),y_j]$$
$$+[D(x_h),x_j][y_h,D(y_j)]+[x_h,D(x_j)][y_h,D(y_j)]$$
$$-[D(y_h),y_j][D(x_h),x_j]-[y_h,D(y_j)][D(x_h),x_j]$$
$$-[D(y_h),y_j][x_h,D(x_j)]-[y_h,D(y_j)][x_h,D(x_j)]$$

By define a Lie algebra we have

$$=D(x_h)x_jD(y_h)y_j-x_jD(x_h)D(y_h)y_j-D(x_h)x_jy_jD(y_h)+x_jD(x_h)y_jD(y_h)$$
$$+x_hD(x_j)D(y_h)y_j-D(x_j)x_hD(y_h)y_j-x_hD(x_j)y_jD(y_h)+D(x_j)x_hy_jD(y_h)$$
$$+D(x_h)x_jy_hD(y_j)-x_jD(x_h)y_hD(y_j)-D(x_h)x_jD(y_j)y_h+x_jD(x_h)D(y_j)y_h$$
$$+x_hD(x_j)y_hD(y_j)-D(x_j)x_hy_hD(y_j)-x_hD(x_j)D(y_j)y_h+D(x_j)x_hD(y_j)y_h$$
$$-D(y_h)y_jD(x_h)x_j+y_jD(y_h)D(x_h)x_j+D(y_h)y_jx_jD(x_h)-y_jD(y_h)x_jD(x_h)$$
$$-y_hD(y_j)D(x_h)x_j+D(y_j)y_hD(x_h)x_j+y_hD(y_j)x_jD(x_h)-D(y_j)y_hx_jD(x_h)$$
$$-D(y_h)y_jx_hD(x_j)+y_jD(y_h)x_hD(x_j)-D(y_h)y_jD(x_j)x_h+y_jD(y_h)D(x_j)x_h$$
$$-y_hD(y_j)x_hD(x_j)+D(y_j)y_hx_hD(x_j)+y_hD(y_j)D(x_j)x_h-D(y_j)y_hD(x_j)x_h$$

By define derivation for example wen combining the part $D(x_h)x_jD(y_h)y_j+x_hD(x_j)D(y_h)y_j$ we have

$$D(x_hx_j)D(y_h)y_j$$

When the process is generalized, we obtain,

$$=D(x_hx_j)D(y_h)y_j-D(x_hx_j)D(y_h)y_j-D(x_hx_j)y_jD(y_h)+D(x_hx_j)y_jD(y_h)$$
$$+D(x_hx_j)y_hD(y_j)-D(x_hx_j)y_hD(y_j)-D(x_hx_j)D(y_j)y_h+D(x_hx_j)D(y_j)y_h$$
$$-D(y_hy_j)D(x_h)x_j+D(y_hy_j)D(x_h)x_j+D(y_hy_j)x_jD(x_h)-D(y_hy_j)x_jD(x_h)$$
$$-D(y_hy_j)x_hD(x_j)+D(y_hy_j)x_hD(x_j)-D(y_hy_j)D(x_j)x_h+D(y_hy_j)D(x_j)x_h$$
$$=D(x_hx_j)D(y_hy_j)-D(x_hx_j)D(y_hy_j)-D(x_hx_j)D(y_hy_j)+D(x_hx_j)D(y_hy_j)$$
$$-D(y_hy_j)D(x_hx_j)+D(y_hy_j)D(x_hx_j)+D(y_hy_j)D(x_hx_j)-D(y_hy_j)D(x_hx_j)$$
$$=0$$

In general, the result is always zero.

