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Cubic arcs in the projective plane over finite field of order four

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Abstract

The main aims of this research is to find the stabilizer groups of a cubic curves over a finite field of order 4, also studying the properties of their groups, and then constructing all different cubic curves, and known which one of them is complete or not. The arcs of degree 2 which are embedding into a cubic curves of even size have been constructed . Also drawing some of them .

Key words: stabilizer groups, arcs, cubic curves.

1- Introduction:-

The subject of this research depends on themes of

- Projective geometry over a finite field;
- Group theory;
- Linear algebra;
- Field theory.

The strategy of this research is to construct the stabilizer groups and finding the linear transformations groups in $PGL(3,q)$ of $PG(2,q)$, where $q=4$ which its element are considering the non-singular matrices $A_n=[a_{ij}], a_{ij}$ in $F_q, i,j=1,2,3$ for some n in $\mathbb{N}$ satisfying $K(tA_n)=K$ for all t in $F_q\backslash\{0\}$ and K be any arc. The set of all matrices A_n, which construct the group, and according to the number of A_n , and its order and then make comparison with the groups in [6], so we can find which one of them similar than it . In another hand, we have found the arcs which are embedding a cubic curves which are splitting into two sets, one of them contain the inflection points and the other does not, the set which does not contain the inflection points is considering the arc of degree two.

The summary history of this theme is shown as follows :

- All theorems and definitions of the research are taken from James Hirshfeld [1];
- In 2010, Najm AL-Seraji [2] studied the cubic curves over finite field of order 17 ;
- In 2011, Emad AL-Zangana [3] showed the cubic curves over finite field of order 19 ;
- In 2013, Emad AL-Zangana [4] described the cubic curves over finite field of orders 2,3,5,7;
- In 2013 , Emad AL-Zangana [5] classified the cubic curves over finite field of order 11,13;

Definition 1.1 [1] :- Denote by S and S^* two subspaces of $P(n,K)$, A projectivity $\beta: S\rightarrow S^*$ is a bijection given by a matrix T, necessarily non-singular, where $P(X)=P(X)\beta$ if $tX^*=XT$, with $t\in K$. Write $\beta=M(T)$; then $\beta=M(\lambda T)$ for any λ in K. The group of projectivities of $PG(n,K)$ is denoted by $PG(n+1,K)$.

Definition 1.2 [1] :- The stabilizer of x in Λ in under the action of G is the group $G_x=\{g\in G|xg=x\}$.

Definition 1.3 [1] :- An $(n;r)$ arc K or arc of degree r in $PG(k,q)$ with $n \geq r+1$ is a set of points with property that every hyperplane meets K in at most r points of K and there is some hyperplane meeting K in exactly r points. An $(n;3)$ -arc is also called an -arc. A $(n;r)$-arc is called complete if it is contained within $(n+1;r)$-arc .

Theorem 1.4 [1] :- A non-singular plane cubic curve with form and nine rational inflexions exists over F_q if and only if $q \equiv 1 \ (mod\ 3)$, and $\mathcal{F}$ then has canonical form $\mathcal{F} = x^3 + y^3 + z^3 - 3cxyz$.

Theorem 1.5 [1] :- A non-singular plane cubic curve with form $\mathcal{F}$ and three rational inflexions exists over F_q for all q . The inflexions are necessary collinear.

i - If the inflexional tangent are concurrent, the canonical forms are as follows:

(a) $q \equiv 1 \ (mod\ 3)$,

$$\mathcal{F} = xy(x+y) + z^3;$$
$$\mathcal{F} = xy(x+y) + cz^3;$$
$$\mathcal{F} = xy(x+y) + c^2z^3;$$

Where c is a primitive of F_q .

ii - If the inflexional tangent are not concurrent, the canonical form is as follows:

$$\mathcal{F} = xyz + e(x+y+z)^3, \qquad e \neq 0,1/27.$$

Theorem 1.6 [1] :- A non-singular plane cubic curve with form $\mathcal{F}$ defined over F_q , $q = p^h$ and at least one rational inflexion has one of following canonical forms.

$p = 2$,

(a) $\mathcal{F} = yz^2 + xyz + x^3 + bx^2y + cxy^2$, where $b = 0$ or a fixed element of trace 1 and $c \neq 0$;

(b) $\mathcal{F} = z^2y + zy^2 + ex^3 + cxy^2 + dy^3$, where $e = 1$ when $(q-1,3) = 1$ and $e = 1, \alpha, \alpha^2$ when $(q-1,3) = 3$, with α a primitive element of F_q; also $d = 0$ or a particular element of trace 1 .

Theorem 1.7 [1]:- A non-singular plane cubic curve with form $\mathcal{F}$ defined overF_q , $q = p^h$, with no rational inflexion has one of following canonical forms.

$q \equiv 1 \ (mod\ 3)$,

(a) $\mathcal{F} = x^3 + \alpha y^3 + \alpha^2 z^3 - 3cxyz$, with α a primitive element of F_q.

(b) $\mathcal{F} = xy^2 + x^2z + eyz^2 - c(x^3 + ey^3 + e^2z^3 - 3exyz)$, with α a primitive element of F_q and $e = \alpha, \alpha^2$.

2. The classification of cubic curves over a finite field of order 4

Let the polynomial $g_1(x) = x^2 + x + 1$ and $F_4 = \frac{F_2[x]}{\langle g_1(x) \rangle}$ which has 4 elements namely $0, 1, \theta, \theta^2$ where θ be x plus the ideal $\langle g_1(x) \rangle$ which generated by polynomial of degree 2 with coefficients in $F_2 = \{0,1\}$. The polynomial $g_2(x) = x^3 + \theta^2x^2 + \theta x + \theta^2$ is primitive over F_4, since $g_2(0) = \theta^2$, $g_2(1) = \theta^2$, $g_2(\theta) = \theta^2$ and $g_2(\theta^2) = \theta$, this means g_2 is irreducible over F_4, also $g_2(\beta^{11}) = g_2(\beta^{44}) = g_2(\beta^{50}) = 0$, where $\beta^{11}, \beta^{44}, \beta^{50}$ in F_{4^3} , this means g_2 is reducible over F_{64}. The companion matrix of $g_2(x) = x^3 + \theta^2x^2 + \theta x + \theta^2$ generated the points and lines of $PG(2,4)$ as follows:

$$P(k) = [1,0,0]C(g)^k = [1,0,0]\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \theta^2 & \theta & \theta^2 \end{pmatrix}^k, k = 0,1,\dots,20$$

With select the points in $PG(2,4)$ which are the third coordinate equal to zero , this means belong to $L_0 = v(z)$ such that $v(z) = tz = z$ for all t in $F_4\setminus\{0\}$, therefore with $P(k) = k , k = 0,1,\dots,20$,we obtain

$L_0 = \{0,1,4,14,16\}$

Moreover,

$$L_k = L_0 C(g)^k = L_0 \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \theta^2 & \theta & \theta^2 \end{pmatrix}^k , k = 0,\dots,20$$

By substituting the points of $PG(2,4)$ in theorem (1.4), we obtain

$\mathcal{F}_1 = x^3 + y^3 + z^3$ $|\mathcal{F}_1| = 9$ … (1)

$\mathcal{F}_2 = x^3 + y^3 + z^3 + xyz$ $|\mathcal{F}_2| = 12$ … (2)

$\mathcal{F}_3 = x^3 + y^3 + z^3 + \theta xyz$ $|\mathcal{F}_3| = 12$ … (3)

$\mathcal{F}_4 = x^3 + y^3 + z^3 + \theta^2 xyz$ $|\mathcal{F}_4| = 12$ … (4)

Also by substituting the points of $PG(2,4)$ in theorem (1.5), we obtain

$\mathcal{F}_5 = xy(x + y) + z^3$ $|\mathcal{F}_5| = 9$ … (5)

$\mathcal{F}_6 = xy(x + y) + \theta z^3$ $|\mathcal{F}_6| = 3$ … (6)

$\mathcal{F}_6$ in equation (6) is drawn in Figure 1 as follows:

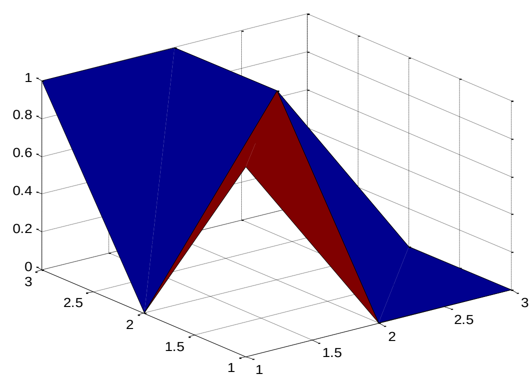

Figure 1 : Drawing of $\mathcal{F}_6$

$\mathcal{F}_7 = xyz + \theta(x + y + z)^3$ $|\mathcal{F}_7| = 6$ … (7)

$\mathcal{F}_8 = xyz + \theta^2(x + y + z)^3$ $|\mathcal{F}_8| = 6$ … (8)

