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ENTROPY IN SIMPLE DYNAMIC

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Abstract
In research we introduce new definition for entropy via n-block maps while the classical way adopts the definition for entropy via n-blocks

Preliminaries

Let (X,T,π) be a right topological transformation group , if $T=Z$ where Z is the group integers then (X,T,π) is called discrete flow.

The alphabet we adopt is $\zeta=\{0,1\}$, and we define the n-block by the function $\beta_n : I_p^q \to \zeta$ where $I_p^q=\{i\in Z : p\le i\le q : \mathrm{p,q}\in \mathrm{Z}\}$ and let B_n be the set all n-blocks. and define the n-block map f that her $f:\mathrm{B_n}\to\zeta$. And define the bi sequence as follows $\alpha : Z\to\zeta$ and let ζ^Z be space all the bi sequences. And is said for discrete flow (ζ^Z,σ) be a full shift if σ shift map. And is said for $(Y,\sigma|Y)$ that her sub shift if $Y\ne\phi, Y\subseteq\zeta^Z$ and Y invariant closed set under impact σ. Now we introduce classical definition for Entropy in dynamic system. if (X,σ_x) discrete flow ,we define the entropy of X as follows:

$h(X)=\lim_{n\to\infty}\frac{1}{n}\log_2|B_n(X)|$ where $|B_n(X)|$: number of blocks by length n .

Definition (1): let X be shift space we define $h_1(X)$ as follows $h_1(X)=\lim_{n\to\infty}\frac{1}{r^n}\log r^{|B_n(X)|}$

such that : r the number simple (number elements ζ) $r^{|B_n(X)|}$ the number n-block maps in sub shift.

Proposition(2): let X be shift space and for all $1\le r\le 10$, $n\ge 2$, $|B_n(X)|>1$ then $h_1\le h$.

Proof : from definition h , h_1

$$h(X)=\lim_{n\to\infty}\tfrac{1}{n}\log|B_n(X)|$$

$$h_1(X)=\lim_{n\to\infty}\frac{1}{r^n}\log r^{|B_n(X)|}$$

we will proof when $1\le r\le 10$ the most researches depend on $n\ge 2$ and $|B_n(X)|>1$

now when $n=2$ $h(X)=\lim_{n\to\infty}\frac{1}{2}\log|B_2(X)|$ and $h_1(X)=\lim_{n\to\infty}\frac{1}{r^2}\log r^{|B_2(X)|}$

this table for several values r and $|B_2(X)|$ explain the relation between h and h_1

r	$\lvert B_2(X)\rvert$	h	h_1
1	1	0	0
2	2	0.150514	0.150514
3	2	0.106026	0.150514
10	2	0.02	0.150514
10	3	0.03	0.238560
10	9	0.09	0.477121

we notice from through the table $h_1 \le h$ when $n = 2$ >

Now we suppose that the relation is correct when $\frac{\log r^{|B_k(X)|}}{r^k} \le \frac{\log|B_k(X)|}{k}$

we will prove that the relation is correct when $n = k+1$

and we will prove $\frac{\log r^{|B_{k+1}(X)|}}{r^{k+1}} \le \frac{\log|B_{k+1}(X)|}{k+1}$

and since $|B_{k+1}(X)| \le |B_k(X)| \cdot |B_1(X)|$......(1) .

Now we will prove for all $k \ge 1$ that $\frac{\log r^{|B_k(X)|}}{r^k} \le \frac{\log|B_k(X)|}{k+1}$

Notice that relation is correct $1 \le r \le 10$ in case $r = 1$ $0 = \log 1 \le \frac{\log|B_k(X)|}{2}$

And in case $k \ge 1, \mathrm{r} = 10$ then $\frac{\log 10^{|B_k(X)|}}{10^k} = \frac{\log 10^{|B_k(X)|} \cdot 1}{10^k} < \frac{1}{k+1} \le \frac{\log|B_k(X)|}{k+1}$

By result for all $k \ge 1, \mathrm{r} \le 10$ then $\frac{\log r^{|B_k(X)|}}{r^k} \le \frac{\log|B_k(X)|}{k+1}$......(2)

Since $|B_1(X)| \le r$ then $\frac{|B_1(X)|}{r} \cdot \frac{\log r^{|B_k(X)|}}{r^k} \le \frac{\log r^{|B_k(X)|}}{r^k}$.......(3)

From (2) and (3) we get on $\frac{|B_1(X)|}{r} \cdot \frac{\log r^{|B_k(X)|}}{r^k} \le \frac{\log|B_k(X)|}{k+1} \le \frac{\log|B_{k+1}(X)|}{k+1}$.......(4)

From (1) and (4)

$$\frac{\log r^{|B_{k+1}(X)|}}{r^{k+1}} \le \frac{\log r^{|B_1(X)|\cdot|B_k(X)|}}{r^{k+1}} = \frac{\cdot|B_1(X)|\log r^{|B_{k+1}(X)|}}{r^{k+1}} \le \frac{\log|B_{k+1}(X)|}{k+1}$$

Example(3)[1]

Let X be the golden Mean Shift then h(X) = 0.20898, h_1(X) = 0.186047227

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Example(4)[1]

Let X be the even shift space ,then $h(X) = 0.20898$, $h_1(X) = 0.186047227$

We notice the even shift and the golden mean shift have the same entropy.

Now we notice the relation between h and h_1 through the follows table:

h_1	h	X Shift space	No. example
0.186047227	0.20898	Golden mean shift	3
0.186047227	0.20898	Even shift space	4

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