Lemma 3.6: let L_{Ext} a Extraction Lie algebra, and $Der(L)$ is closed under the usual Lie bracket

Let $D_1,D_2\in Der(L_k)$ and let $x,y\in L_{Ext}$ then $x,y\in L_k$

We show that

$$[D_1D_2]([x,y]) = [[D_1D_2](x),y] + [x,[D_1D_2](y)]$$

$$[D_1,D_2]([x,y]) = (D_1 \circ D_2 - D_2 \circ D_1)[x,y]$$

$$= D_1 \circ D_2[x,y] - D_2 \circ D_1[x,y]$$

$$= D_1D_2[x,y] - D_2D_1[x,y]$$

$$= D_1([D_2(x),y] + [x,D_2(y)]) - D_2([D_1(x),y] + [x,D_1(y)])$$

$$= D_1[D_2(x),y] + D_1[x,D_2(y)] - D_2[D_1(x),y] - D_2[x,D_1(y)]$$

$$= [D_1D_2(x),y] + [D_2(x),D_1(y)] + [D_1(x),D_2(y)] + [x,D_1D_2(y)]$$

$$-[D_2D_1(x),y] - [D_1(x),D_2(y)] - [D_2x,D_1(y)] - [x,D_2D_1(y)]$$

$$= [D_1D_2(x),y] - [D_2D_1(x),y] + [x,D_1D_2(y)] - [x,D_2D_1(y)]$$

$$= [D_1D_2(x) - D_2D_1(x),y] + [x,D_1D_2(y) - D_2D_1(y)]$$

$$= [(D_1D_2 - D_2D_1)(x),y] + [x,(D_1D_2 - D_2D_1)(y)]$$

$$= [[D_1D_2](x),y] + [x,[D_1D_2](y)]$$

Then $[D_1,D_2]([x,y]) = [[D_1D_2](x),y] + [x,[D_1D_2](y)]$

Proposition 3.7: if L is direct sum Lie algebra and L_k sub Lie algebra of L ,$x \in L_{Ext} \subseteq L_k$ such that .

$$x = [x_i,x_j]$$

Then $ad(L_{Ext})$ is ideal of $Der(L_k)$ and $[D,adx] = ad(d(x))$

For all $y \in L_k$ we have

$$[D,adx](y) = Dadx(y) - adxD(y)$$

$$= D([x,y]) - [x,D(y)]$$

$$= D\big([[x_i,x_j],y]\big) - \big[[x_i,x_j],D(y)\big]$$

$$= \big[[D(x_i),x_j],y\big] + \big[[x_i,D(x_j)],y\big] + \big[[x_i,x_j],D(y)\big] - \big[[x_i,x_j],D(y)\big]$$

$$= \big[[D(x_i),x_j],y\big] + \big[[x_i,D(x_j)],y\big]$$

$$= -\big[y,[D(x_i),x_j]\big] - \big[y,[x_i,D(x_j)]\big]$$

$$= -(\big[y,[D(x_i),x_j]\big] + \big[y,[x_i,D(x_j)]\big])$$

$$= -\big[y,[D(x_i),x_j] + [x_i,D(x_j)]\big]$$

$$= -\big[y,D([x_i,x_j])\big]$$

$$= \big[D([x_i,x_j]),y\big]$$

$$= ad\left(D\big([x_i,x_j]\big)\right)(y)$$

$$= ad(D(x))(y)$$

By define $ad(L_k)$ then $[D, adx](y)$ is inner derivation and $ad(L_{Ext})$ is ideal of $Der(L_k)$ and

$$[D, adx] = ad(D(x))$$

Proposition 3.8 if L_k sub Lie algebra and $D \in$ homomorphism and $Z(L_k) = 0$ $\partial: Der(L_k) \to End(L_k)$ $\partial(D) = \partial_D$. For all $x \in [x_i, x_j]$ and $D \in Der([x_i, x_j])$ then $[D, adx] = ad(\partial_D(x))$