Also by substituting the points of $PG(2,4)$ in theorem (1.6), we obtain

$\mathcal{F}_9 = yz^2 + xyz + x^3 + xy^2$, $|\mathcal{F}_9| = 8$ … (9)

$\mathcal{F}_{10} = yz^2 + xyz + x^3 + x^2y + xy^2$, $|\mathcal{F}_{10}| = 8$ … (10)

$\mathcal{F}_{11} = yz^2 + xyz + x^3 + \theta xy^2$, $|\mathcal{F}_{11}| = 4$ … (11)

$\mathcal{F}_{12} = yz^2 + xyz + x^3 + \theta^2 xy^2$, $|\mathcal{F}_{12}| = 4$ … (12)

$\mathcal{F}_{13} = yz^2 + xyz + x^3 + x^2y + \theta xy^2$, $|\mathcal{F}_{13}| = 4$ … (13)

$\mathcal{F}_{14} = yz^2 + xyz + x^3 + x^2y + \theta^2 xy^2$, $|\mathcal{F}_{14}| = 4$ … (14)

$\mathcal{F}_{15} = yz^2 + xyz + x^3 + \theta x^2y + xy^2$, $|\mathcal{F}_{15}| = 2$ … (15)

$\mathcal{F}_{15}$ in equation (15) is drawn in Figure 2 as follows:

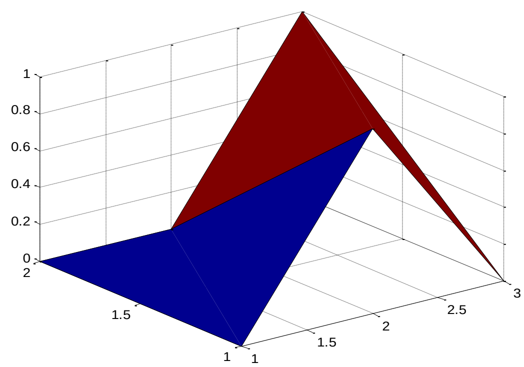

Figure 2 : Drawing of $\mathcal{F}_{15}$

$\mathcal{F}_{16} = yz^2 + xyz + x^3 + \theta x^2y + \theta xy^2$, $|\mathcal{F}_{16}| = 6$ … (16)

$\mathcal{F}_{17} = yz^2 + xyz + x^3 + \theta x^2y + \theta^2 xy^2$, $|\mathcal{F}_{17}| = 6$ … (17)

$\mathcal{F}_{18} = yz^2 + xyz + x^3 + \theta^2 x^2y + \theta xy^2$, $|\mathcal{F}_{18}| = 6$ … (18)

$\mathcal{F}_{19} = yz^2 + xyz + x^3 + \theta^2 x^2y + \theta^2 xy^2$, $|\mathcal{F}_{19}| = 6$ … (19)

$\mathcal{F}_{20} = z^2y + zy^2 + x^3 + \theta y^3$, $|\mathcal{F}_{20}| = 1$ … (20)

$\mathcal{F}_{21} = z^2y + zy^2 + x^3 + xy^2 + \theta y^3$, $|\mathcal{F}_{21}| = 5$ … (21)

$\mathcal{F}_{22} = z^2y + zy^2 + x^3 + xy^2 + \theta^2 y^3$, $|\mathcal{F}_{22}| = 5$ … (22)

$\mathcal{F}_{23} = z^2y + zy^2 + x^3 + \theta xy^2 + \theta y^3$, $|\mathcal{F}_{23}| = 5$ … (23)

$\mathcal{F}_{24} = z^2y + zy^2 + x^3 + \theta xy^2 + \theta^2 y^3$, $|\mathcal{F}_{24}| = 5$ … (24)

$\mathcal{F}_{25} = z^2y + zy^2 + x^3 + \theta^2 xy^2 + \theta y^3$, $|\mathcal{F}_{25}| = 5$ … (25)

$\mathcal{F}_{26} = z^2y + zy^2 + x^3 + \theta^2 xy^2 + \theta^2 y^3$, $|\mathcal{F}_{26}| = 5$ … (26)

$\mathcal{F}_{27} = z^2y + zy^2 + x^3 + \theta xy^2 + \theta y^3$, $|\mathcal{F}_{27}| = 7$ … (27)

$\mathcal{F}_{28} = z^2y + zy^2 + \theta x^3 + \theta^2 y^3$, $|\mathcal{F}_{28}| = 7$ … (28)

$\mathcal{F}_{29} = z^2y + zy^2 + \theta x^3 + xy^2 + \theta y^3$, $|\mathcal{F}_{29}| = 3$ … (29)

$\mathcal{F}_{30} = z^2y + zy^2 + \theta x^3 + xy^2 + \theta^2 y^3$, $|\mathcal{F}_{30}| = 3$ … (30)

$\mathcal{F}_{31} = z^2y + zy^2 + \theta x^3 + \theta xy^2 + \theta y^3$, $|\mathcal{F}_{31}| = 3$ … (31)

$\mathcal{F}_{32} = z^2y + zy^2 + \theta x^3 + \theta xy^2 + \theta^2 y^3$, $|\mathcal{F}_{32}| = 3$ … (32)

$\mathcal{F}_{33} = z^2y + zy^2 + \theta x^3 + \theta^2 xy^2 + \theta y^3$, $|\mathcal{F}_{33}| = 3$ … (33)

$\mathcal{F}_{34} = z^2y + zy^2 + \theta x^3 + \theta^2 xy^2 + \theta^2 y^3$, $|\mathcal{F}_{34}| = 3$ … (34)

$\mathcal{F}_{35} = z^2y + xyz + x^3 + \theta x^2y + y^3$, $|\mathcal{F}_{35}| = 2$ … (35)

$\mathcal{F}_{36} = z^2y + xyz + x^3 + \theta x^2y + \theta y^3$, $|\mathcal{F}_{36}| = 6$ … (36)

$\mathcal{F}_{37} = z^2y + xyz + x^3 + \theta x^2y + \theta^2 y^3$, $|\mathcal{F}_{38}| = 6$ … (37)

$\mathcal{F}_{38} = z^2y + xyz + x^3 + \theta^2 x^2y + \theta y^3$, $|\mathcal{F}_{38}| = 6$ … (38)

$\mathcal{F}_{39} = z^2y + xyz + x^3 + \theta^2 x^2y + \theta^2 y^3$, $|\mathcal{F}_{39}| = 6$ … (39)

Also by substituting the points of $PG(2,4)$ in theorem (1.7), we obtain

$\mathcal{F}_{40} = x^3 + \theta y^3 + \theta^2 z^3$, $|\mathcal{F}_{40}| = 9$ … (40)

$\mathcal{F}_{41} = xy^2 + x^2z + \theta yz^2$, $|\mathcal{F}_{41}| = 3$ …(41)

$\mathcal{F}_{41}$ in equation (41) is drawn in Figure 3 as follows:

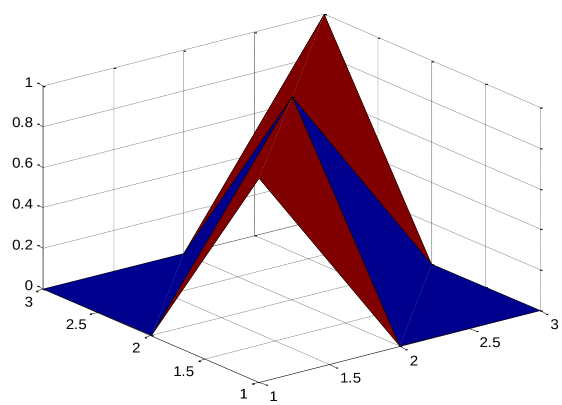

Figure 3 : Drawing of $\mathcal{F}_{41}$

$\mathcal{F}_{42} = xy^2 + x^2z + \theta yz^2 + (x^3 + \theta y^3 + \theta^2 z^3 + \theta xyz)$, $|\mathcal{F}_{42}| = 6$ …(42)

$\mathcal{F}_{43} = xy^2 + x^2z + \theta yz^2 + \theta(x^3 + \theta y^3 + \theta^2 z^3 + \theta xyz)$, $|\mathcal{F}_{43}| = 6$ …(43)

$\mathcal{F}_{44} = xy^2 + x^2z + \theta yz^2 + \theta^2(x^3 + \theta y^3 + \theta^2 z^3 + \theta xyz)$, $|\mathcal{F}_{44}| = 6$ …(44)

$\mathcal{F}_{45} = xy^2 + x^2z + \theta^2 yz^2 + (x^3 + \theta^2 y^3 + \theta^4 z^3 + \theta^2 xyz)$, $|\mathcal{F}_{45}| = 6$ …(45)

$\mathcal{F}_{46} = xy^2 + x^2z + \theta^2 yz^2 + \theta(x^3 + \theta^2 y^3 + \theta^4 z^3 + \theta^2 xyz)$, $|\mathcal{F}_{46}| = 6$ …(46)

$\mathcal{F}_{47} = xy^2 + x^2z + \theta^2 yz^2 + \theta^2(x^3 + \theta^2 y^3 + \theta^4 z^3 + \theta^2 xyz)$, $|\mathcal{F}_{47}| = 6$ …(47)

$\mathcal{F}_{48} = xy^2 + x^2z + x^3$, $|\mathcal{F}_{48}| = 9$ …(48)

$\mathcal{F}_{49} = xy^2 + x^2z + \theta x^3$, $|\mathcal{F}_{49}| = 9$ … (49)

$\mathcal{F}_{50} = xy^2 + x^2z + \theta^2 x^3$, $|\mathcal{F}_{50}| = 9$ ... (50)

$\mathcal{F}_{51} = xy^2 + x^2z + yz^2$, $|\mathcal{F}_{51}| = 9$ … (51)

The transformation matrix between $\mathcal{F}_2$ in equation (2) and $\mathcal{F}_3$ in equation (3) is given as following :

$$\mathcal{F}_2 \xrightarrow{\begin{pmatrix} 0 & 0 & \theta \\ 0 & 1 & 0 \\ \theta & 0 & 0 \end{pmatrix}} \mathcal{F}_3$$

The transformation matrix between $\mathcal{F}_2$ in equation (2) and $\mathcal{F}_4$ in equation (4) is given as following

$$\mathcal{F}_2 \xrightarrow{\begin{pmatrix} 0 & 0 & \theta \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}} \mathcal{F}_4$$

The transformation matrix between $\mathcal{F}_{29}$ in equation (29) and $\mathcal{F}_{30}$ in equation (30) is given as following

$$\mathcal{F}_{29} \xrightarrow{\begin{pmatrix} 0 & \theta & 0 \\ \theta & 0 & 1 \\ 0 & 0 & \theta \end{pmatrix}} \mathcal{F}_{30}$$

The transformation matrix between $\mathcal{F}_{29}$ in equation (29) and $\mathcal{F}_{31}$ in equation (31) is given as following

$$\mathcal{F}_{29} \xrightarrow{\begin{pmatrix} 0 & \theta & 0 \\ \theta & 1 & 0 \\ 0 & 0 & \theta^2 \end{pmatrix}} \mathcal{F}_{31}$$

The transformation matrix between $\mathcal{F}_{29}$ in equation (29) and $\mathcal{F}_{32}$ in equation (32) is given as following

$$\mathcal{F}_{29} \xrightarrow{\begin{pmatrix} 0 & \theta & 0 \\ \theta & 1 & \theta \\ 0 & 0 & \theta^2 \end{pmatrix}} \mathcal{F}_{32}$$

The transformation matrix between $\mathcal{F}_{29}$ in equation (29) and $\mathcal{F}_{33}$ in equation (33) is given as following

$$\mathcal{F}_{29} \xrightarrow{\begin{pmatrix} 0 & \theta & 0 \\ \theta & \theta^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \mathcal{F}_{33}$$

The transformation matrix between $\mathcal{F}_{29}$ in equation (29) and $\mathcal{F}_{34}$ in equation (34) is given as following

$$\mathcal{F}_{29} \xrightarrow{\begin{pmatrix} 0 & \theta & 0 \\ \theta & \theta^2 & \theta \\ 0 & 0 & 1 \end{pmatrix}} \mathcal{F}_{34}$$

The transformation matrix between $\mathcal{F}_7$ in equation (7) and $\mathcal{F}_8$ in equation (8) is given as following

$$\mathcal{F}_7 \xrightarrow{\begin{pmatrix} 0 & 0 & \theta \\ 0 & \theta & 0 \\ 1 & \theta^2 & \theta^2 \end{pmatrix}} \mathcal{F}_8$$

The transformation matrix between $\mathcal{F}_9$ in equation (9) and $\mathcal{F}_{10}$ in equation (10) is given as following

$$\mathcal{F}_9 \xrightarrow{\begin{pmatrix} \theta & 0 & 1 \\ 0 & \theta & 0 \\ 0 & 0 & \theta \end{pmatrix}} \mathcal{F}_{10}$$

The transformation matrix between $\mathcal{F}_{11}$ in equation (11) and $\mathcal{F}_{12}$ in equation (12) is given as following

$$\mathcal{F}_{11} \xrightarrow{\begin{pmatrix} 0 & \theta & \theta \\ 0 & 1 & 0 \\ \theta & 1 & 0 \end{pmatrix}} \mathcal{F}_{12}$$

The transformation matrix between $\mathcal{F}_{11}$ in equation (11) and $\mathcal{F}_{13}$ in equation (13) is given as following

$$\mathcal{F}_{11} \xrightarrow{\begin{pmatrix} 0 & \theta & \theta \\ 0 & 1 & 0 \\ \theta & \theta^2 & 1 \end{pmatrix}} \mathcal{F}_{13}$$

The transformation matrix between $\mathcal{F}_{11}$ in equation (11) and $\mathcal{F}_{14}$ in equation (14) is given as following

$$\mathcal{F}_{11} \xrightarrow{\begin{pmatrix} 0 & \theta & \theta \\ 0 & 1 & 0 \\ \theta & 1 & 1 \end{pmatrix}} \mathcal{F}_{14}$$

The transformation matrix between $\mathcal{F}_{15}$ in equation (15) and $\mathcal{F}_{35}$ in equation (35) is given as following

$$\mathcal{F}_{15} \xrightarrow{\begin{pmatrix} \theta & 0 & 0 \\ 0 & 0 & \theta \\ 0 & \theta & \theta \end{pmatrix}} \mathcal{F}_{35}$$

The transformation matrix between $\mathcal{F}_{16}$ in equation (16) and $\mathcal{F}_{17}$ in equation (17) is given as following

$$\mathcal{F}_{16} \xrightarrow{\begin{pmatrix} 0 & \theta & \theta \\ \theta^2 & 1 & \theta^2 \\ 1 & 1 & 1 \end{pmatrix}} \mathcal{F}_{17}$$

The transformation matrix between $\mathcal{F}_{16}$ in equation (16) and $\mathcal{F}_{18}$ in equation (18) is given as following

$$\mathcal{F}_{16} \xrightarrow{\begin{pmatrix} 0 & \theta & \theta \\ \theta & \theta & 0 \\ \theta^2 & \theta & \theta^2 \end{pmatrix}} \mathcal{F}_{18}$$

The transformation matrix between $\mathcal{F}_{16}$ in equation (16) and $\mathcal{F}_{19}$ in equation (19) is given as following

$$\mathcal{F}_{16} \xrightarrow{\begin{pmatrix} 0 & \theta & \theta \\ \theta^2 & 1 & \theta \\ 1 & 1 & \theta \end{pmatrix}} \mathcal{F}_{19}$$

The transformation matrix between $\mathcal{F}_{16}$ in equation (16) and $\mathcal{F}_{36}$ in equation (36) is given as following

$$\mathcal{F}_{16} \xrightarrow{\begin{pmatrix} 0 & \theta & 0 \\ 1 & \theta & 0 \\ \theta & \theta & 1 \end{pmatrix}} \mathcal{F}_{36}$$

The transformation matrix between $\mathcal{F}_{16}$ in equation (16) and $\mathcal{F}_{37}$ in equation (37) is given as following

$$\mathcal{F}_{16} \xrightarrow{\begin{pmatrix} 0 & \theta & 0 \\ 1 & 1 & 0 \\ \theta & 1 & \theta^2 \end{pmatrix}} \mathcal{F}_{37}$$

The transformation matrix between $\mathcal{F}_{16}$ in equation (16) and $\mathcal{F}_{38}$ in equation (38) is given as following

$$\mathcal{F}_{16} \xrightarrow{\begin{pmatrix} 0 & \theta & 0 \\ 1 & \theta & \theta \\ \theta & \theta & \theta \end{pmatrix}} \mathcal{F}_{38}$$

The transformation matrix between $\mathcal{F}_{16}$ in equation (16) and $\mathcal{F}_{39}$ in equation (39) is given as following

$$\mathcal{F}_{16} \xrightarrow{\begin{pmatrix} 0 & \theta & 0 \\ 1 & 1 & 0 \\ \theta & 1 & 0 \end{pmatrix}} \mathcal{F}_{39}$$

The transformation matrix between $\mathcal{F}_{21}$ in equation (21) and $\mathcal{F}_{22}$ in equation (22) is given as following

$$\mathcal{F}_{21} \xrightarrow{\begin{pmatrix} \theta & 0 & 0 \\ 0 & \theta & 1 \\ 0 & 0 & \theta \end{pmatrix}} \mathcal{F}_{22}$$

The transformation matrix between $\mathcal{F}_{21}$ in equation (21) and $\mathcal{F}_{23}$ in equation (23) is given as following

$$\mathcal{F}_{21} \xrightarrow{\begin{pmatrix} \theta & 0 & 0 \\ 0 & \theta^2 & 0 \\ 0 & 0 & \theta^2 \end{pmatrix}} \mathcal{F}_{23}$$