By Proposition (3.7) if L is direct sum Lie algebra and L_k sub Lie algebra of L and $D \in Der([x_i, x_j])$ We're able to define endomorphism ∂_D on L_k such that $x = [x_i, x_j] \in [L_i, L_j] \subseteq L_k$

$$\partial_D(x) = [D(x_i), x_j] + [x_i, D(x_j)]$$

In fact the definition is independent of the form of expression of x for proving it

$$\alpha = [D(x_i), x_j] + [x_i, D(x_j)]$$

If x can be expressed in the form

Let

$$x = [y_i, y_j]$$

And

$$\beta = [D(y_i), y_j] + [y_i, D(y_j)]$$

Since $D \in Der([x_i, x_j])$ and $z \in L_k$ we have

$$[z, \alpha] = [D(x_i), x_j] + [x_i, D(x_j)] = [z, \beta]$$

Hanse $[z, \alpha - \beta] = 0$. This means that $\alpha - \beta \in Z(L_k)$ sinse $Z(L_k) = 0 \to \alpha = \beta$

Hanes ∂_D define the rests of the Proposition follow form the proof Proposition:(1).

Using ∂_D and the Proposition:(3.7) we have the following Proposition

Proposition 3.9: if L is dierct sum Lie algebra and L_k sub Lie algebra of L ,$x \in L_{Ext} \subseteq L_k$ for all $D \in Der(L_{Ext})$ then $\partial_D \in Der(L_{Ext})$

Let $D \in Der(L_{Ext})$ and $x \in L_{Ext}$ such that $x = [x_i, x_j]$ then

$$[D, ad(x)] = ad\partial_D(x)$$

$$[D, ad(x)] = [D, ad[x_i, x_j]] = [D, [adx_i, adx_j]]$$

$$= -[adx_i, [D, ady]] - [ady, [adx_i, D]]$$

$$= [[D, adx_i], adx_j] + [adx_i, [D, adx_j]]$$

$$= [ad\partial_D(x_i), adx_j] + [adx, ad\partial_D(x_j)]$$

$$= ad([\partial_D(x_i), x_j] + [x_i, \partial_D(x_j)])$$

$$= ad\partial_D[x_i, x_j]$$

$$= ad\partial_D(x)$$

Hanse $[D, adx] = ad\partial_D x$ and $Z(L_k) = 0$, $\partial_D([x_i, x_j]) = [\partial_D(x_i), x_j] + [x_i, \partial_D(x_j)]$ by the arbitrary of $\partial_D, x \in Der([L_h, L_j])$.

Proposition 3.10: A base ring R ,and if L direct sum Lie algebra and L_k sub Lie algebra of L , and $D \in Z(L_k)$

Let $x \in L_{Ext}$ and $D \in Der(L_{Ext})$

Prove that $[D, ad(x)] \subseteq Z(L_k)$

Let $x \in L_{Ext}$ and $D \in Der(L_{Ext})$

$$[D, ad(x)](y) = Dadx(y) - adxD(y)$$

$$= D[x, y] - [x, D(y)]$$

$$= [D(x), y] + [x, D(y)] - [x, D(y)]$$

By Proposition(3.9) $Z(L_k)$ is trivial

$$[0, y] + [x, 0] - [x, 0] = 0$$

Then

$$[D, ad(x)] = 0$$

. By proposition(3.7) $[D, ad(x)] = adD(x)$ we have $adD(x) = 0$

Theorem 3.11: if L is direct sum Lie algebra and L_k sub Lie algebra of L , L_k has trivial center then $Der(L_k) = Der(L_{Ext})$

Let $x \in L_k$ and $D \in Der(L_k)$

By Proposition(3.8) $\alpha = \beta$, $\alpha - \beta \in Z(L_k)$

By Proposition(3.9) $[D, adx] = ad\partial_D x$ and $Z(L_k) = 0$

By Proposition(3.10) $D \in Z(L_k)$ and $[D, ad(x)] = 0$

We have $Der(L_k) \in Z(L_k)$ and $Der(L_k)$

$D \in Der(L_{Ext})$

Hence $Der(L_k) = Der(L_{Ext})$

4. Conclusions

We introduce successfully the definition Extraction Lie algebra in this paper and exhibit an application based on a pre-existing example in the reference [8]. We also consider its Extraction derivation, its applicability and its underlying relationships with other components, and the particular component which it is contained in."

References

[1] Allcock, Jonathan, et al. "On generating direct powers of dynamical Lie algebras." arXiv preprint arXiv:2506.05733 (2025).