The transformation matrix between $\mathcal{F}_{21}$ in equation (21) and $\mathcal{F}_{24}$ in equation (24) is given as following

$$\mathcal{F}_{21} \xrightarrow{\begin{pmatrix} \theta & 0 & 0 \\ 0 & \theta^2 & \theta \\ 0 & 0 & \theta^2 \end{pmatrix}} \mathcal{F}_{24}$$

The transformation matrix between $\mathcal{F}_{21}$ in equation (21) and $\mathcal{F}_{25}$ in equation (25) is given as following

$$\mathcal{F}_{21} \xrightarrow{\begin{pmatrix} \theta & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \mathcal{F}_{25}$$

The transformation matrix between $\mathcal{F}_{21}$ in equation (21) and $\mathcal{F}_{26}$ in equation (26) is given as following

$$\mathcal{F}_{21} \xrightarrow{\begin{pmatrix} \theta & 0 & 0 \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix}} \mathcal{F}_{26}$$

The transformation matrix between $\mathcal{F}_{27}$ in equation (27) and $\mathcal{F}_{28}$ in equation (28) is given as following

$$\mathcal{F}_{27} \xrightarrow{\begin{pmatrix} \theta & 0 & 0 \\ 0 & \theta & 1 \\ 0 & 0 & \theta \end{pmatrix}} \mathcal{F}_{28}$$

The transformation matrix between $\mathcal{F}_{42}$ in equation (42) and $\mathcal{F}_{43}$ in equation (43) is given as following

$$\mathcal{F}_{42} \xrightarrow{\begin{pmatrix} 0 & 0 & \theta \\ \theta & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}} \mathcal{F}_{43}$$

The transformation matrix between $\mathcal{F}_{42}$ in equation (42) and $\mathcal{F}_{44}$ in equation (44) is given as following

$$\mathcal{F}_{42} \xrightarrow{\begin{pmatrix} 0 & 0 & \theta \\ 1 & 0 & 0 \\ \theta & \theta & \theta \end{pmatrix}} \mathcal{F}_{44}$$

The transformation matrix between $\mathcal{F}_{42}$ in equation (42) and $\mathcal{F}_{45}$ in equation (45) is given as following

$$\mathcal{F}_{42} \xrightarrow{\begin{pmatrix} 0 & 0 & \theta \\ 0 & \theta & 0 \\ \theta & 0 & 0 \end{pmatrix}} \mathcal{F}_{45}$$

The transformation matrix between $\mathcal{F}_{42}$ in equation (42) and $\mathcal{F}_{46}$ in equation (46) is given as following

$$\mathcal{F}_{42} \xrightarrow{\begin{pmatrix} 0 & 0 & \theta \\ 0 & \theta^2 & 0 \\ 1 & 0 & 0 \end{pmatrix}} \mathcal{F}_{46}$$

The transformation matrix between $\mathcal{F}_{42}$ in equation (42) and $\mathcal{F}_{47}$ in equation (47) is given as following

$$\mathcal{F}_{42} \xrightarrow{\begin{pmatrix} 0 & 0 & \theta \\ 0 & 1 & 0 \\ \theta^2 & 0 & 0 \end{pmatrix}} \mathcal{F}_{47}$$

The transformation matrix between $\mathcal{F}_{48}$ in equation (48) and $\mathcal{F}_{49}$ in equation (49) is given as following :

$$\mathcal{F}_{48} \xrightarrow{\begin{pmatrix} \theta & 0 & 0 \\ 0 & 0 & \theta^2 \\ 0 & 1 & 0 \end{pmatrix}} \mathcal{F}_{49}$$

The transformation matrix between $\mathcal{F}_{48}$ in equation (48) and $\mathcal{F}_{50}$ in equation (50) is given as following :

$$\mathcal{F}_{48} \xrightarrow{\begin{pmatrix} \theta & 0 & 0 \\ 0 & 0 & 1 \\ 0 & \theta^2 & 0 \end{pmatrix}} \mathcal{F}_{50}$$

Therefore, the number of different cubic curves is shown in following theorem as follows :

Theorem 2.1 :- on $PG(2,4)$ there are precisely 18 distinct cubic curves which given in Table 1

Where $\aleph$ represents the infliction points.

Table 1 : The distinct cubic curves in $PG(2,4)$

$\aleph$	No	Canonical form	Size	Description	Maximum arc
9	1	$\mathcal{F}_1 = x^3 + y^3 + z^3$	9	complete	−
	2	$\mathcal{F}_2 = x^3 + y^3 + z^3 + xyz$	12	complete	−
3	3	$\mathcal{F}_5 = xy(x+y) + z^3$	9	complete	−
	4	$\mathcal{F}_6 = xy(x+y) + \theta z^3$	3	incomplete	9
	5	$\mathcal{F}_7 = xyz + \theta(x+y+z)^3$	6	incomplete	9
1	6	$\mathcal{F}_9 = yz^2 + xyz + x^3 + xy^2$	8	incomplete	9
	7	$\mathcal{F}_{11} = yz^2 + xyz + x^3 + \theta xy^2$	4	incomplete	9
	8	$\mathcal{F}_{15} = yz^2 + xyz + x^3 + \theta x^2y + xy^2$	2	incomplete	9
	9	$\mathcal{F}_{16} = yz^2 + xyz + x^3 + \theta x^2y + \theta xy^2$	6	incomplete	9
	10	$\mathcal{F}_{20} = z^2y + zy^2 + x^3 + \theta y^3$	1	incomplete	9
	11	$\mathcal{F}_{21} = z^2y + zy^2 + x^3 + xy^2 + \theta y^3$	5	incomplete	9
	12	$\mathcal{F}_{27} = z^2y + zy^2 + x^3 + \theta xy^2 + \theta y^3$	7	incomplete	9
	13	$\mathcal{F}_{29} = z^2y + zy^2 + \theta x^3 + xy^2 + \theta y^3$	3	incomplete	9
0	14	$\mathcal{F}_{40} = x^3 + \theta y^3 + \theta^2 z^3$	9	complete	−
	15	$\mathcal{F}_{41} = xy^2 + x^2z + \theta yz^2$	3	incomplete	9
	16	$\mathcal{F}_{42} = xy^2 + x^2z + \theta yz^2 + (x^3 + \theta y^3 + \theta^2 z^3 + \theta xyz)$	6	incomplete	9
	17	$\mathcal{F}_{48} = xy^2 + x^2z + x^3$	9	complete	−
	18	$\mathcal{F}_{51} = xy^2 + x^2z + yz^2$	9	complete	−

The number of distinct cubic curves is 18 see table 1 , one of them is given as following : $\mathcal{F}_1 = x^3 + y^3 + z^3$. The points of $PG(2,4)$ on $\mathcal{F}_1$ are $[1,1,0],[0,1,1],[\theta,0,1],[1,0,1],[\theta^2,0,1]$, $[\theta^2 1,0],[0,\theta^2,1],[\theta,1,0],[0,\theta,1]$. After calculations and help the computer, we are note that the number of matrices which are stabilizing of $\mathcal{F}_1$ and their order is 216, and we can not write them, because they are too much.

Therefore, the stabilizer group of $\mathcal{F}_1$ which is denoted by $G_{\mathcal{F}_1}$ contains

- 9 matrices of order 2;
- 80 matrix of order 3;
- 54 matrix of order 4;
- 72 matrix of order 6;
- The identity matrix.

Another one of the cubic curves which given in Table 1 is $\mathcal{F}_2 = x^3 + y^3 + z^3 + xyz$ .The points of $PG(2,4)$ on $\mathcal{F}_2$ are $[1,1,0],[0,1,1],[\theta,0,1],[1,0,1],[1,1,1],[\theta^2 0,1],[\theta^2,1,0],[0,\theta^2,1],[\theta,1,0],[0,\theta,1],[\theta^2,\theta,1],[\theta,\theta^2,1]$. After calculations and help the computer, we are note that the number of matrices which are stabilizing of $\mathcal{F}_2$ and their order is 54, and we can not write them, because they are too much.

Therefore, the stabilizer groups of $\mathcal{F}_2$ which is denoted by $G_{\mathcal{F}_2}$ contains

- 9 matrices of order 2;
- 26 matrix of order 3;
- 18 matrix of order 6;
- The identity matrix .

Form [6], $G_{\mathcal{F}_2}$ is isomorphic to $Z_6 \times Z_3 \times Z_3$, that is $G_{\mathcal{F}_2} \cong Z_6 \times Z_3 \times Z_3$.

Another one of the cubic curves which given in Table 1 is $\mathcal{F}_5 = xy(x+y) + z^3$ .The points of $PG(2,4)$ on $\mathcal{F}_5$ are $[1,0,0],[0,1,0],[1,\theta^2,1],[1,1,0],[\theta,1,1],[1,\theta,1],[\theta^2,\theta,1],[\theta^2,1,1],[\theta,\theta^2,1]$. After calculations and help the computer, we are note that the number of matrices which are stabilizing of $\mathcal{F}_5$ and their order is 216, and we can not write them, because they are too much.

Therefore, the stabilizer group of $\mathcal{F}_5$ which is denoted by $G_{\mathcal{F}_5}$ contains

- 9 matrices of order 2;
- 80 matrix of order 3;
- 54 matrix of order 4;
- 72 matrix of order 6;
- The identity matrix .

Another one of the cubic curves which given in Table 1 is $\mathcal{F}_6 = xy(x+y) + \theta z^3$ .The points of $PG(2,4)$ on $\mathcal{F}_6$ are $[1,0,0],[0,1,0],[1,1,0]$. After calculations and help the computer, we are note that the number of matrices which are stabilizing of $\mathcal{F}_6$ and their order is 288, and we can not write them, because they are too much .

Therefore, the stabilizer group of $\mathcal{F}_6$ which is denoted by $G_{\mathcal{F}_6}$ contains

- 27 matrix of order 2;
- 80 matrix of order 3;
- 36 matrix of order 4;
- 144 matrix of order 6;
- The identity matrix .

Another one of the cubic curves which given in Table 1 is $\mathcal{F}_7 = xyz + \theta(x+y+z)^3$. The points of $PG(2,4)$ on $\mathcal{F}_7$ are $[1,1,0],[0,1,1],[\theta,1,1],[1,0,1],[1,\theta,1],[\theta^2,\theta^2,1]$,. To find the stabilizer group of $\mathcal{F}_7$, we are doing calculations by help the computer, thus the transformation matrices which stabilizing of $\mathcal{F}_7$ and their order are shown as follows :

$$\begin{pmatrix}0&0&\theta^2\\\theta^2&0&0\\0&\theta^2&0\end{pmatrix}:3,\ \begin{pmatrix}0&0&\theta^2\\1&\theta&\theta\\\theta&1&\theta\end{pmatrix}:3,\ \begin{pmatrix}0&0&\theta^2\\0&\theta^2&0\\\theta^2&0&0\end{pmatrix}:2,\ \begin{pmatrix}0&0&\theta^2\\\theta&1&\theta\\1&\theta&\theta\end{pmatrix}:4$$

$$\begin{pmatrix}\theta^2&0&0\\0&0&\theta^2\\0&\theta^2&0\end{pmatrix}:2,\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}1&0&0\\\theta^2&\theta&\theta^2\\\theta^2&\theta^2&\theta\end{pmatrix}:2,\begin{pmatrix}\theta^2&0&0\\\theta&\theta&1\\\theta&1&\theta\end{pmatrix}:2$$

$$\begin{pmatrix}1&\theta&\theta\\0&0&\theta^2\\\theta&1&\theta\end{pmatrix}:4,\ \begin{pmatrix}\theta&\theta^2&\theta^2\\0&1&0\\\theta^2&\theta^2&\theta\end{pmatrix}:2,\ \begin{pmatrix}\theta&\theta^2&\theta^2\\\theta^2&\theta&\theta^2\\0&0&1\end{pmatrix}:2,\ \begin{pmatrix}1&\theta&\theta\\\theta&\theta&1\\0&\theta^2&0\end{pmatrix}:4$$

$$\begin{pmatrix}0&\theta^2&0\\0&0&\theta^2\\\theta^2&0&0\end{pmatrix}:3,\ \begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}:2,\ \begin{pmatrix}0&1&0\\\theta&\theta^2&\theta^2\\\theta^2&\theta^2&\theta\end{pmatrix}:4,\ \begin{pmatrix}0&\theta^2&0\\\theta&\theta&1\\1&\theta&\theta\end{pmatrix}:3$$

$$\begin{pmatrix}\theta&1&\theta\\0&0&\theta^2\\1&\theta&\theta\end{pmatrix}:3,\ \begin{pmatrix}\theta^2&\theta&\theta^2\\1&0&0\\\theta^2&\theta^2&\theta\end{pmatrix}:4,\ \begin{pmatrix}\theta^2&\theta&\theta^2\\\theta&\theta^2&\theta^2\\0&0&1\end{pmatrix}:2,\ \begin{pmatrix}\theta&1&\theta\\\theta&\theta&1\\\theta^2&0&0\end{pmatrix}:3$$

$$\begin{pmatrix}\theta&\theta&1\\\theta^2&0&0\\\theta&1&\theta\end{pmatrix}:3,\ \begin{pmatrix}\theta&\theta&1\\1&\theta&\theta\\0&\theta^2&0\end{pmatrix}:3,\ \begin{pmatrix}\theta&\theta&1\\0&\theta^2&0\\1&\theta&\theta\end{pmatrix}:2,\ \begin{pmatrix}\theta&\theta&1\\\theta&1&\theta\\\theta^2&0&0\end{pmatrix}:4$$

Therefore, the stabilizer groups of $\mathcal{F}_7$ which is denoted by $G_{\mathcal{F}_7}$ contains

- 9 matrices of order 2;
- 8 matrices of order 3;
- 6 matrices of order 4;
- The identity matrix .

Form [6], $G_{\mathcal{F}_7}$ is isomorphic to $\boldsymbol{S_4}$, that is $G_{\mathcal{F}_7} \cong \boldsymbol{S_4}$.

Let $\mathcal{F}_7^* = [\theta,1,1],[1,\theta,1],[\theta^2,\theta^2,1]$ be a subset of $\mathcal{F}_7$ which is forming by partition the $\mathcal{F}_7$ into two sets such that $\mathcal{F}_7^*$ dose not contains the inflection points of $\mathcal{F}_7$, so we note that $\mathcal{F}_7^*$ represents an arc of degree two . After the calculation and help the computer, we are obtained that the number of matrices which are stabilizing of $\mathcal{F}_7^*$ and their order is 54, and we can not write them, because they are too much

Therefore, the stabilizer group of $\mathcal{F}_7^*$ which is denoted by $G_{\mathcal{F}_7^*}$ contains

- 9 matrices of order 2;
- 26 matrix of order 3;
- 18 matrix of order 6;
- The identity matrix.

Form [6], $G_{\mathcal{F}_7^*}$ is isomorphic to $\boldsymbol{Z_6} \times \boldsymbol{Z_3} \times \boldsymbol{Z_3}$, that is $G_{\mathcal{F}_7^*} \cong \boldsymbol{Z_6} \times \boldsymbol{Z_3} \times \boldsymbol{Z_3}$.

Journal of Iraqi Al-Khwarizmi Society (JIKhS) Year 2018 pages: 81-98
Volume:2 Special issue of the first international scientific conference of Iraqi Al-Khawarizmi Society 28-29 March 2018

Another one of the cubic curves which given in Table 1 is $\mathcal{F}_9 = yz^2 + xyz + x^3 + xy^2$.The points of $PG(2,4)$ on $\mathcal{F}_9$ are $[0,1,0],[0,0,1],[1,1,0],[\theta,1,1],[1,1,1],[\theta^2,\theta,1],[\theta^2 1,1],[\theta,\theta^2 1]$. To find the stabilizer group of $\mathcal{F}_9$, we are doing calculations by help the computer, thus the transformation matrices which stabilizing of $\mathcal{F}_9$ and their order are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\ \begin{pmatrix}\theta&0&\theta\\0&\theta&0\\0&0&\theta\end{pmatrix}:2$$

Therefore, the stabilizer groups of $\mathcal{F}_9$ which is denoted by $G_{\mathcal{F}_9}$ contains

- One matrix of order 2;
- The identity matrix.

Form [6], $G_{\mathcal{F}_9}$ is isomorphic to Z_2 , that is $G_{\mathcal{F}_9} \cong Z_2$.

Let $\mathcal{F}_9^* = [0,1,0],[1,1,0],[1,1,1],[\theta^2,1,1]$ be a subset of $\mathcal{F}_9$ which is forming by partition the $\mathcal{F}_9$ into two sets such that $\mathcal{F}_9^*$ dose not contains the inflection points of $\mathcal{F}_9$, so we note that $\mathcal{F}_9^*$ represents an are of degree two. Also, to find the stabilizer group and their order of $\mathcal{F}_9^*$, by some calculation, we obtain