[2] Barati, M., F. Saeedi, and M. R. Alemi. "Derivation Lie algebras of semidirect sums." Rendiconti del Circolo Matematico di Palermo Series 2 69.2 (2020): 653-663.

[3] Chevalley, Claude. Theory of Lie groups. Courier Dover Publications, 2018.

[4] Dilxat, Munayim, Shoulan Gao, and Dong Liu. "Super-biderivations and post-Lie superalgebras on some Lie superalgebras." arXiv preprint arXiv:2206.05925 (2022).

[5] Frappat, L., A. Sciarrino, and P. Sorba. Dictionary on Lie algebras and superalgebras. Academic Press, Inc., San Diego, CA (2000).

[6] Guo, Fukui, and Yufeng Zhang. "A new loop algebra and a corresponding integrable hierarchy, as well as its integrable coupling." Journal of Mathematical Physics 44.12 (2003): 5793-5803.

[7] Jacobson, Nathan. "Abstract derivation and Lie algebras." Transactions of the American Mathematical Society 42.2 (1937): 206-224.

[8] Jacobson, Nathan. Lie algebras. Courier Corporation, 2013.

[9] Jacobson, Nathan. Nathan Jacobson Collected Mathematical Papers: Volume 2 (1947–1965). Springer Science & Business Media, 2013.

[10] Kac, Victor G. "Lie superalgebras." Advances in Mathematics 26.1 (1977): 8-96.

[11] La Rosa, Gianmarco, and Manuel Mancini. "Derivations of two-step nilpotent algebras." Communications in Algebra 51.12 (2023): 4928-4948.

[12] Liu, Xuewen, Xiangqian Guo, and Kaiming Zhao. "Biderivations of the block Lie algebras." Linear Algebra and its Applications 538 (2018): 43-55.

[13] Ma, Wen-Xiu. "Enlarging spectral problems to construct integrable couplings of soliton equations." Physics Letters A 316.1-2 (2003): 72-76.

[14] Ma, Wen-Xiu. "Integrable couplings of soliton equations by perturbations I. A general theory and application to the KdV hierarchy." arXiv preprint solv-int/9912004 (1999).

[15] Ma, Wen-Xiu. "Integrable couplings of vector AKNS soliton equations." Journal of Mathematical Physics 46.3 (2005).

[16] Ma, Wen-Xiu, and Benno Fuchssteiner. "Integrable theory of the perturbation equations." Chaos, Solitons & Fractals 7.8 (1996): 1227-1250.

[17] Ma, Wen-Xiu, and Benno Fuchssteiner. "The bi-Hamiltonian structure of the perturbation equations of the KdV hierarchy." Physics Letters A 213.1-2 (1996): 49-55.

[18] Ma, Wen-Xiu, Xi-Xiang Xu, and Yufeng Zhang. "Semi-direct sums of Lie algebras and continuous integrable couplings." Physics Letters A 351.3 (2006): 125-130.

[19] Ma, Wen-Xiu, Xi-Xiang Xu, and Yufeng Zhang. "Semidirect sums of Lie algebras and discrete integrable couplings." Journal of Mathematical Physics 47.5 (2006).

[20] Shalev, Aner. "Simple Lie algebras and Lie algebras of maximal class." Archiv der Mathematik 63.4 (1994): 297-301.

[21] Tang, Xiaomin. "Biderivations, linear commuting maps and commutative post-Lie algebra structures on W-algebras." Communications in Algebra 45.12 (2017): 5252-5261.

[22] Wang, Yu. "Lie superderivations of superalgebras." Linear and Multilinear Algebra 64.8 (2016): 1518-1526.

[23] Zhang, Runxuan, and Yongzheng Zhang. "Generalized derivations of Lie superalgebras." Communications in Algebra® 38.10 (2010): 3737-3751.

[24] Zhang, Yufeng. "A generalized multi-component Glachette–Johnson (GJ) hierarchy and its integrable coupling system." Chaos, Solitons & Fractals 21.2 (2004): 305-310.