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}1&0&0\\0&1&0\\\theta&0&1\end{pmatrix}:2,\begin{pmatrix}1&0&0\\\theta&\theta^2&\theta^2\\1&1&\theta^2\end{pmatrix}:2,\begin{pmatrix}1&0&0\\\theta&\theta^2&\theta^2\\\theta^2&\theta&\theta^2\end{pmatrix}:4$$

$$\begin{pmatrix}1&0&0\\1&1&0\\1&0&1\end{pmatrix}:2,\begin{pmatrix}1&0&0\\1&1&0\\\theta^2&0&1\end{pmatrix}:2,\begin{pmatrix}1&0&0\\\theta^2&\theta^2&\theta^2\\0&1&\theta^2\end{pmatrix}:4,\begin{pmatrix}1&0&0\\\theta^2&\theta^2&\theta^2\\\theta&1&\theta^2\end{pmatrix}:2$$

$$\begin{pmatrix}1&1&\theta\\0&\theta^2&0\\0&\theta^2&\theta\end{pmatrix}:3,\begin{pmatrix}1&1&\theta\\0&\theta^2&0\\\theta&1&1\end{pmatrix}:2,\begin{pmatrix}1&1&\theta\\\theta&\theta&\theta\\1&0&\theta^2\end{pmatrix}:3,\begin{pmatrix}1&1&0\\\theta&\theta&\theta\\\theta^2&\theta&0\end{pmatrix}:2,$$

$$\begin{pmatrix}1&1&0\\1&\theta&\theta\\1&\theta&0\end{pmatrix}:4,\begin{pmatrix}1&1&\theta\\1&\theta&\theta\\\theta^2&0&\theta^2\end{pmatrix}:3,\begin{pmatrix}1&1&\theta\\\theta^2&\theta^2&0\\0&1&1\end{pmatrix}:4,\begin{pmatrix}1&1&\theta\\\theta^2&\theta^2&0\\\theta&\theta^2&\theta\end{pmatrix}:3$$

$$\begin{pmatrix}1&\theta^2&1\\0&\theta&0\\0&\theta^2&\theta^2\end{pmatrix}:3,\begin{pmatrix}1&\theta^2&1\\0&\theta&0\\\theta&\theta&1\end{pmatrix}:2,\begin{pmatrix}1&\theta^2&1\\\theta&\theta&0\\1&\theta^2&\theta^2\end{pmatrix}:3,\begin{pmatrix}1&\theta^2&1\\\theta&\theta&0\\\theta^2&\theta&1\end{pmatrix}:4$$

$$\begin{pmatrix}1&\theta^2&1\\1&1&1\\1&0&\theta\end{pmatrix}:4,\begin{pmatrix}1&\theta^2&1\\1&1&1\\\theta^2&1&0\end{pmatrix}:3,\begin{pmatrix}1&\theta^2&1\\\theta^2&1&1\\0&0&\theta\end{pmatrix}:2,\begin{pmatrix}1&\theta^2&1\\\theta^2&1&1\\\theta&1&0\end{pmatrix}:3$$

Therefore, the stabilizer group of $\mathcal{F}_9^*$ which is denoted by $G_{\mathcal{F}_9^*}$ contains

- 9 matrices of order 2;
- 8 matrices of order 3;
- 6 matrices of order 4;
- The identity matrix.

Form [6], $G_{\mathcal{F}_9^*}$ is isomorphic to S_4, that is $G_{\mathcal{F}_9^*} \cong S_4$.

Journal of Iraqi Al-Khwarizmi Society (JIKhS) Year 2018 pages: 81-98
Volume:2 Special issue of the first international scientific conference of Iraqi Al-Khawarizmi Society 28-29 March 2018

Another one of the cubic curves which given in Table 1 is $\mathcal{F}_{11} = yz^2 + xyz + x^3 + \theta xy^2$.The points of $PG(2,4)$ on $\mathcal{F}_{11}$ are $[0,1,0]$,$[0,0,1]$,$[1,\theta,1]$,$[\theta^2,1,0]$. To find the stabilizer group of $\mathcal{F}_{11}$, we are doing calculations by help the computer, thus the transformation matrices which stabilizing of $\mathcal{F}_{11}$ and their order are shown as follows :

$$\begin{pmatrix}0&\theta&\theta\\0&1&0\\\theta&\theta^2&0\end{pmatrix}:6\ ,\begin{pmatrix}0&\theta&\theta\\0&1&0\\\theta&\theta^2&\theta\end{pmatrix}:3\ ,\begin{pmatrix}0&\theta&1\\0&1&0\\1&\theta&0\end{pmatrix}:2\ ,\begin{pmatrix}0&\theta&1\\0&1&0\\1&\theta&1\end{pmatrix}:3$$

$$\begin{pmatrix}0&\theta&\theta^2\\0&1&0\\\theta^2&1&0\end{pmatrix}:6\ ,\begin{pmatrix}0&\theta&\theta^2\\0&1&0\\\theta^2&1&\theta^2\end{pmatrix}:3\ ,\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}\theta&0&0\\0&\theta&0\\\theta&\theta^2&\theta\end{pmatrix}:2$$

$$\begin{pmatrix}\theta&0&\theta\\0&\theta&0\\0&0&\theta\end{pmatrix}:2\ ,\begin{pmatrix}\theta&0&\theta\\0&\theta&0\\\theta&\theta^2&0\end{pmatrix}:3\ ,\begin{pmatrix}\theta&\theta&0\\0&\theta^2&0\\0&0&\theta\end{pmatrix}:3\ ,\begin{pmatrix}\theta&\theta&0\\0&\theta^2&0\\\theta&\theta^2&\theta\end{pmatrix}:6$$

$$\begin{pmatrix}\theta&\theta&\theta\\0&\theta^2&0\\0&0&\theta\end{pmatrix}:6\ ,\begin{pmatrix}\theta&\theta&\theta\\0&\theta^2&0\\\theta&\theta^2&0\end{pmatrix}:3\ ,\begin{pmatrix}\theta&1&0\\0&1&0\\0&0&\theta\end{pmatrix}:3\ ,\begin{pmatrix}\theta&1&0\\0&1&0\\\theta&\theta^2&\theta\end{pmatrix}:6$$

$$\begin{pmatrix}\theta&1&\theta\\0&1&0\\0&0&\theta\end{pmatrix}:6\ ,\begin{pmatrix}\theta&1&\theta\\0&1&0\\\theta&\theta^2&0\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{11}$ which is denoted by $G_{\mathcal{F}_{11}}$ contains

- 3 matrices of order 2;
- 8 matrices of order 3;
- 6 matrices of order 6;
- The identity matrix .

Form [6], $G_{\mathcal{F}_{11}}$ is isomorphic to $\boldsymbol{S_3} \times \boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{11}} \cong \boldsymbol{S_3} \times \boldsymbol{Z_3}$.

Let $\mathcal{F}_{11}^{*} = [0,1,0], [0,0,1]$ be a subset of $\mathcal{F}_{11}$ which is forming by partition the $\mathcal{F}_{11}$ into two sets such that $\mathcal{F}_{11}^{*}$ dose not contains the inflection points of $\mathcal{F}_{11}$, so we note that $\mathcal{F}_{11}^{*}$ represents an arc of degree two. After the calculation and help the computer, we are obtained that the number of matrices which are stabilizing of $\mathcal{F}_{11}^{*}$ and their order is 288 , and we can not write them , because they are too much

Therefore, the stabilizer group of $\mathcal{F}_{11}^{*}$ which is denoted by $G_{\mathcal{F}_{11}^{*}}$ contains

- 27 matrix of order 2;
- 80 matrix of order 3;
- 36 matrix of order 4;
- 144 matrix of order 6;
- The identity matrix .

Drawing of $\mathcal{F}_{11}^{*}$ is given in figure 4 as following :

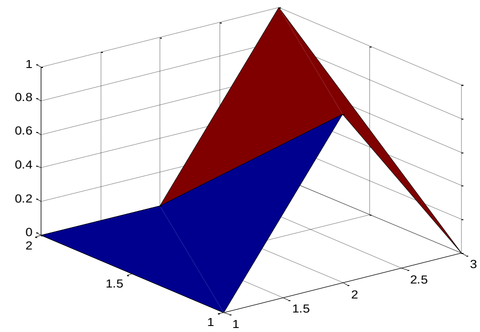

Figure 4 : drawing of $\mathcal{F}_{11}^{*}$

Another one of the cubic curves which given in Table 1 is $\mathcal{F}_{15} = yz^2 + xyz + x^3 + \theta x^2 y + xy^2$.The points of $PG(2,4)$ on $\mathcal{F}_{15}$ are $[0,1,0]$,$[0,0,1]$,. After calculations and help the computer, we are note that the number of matrices which are stabilizing of $\mathcal{F}_{15}$ and their order is 288, and we can not write them, because they are too much.

Therefore, the stabilizer group of $\mathcal{F}_{15}$ which is denoted by $G_{\mathcal{F}_{15}}$ contains

- 27 matrix of order 2;
- 80 matrix of order 3;
- 36 matrix of order 4;
- 144 matrix of order 6;
- The identity matrix .

Another one of the cubic curves which given in Table 1 is $\mathcal{F}_{16} = yz^2 + xyz + x^3 + \theta x^2 y + \theta xy^2$ .The points of $PG(2,4)$ on $\mathcal{F}_{16}$ are $[0,1,0], [0,0,1], [\theta,1,1], [\theta,\theta,1], [\theta^2,\theta^2,1], [\theta^2,\theta,1]$. To find the stabilizer group of $\mathcal{F}_{16}$, we are doing calculations by help the computer, thus the transformation matrices which stabilizing of $\mathcal{F}_{16}$ and their order are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1, \begin{pmatrix}1&\theta&0\\0&1&0\\0&0&1\end{pmatrix}:2, \begin{pmatrix}1&\theta&1\\0&1&0\\0&0&1\end{pmatrix}:2, \begin{pmatrix}1&0&1\\0&1&0\\0&0&1\end{pmatrix}:2$$

$$\begin{pmatrix}1&\theta&1\\0&0&\theta^2\\0&\theta&0\end{pmatrix}:2, \begin{pmatrix}1&0&1\\0&0&\theta^2\\0&\theta&0\end{pmatrix}:4, \begin{pmatrix}1&0&0\\0&0&\theta^2\\0&\theta&0\end{pmatrix}:2, \begin{pmatrix}1&\theta&0\\0&0&\theta^2\\0&\theta&0\end{pmatrix}:4$$

$$\begin{pmatrix}1&0&0\\\theta^2&\theta&\theta\\1&1&\theta\end{pmatrix}:2, \begin{pmatrix}1&\theta&1\\\theta^2&\theta&\theta\\1&1&\theta\end{pmatrix}:4, \begin{pmatrix}0&1&1\\1&\theta^2&\theta^2\\\theta&\theta&\theta^2\end{pmatrix}:3, \begin{pmatrix}0&\theta&\theta^2\\1&\theta^2&\theta^2\\\theta&\theta&\theta^2\end{pmatrix}:3$$

$$\begin{pmatrix}1&\theta&0\\\theta^2&\theta^2&\theta\\1&\theta^2&\theta\end{pmatrix}:3, \begin{pmatrix}1&0&1\\\theta^2&\theta^2&\theta\\1&\theta^2&\theta\end{pmatrix}:3, \begin{pmatrix}0&1&1\\1&1&\theta^2\\\theta&1&\theta^2\end{pmatrix}:2, \begin{pmatrix}0&\theta&\theta^2\\1&1&\theta^2\\\theta&1&\theta^2\end{pmatrix}:4$$

$$\begin{pmatrix}1&\theta&1\\\theta^2&\theta^2&1\\1&\theta^2&\theta^2\end{pmatrix}:4, \begin{pmatrix}1&0&0\\\theta^2&\theta^2&1\\1&\theta^2&\theta^2\end{pmatrix}:2, \begin{pmatrix}0&\theta&\theta^2\\1&1&\theta\\\theta&1&1\end{pmatrix}:3, \begin{pmatrix}0&1&1\\1&1&\theta\\\theta&1&1\end{pmatrix}:3$$

$$\begin{pmatrix}1&0&1\\\theta^2&\theta&1\\1&1&\theta^2\end{pmatrix}:3,\begin{pmatrix}1&\theta&0\\\theta^2&\theta&1\\1&1&\theta^2\end{pmatrix}:3,\begin{pmatrix}0&\theta&\theta^2\\1&\theta^2&\theta\\\theta&\theta&1\end{pmatrix}:2,\begin{pmatrix}0&1&1\\1&\theta^2&\theta\\\theta&\theta&1\end{pmatrix}:4$$

Therefore, the stabilizer groups of $\mathcal{F}_{16}$ which is denoted by $G_{\mathcal{F}_{16}}$ contains

- 9 matrices of order 2;
- 8 matrices of order 3;
- 6 matrices of order 4;
- The identity matrix .

Form [6], $G_{\mathcal{F}_{16}}$ is isomorphic to S_4 , that is $G_{\mathcal{F}_{16}} \cong S_4$.

Let $\mathcal{F}_{16}^* = [0,1,0],[\theta,\theta,1],[\theta^2,\theta^2,1]$ be a subset of $\mathcal{F}_{16}$ which is forming by partition the $\mathcal{F}_{16}$ into two sets such that $\mathcal{F}_{16}^*$ dose not contains the inflection points of $\mathcal{F}_{16}$, so we note that $\mathcal{F}_{16}^*$ represents an arc of degree two. After the calculation and help the computer, we are obtained that the number of matrices which are stabilizing of $\mathcal{F}_{16}^*$and their order is 54, and we can not write them, because they are too much

Therefore, the stabilizer group of $\mathcal{F}_{16}^*$ which is denoted by $G_{\mathcal{F}_{16}^*}$ contains

- 9 matrices of order 2;
- 26 matrix of order 3;
- 18 matrix of order 6;
- The identity matrix.

Form [6], $G_{\mathcal{F}_{16}^*}$ is isomorphic to $Z_6 \times Z_3 \times Z_3$, that is $G_{\mathcal{F}_{16}^*} \cong Z_6 \times Z_3 \times Z_3$.

Another one of the cubic curves which given in Table 1 is
$\mathcal{F}_{20} = z^2y + zy^2 + x^3 + \theta y^3$ .The points of $PG(2,4)$ on $\mathcal{F}_{20}$ are $[0,0,1]$,. After calculations and help the computer , we are note that the number of matrices which are stabilizing of $\mathcal{F}_{20}$ and their order is 2880, and we can not write them, because they are too much.

Therefore, the stabilizer group of $\mathcal{F}_{20}$ which is denoted by $G_{\mathcal{F}_{20}}$ contains

- 73 matrix of order 2;
- 512 matrix of order 3;
- 180 matrix of order 4;
- 384 matrix of order 5;
- 958 matrix of order 6;
- 772 matrix of order 15;
- The identity matrix.

Another one of the cubic curves which given in Table 1 is $\mathcal{F}_{21} = z^2y + zy^2 + x^3 + xy^2 + \theta y^3$ .The points of $PG(2,4)$ on $\mathcal{F}_{21}$ are $[0,0,1],[1,\theta^2,1],[\theta^2,1,0],[\theta^2,\theta,1],[\theta^2,1,1]$. To find the stabilizer group of $\mathcal{F}_{21}$, we are doing calculations by help the computer, thus the transformation matrices which stabilizing of $\mathcal{F}_{21}$ and their order are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}1&0&1\\0&1&\theta\\0&0&1\end{pmatrix}:2,\begin{pmatrix}1&0&0\\1&1&\theta\\0&0&1\end{pmatrix}:2,\begin{pmatrix}1&0&1\\1&1&0\\0&0&1\end{pmatrix}:4$$

$$\begin{pmatrix}1&0&1\\0&1&\theta^2\\0&0&1\end{pmatrix}:2,\begin{pmatrix}1&0&0\\0&1&1\\0&0&1\end{pmatrix}:2,\begin{pmatrix}1&0&1\\1&1&1\\0&0&1\end{pmatrix}:4,\begin{pmatrix}1&0&0\\1&1&\theta^2\\0&0&1\end{pmatrix}:2$$

Therefore, the stabilizer groups of $\mathcal{F}_{21}$ which is denoted by $G_{\mathcal{F}_{21}}$ contains

- 5 matrices of order 2;
- 2 matrices of order 4;
- The identity matrix.

Form [6], $G_{\mathcal{F}_{21}}$ is isomorphic to $\boldsymbol{D_4}$, that is $G_{\mathcal{F}_{21}} \cong \boldsymbol{D_4}$.

Another one of the cubic curves which given in Table 1 is $\mathcal{F}_{27} = z^2y + zy^2 + x^3 + \theta xy^2 + \theta y^3$.The points of $PG(2,4)$ on $\mathcal{F}_{27}$ are $[0,0,1],[1,1,0],[\theta,1,1],[1,1,1],[\theta^2,1,0],[\theta,1,0],[\theta^2,1,1]$. To find the stabilizer group of $\mathcal{F}_{27}$, we are doing calculations by help the computer, thus the transformation matrices which stabilizing of $\mathcal{F}_{27}$ and their order are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1\ ,\ \begin{pmatrix}\theta^2&0&0\\0&\theta&\theta\\0&0&\theta\end{pmatrix}:6\ ,\ \begin{pmatrix}1&0&0\\0&1&1\\0&0&1\end{pmatrix}:2\ ,$$

$$\begin{pmatrix}\theta&0&0\\0&\theta^2&0\\0&0&\theta^2\end{pmatrix}:3\ \begin{pmatrix}\theta^2&0&0\\0&\theta&0\\0&0&\theta\end{pmatrix}:3\ ,\ \begin{pmatrix}\theta&0&0\\0&\theta^2&\theta^2\\0&0&\theta^2\end{pmatrix}:6$$

Therefore, the stabilizer groups of $\mathcal{F}_{27}$ which is denoted by $G_{\mathcal{F}_{27}}$ contains

- One matrix of order 2;
- 2 matrices of order 3;
- 2 matrices of order 6;
- The identity matrix.

Form [6], $G_{\mathcal{F}_{27}}$ is isomorphic to $\boldsymbol{Z_6}$, that is $G_{\mathcal{F}_{27}} \cong \boldsymbol{Z_6}$.

Another one of the cubic curves which given in Table 1 is $\mathcal{F}_{29} = z^2y + zy^2 + \theta x^3 + xy^2 + \theta y^3$ .The points of $PG(2,4)$ on $\mathcal{F}_{29}$ are $[0,0,1],[\theta,\theta,1],[\theta^2,\theta^2,1]$. After calculations and help the computer, we are note that the number of matrices which are stabilizing of $\mathcal{F}_{29}$ and their order is 288, and we can not write them, because they are too much .

Therefore, the stabilizer group of $\mathcal{F}_{29}$ which is denoted by $G_{\mathcal{F}_{29}}$ contains

- 27 matrix of order 2 ;
- 80 matrix of order 3 ;
- 36 matrix of order 4 ;
- 144 matrix of order 6 ;
- The identity matrix ;

Another one of the cubic curves which given in Table 1 is $\mathcal{F}_{40} = x^3 + \theta y^3 + \theta^2 z^3$.The points of $PG(2,4)$ on $\mathcal{F}_{40}$ are $[1,\theta^2,1],[\theta,1,1],[1,1,1],[\theta,\theta,1],[1,\theta,1],[\theta^2,\theta^2,1],[\theta^2,\theta,1],[\theta^2,1,1],[\theta,\theta^2,1]$. After calculations and help the computer, we are note that the number of matrices which are stabilizing of $\mathcal{F}_{40}$ and their order is 54, and we can not write them , because they are too much.

Therefore , the stabilizer groups of $\mathcal{F}_{40}$ which is denoted by $G_{\mathcal{F}_{40}}$ contains

- 9 matrices of order 2;
- 26 matrix of order 3;
- 18 matrix of order 6;
- The identity matrix.

Form [6], $G_{\mathcal{F}_{40}}$ is isomorphic to $\boldsymbol{Z_6} \times \boldsymbol{Z_3} \times \boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{40}} \cong \boldsymbol{Z_6} \times \boldsymbol{Z_3} \times \boldsymbol{Z_3}$.

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Another one of the cubic curves which given in Table 1 is $\mathcal{F}_{41} = xy^2 + x^2z + \theta yz^2$.The points of $PG(2,4)$ on $\mathcal{F}_{41}$ are $[1,0,0],[0,1,0],[0,0,1]$. After calculations and help the computer, we are note that the number of matrices which are stabilizing of $\mathcal{F}_{41}$ and their order is 54, and we can not write them , because they are too much .

Therefore, the stabilizer groups of $\mathcal{F}_{41}$ which is denoted by $G_{\mathcal{F}_{41}}$ contains

- 9 matrices of order 2;
- 26 matrix of order 3;
- 18 matrix of order 6;
- The identity matrix.

Form [6], $G_{\mathcal{F}_{41}}$ is isomorphic to $\boldsymbol{Z_6} \times \boldsymbol{Z_3} \times \boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{41}} \cong \boldsymbol{Z_6} \times \boldsymbol{Z_3} \times \boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table 1 is $\mathcal{F}_{42} = xy^2 + x^2z + \theta yz^2 + (x^3 + \theta y^3 + \theta^2 z^3 + \theta xyz)$ .The points of $PG(2,4)$ on $\mathcal{F}_{42}$ are $[\theta,1,1],[1,1,1],[\theta,\theta,1],[\theta^2,0,1],[\theta^2,1,0],[0,\theta^2,1]$. To find the stabilizer group of $\mathcal{F}_{42}$, we are doing calculations by help the computer, thus the transformation matrices which stabilizing of $\mathcal{F}_{42}$ and their order are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1\ ,\ \begin{pmatrix}0&0&\theta\\\theta^2&0&0\\0&\theta^2&0\end{pmatrix}:3,\ \begin{pmatrix}0&\theta&0\\0&0&\theta\\\theta^2&0&0\end{pmatrix}:3$$

Therefore, the stabilizer groups of $\mathcal{F}_{42}$ which is denoted by $G_{\mathcal{F}_{42}}$ contains

- 2 matrices of order 3;
- The identity matrix.

Form [6], $G_{\mathcal{F}_{42}}$ is isomorphic to $\boldsymbol{Z_3}$, that is $G_{\mathcal{F}_{42}} \cong \boldsymbol{Z_3}$.

Another one of the cubic curves which given in Table 1 is $\mathcal{F}_{48} = xy^2 + x^2z + x^3$The points of $PG(2,4)$ on $\mathcal{F}_{48}$ are $[0,1,0],[0,0,1],[1,1,0],[0,1,1],[\theta,1,1],[1,0,1],[0,\theta^2,1],[0,\theta,1],[\theta^2,1,1]$. To find the stabilizer group of $\mathcal{F}_{48}$, we are doing calculations by help the computer, thus the transformation matrices which stabilizing of $\mathcal{F}_{48}$ and their order are shown as follows :

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}:1,\begin{pmatrix}\theta^2&1&0\\0&\theta&0\\0&0&1\end{pmatrix}:3,\begin{pmatrix}\theta&1&0\\0&\theta^2&0\\0&0&1\end{pmatrix}:3,\begin{pmatrix}\theta^2&1&\theta\\0&\theta^2&0\\0&0&\theta^2\end{pmatrix}:2$$

$$\begin{pmatrix}\theta^2&\theta^2&\theta\\0&1&0\\0&0&\theta\end{pmatrix}:3,\begin{pmatrix}1&0&\theta^2\\0&\theta^2&0\\0&0&\theta\end{pmatrix}:3,\begin{pmatrix}\theta&1&\theta\\0&1&0\\0&0&\theta^2\end{pmatrix}:3,\begin{pmatrix}\theta&\theta^2&\theta\\0&\theta^2&0\\0&0&1\end{pmatrix}:3$$

$$\begin{pmatrix}\theta&\theta&\theta\\0&\theta&0\\0&0&\theta\end{pmatrix}:2,\begin{pmatrix}1&0&\theta\\0&\theta&0\\0&0&\theta^2\end{pmatrix}:3,\begin{pmatrix}1&1&\theta\\0&\theta^2&0\\0&0&\theta\end{pmatrix}:3,\begin{pmatrix}\theta^2&\theta&1\\0&\theta^2&0\\0&0&\theta^2\end{pmatrix}:2$$

$$\begin{pmatrix}\theta&\theta&\theta^2\\0&0&\theta^2\\0&1&0\end{pmatrix}:2,\begin{pmatrix}\theta&\theta&1\\0&0&1\\0&\theta^2&0\end{pmatrix}:2,\begin{pmatrix}\theta&\theta&\theta\\0&0&\theta\\0&\theta&0\end{pmatrix}:2,\begin{pmatrix}\theta^2&\theta&\theta^2\\0&0&1\\0&\theta&0\end{pmatrix}:2$$

$$\begin{pmatrix}\theta^2&\theta&1\\0&0&\theta^2\\0&\theta^2&0\end{pmatrix}:4,\begin{pmatrix}1&\theta^2&0\\0&0&\theta^2\\0&\theta&0\end{pmatrix}:4,\begin{pmatrix}\theta^2&0&1\\0&0&\theta\\0&1&0\end{pmatrix}:4,\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}:2$$

$$\begin{pmatrix}\theta & 0 & 1\\ 0 & 0 & \theta^2\\ 0 & 1 & 0\end{pmatrix}:4,\begin{pmatrix}\theta^2 & 1 & \theta\\ 0 & 0 & \theta^2\\ 0 & \theta^2 & 0\end{pmatrix}:4,\begin{pmatrix}1 & \theta & 1\\ 0 & 0 & \theta^2\\ 0 & \theta & 0\end{pmatrix}:2,\begin{pmatrix}1 & \theta & 0\\ 0 & 0 & \theta\\ 0 & \theta^2 & 0\end{pmatrix}:4$$

Therefore, the stabilizer groups of $\mathcal{F}_{48}$ which is denoted by $G_{\mathcal{F}_{48}}$ contains

- 9 matrices of order 2;
- 8 matrices of order 3;
- 6 matrices of order 4;
- The identity matrix.

Form [6], $G_{\mathcal{F}_{48}}$ is isomorphic to S_4 , that is $G_{\mathcal{F}_{48}} \cong S_4$.

Another one of the cubic curves which given in Table 1 is $\mathcal{F}_{51} = xy^2 + x^2z + yz^2$.The points of $PG(2,4)$ on $\mathcal{F}_{51}$ are $[1,0,0],[0,1,0],[0,0,1],[1,\theta^2,1],[\theta,1,1],[\theta,\theta,1],[1,\theta,1],[\theta^2,\theta^2,1],[\theta^2,1,1]$.After calculations and help the computer, we are note that the number of matrices which are stabilizing of $\mathcal{F}_{51}$ and their order is 54, and we can not write them, because they are too much.

Therefore , the stabilizer groups of $\mathcal{F}_{51}$ which is denoted by $G_{\mathcal{F}_{51}}$ contains

- 9 matrices of order 2;
- 26 matrices of order 3;
- 18 matrices of order 6;
- The identity matrix.

Form [6], $G_{\mathcal{F}_{51}}$ is isomorphic to $Z_6 \times Z_3 \times Z_3$, that is $G_{\mathcal{F}_{51}} \cong Z_6 \times Z_3 \times Z_3$.

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الخلاصة :

الاهداف الرئيسية لهذا البحث هو لإيجاد الزمر المثبتة للمنحنيات المكعبة حول الحقل المنتهي من الرتبة 4، ودراسة الخواص لهذه الزمر، وكذلك تشكيل كل المنحنيات المكعبة المختلفة، ومعرفة اي واحده منها هو كامل او لا. الاقواس من الدرجة الثانية والتي غمرت في منحنيات مكعبة ذات حجم زوجي تم تشكيلها. ورسم بعضها.