Review: Nonlinear Differential Equations by Applying the Method of Homogeneous Balance

Authors Names

Muthanna Mishlish [a]
Abdulsalam F. Talak [b]
Israa A. Ibrahim [c]

Publication date: 7 /10 /2026

Keywords: *Nonlinear differential equations, Fractional partial differential equations, α-Differentiable, Homogeneous balance.*

ABSTRACT

The Homogeneous Balance Method (HBM) for solving nonlinear differential equations is thoroughly reviewed in this paper, along with its theoretical formulation and analytical technique for solving and analyzing initial boundary value problems represented by fractional partial differential equations (FPDEs). Although it can be challenging to find exact or close analytical solutions for these equations, especially when dealing with nonlinear problems, FPDEs are useful in explaining complex scientific events involving heritable traits. In order to resolve nonlinear health issues (FPDE), the homogeneous balancing method (HBM) is studied and developed in a novel way. HB provides a straightforward method that uses a closed-form solution to solve both linear and nonlinear fractional models. A number of fractional calculus concepts are included in this work's HBM, including fractional derivative techniques, which include rational, trigonometric, and hyperbolic solutions as well as examples of nonlinear FPDEs that are solved and analytically examined to demonstrate how the method operates. The results of the extended version of the HBM are examined and compared with those documented in the literature to demonstrate the efficacy of the method. Overall, this study suggests a practical and effective analytical method for resolving nonlinear FPDEs by extending the HB technique with result analysis.

1. Introduction

Patrial differential equation (PDE) have been developed and used in many areas of mathematical physics and engineering, and many events in sciences are modeled by partial differential equation. In physics for example, the heat flow is the best described for PDEs. In addition, for the most part in physical phenomena like fluid dynamics, electricity, plasma PDEs are used in physics and numerous other models to describe how physical systems behave, we use the laws of physics to describe these system, when we went to understand the behavior of physics to characterize this system, and many of these laws are written in the style of PDEs. Also, many of these laws happen to be linear PDEs or nonlinear PDEs. This study focuses on nonlinear PDE, most phenomena can be modeled using nonlinear PDEs. Solving nonlinear PDEs is still very difficult either numerically or analytically [1].

Therefore, it is necessary to develop and improve some nonlinear analytically methods to solve nonlinear PDEs [2],[3],[4],[5].

There are many methods for obtaining exact solution to nonlinear PDEs, such as the tanh-coth method [6], sine- cosine method [7], expel-function method [8], Jacobi elliptic function method [8], and

[a]Muthanna Mishlish,Department of Mathematics, Faculty of Education for pure sciences, University of Anbar, Anbar, Iraq, E-Mail:muthana.kh.m@uoanbar.edu.iq
[b]Abdulsalam F. Talak, Department of Mathematics, Faculty of Education for pure sciences, University of Anbar, Anbar, Iraq, E-Mail:abd19u2007@uoanbar.edu.iq
[c]Israa A. Ibrahim, Department of Mathematics, College of Open Education, Kirkuk Education Directorate, Kirkuk, Iraq, E-Mail:esraabdullah3@gmail.com

$(\frac{G'}{G})$ −expansion method [9] and others approaches. All methods have limitation in their application. In fact, no unified method can be used to handle all types of nonlinear PDEs.

Furthermore, even though fractional calculus isn't always required or appropriate for every application, it is a helpful mathematical instrument for describing systems with long-lasting memory effects. In many real-life scenarios, traditional calculation is still necessary, and fractional calculus calls for expertise and careful thought. The necessity of resolving complicated models in fractional calculus theory is highlighted by this observation. Using this theory to accurately model complicated real-world events is only one step; solving these models also necessitates a deep understanding of the technological and physical aspects of the issue at hand [10]. Using the answers to these models, researchers can forecast outcomes and create interventions to improve the system, which can offer insightful information about the behavior of the system being studied. However, solving these models can be challenging due to the intricate dynamics involved. Analytical techniques and algorithms have been developed to address these models, and researchers are working to improve these methods. For advancement in many disciplines, such as engineering, physics, and economics, accurate solutions to complex models are essential and the homogeneous balancing method's popularity and importance in computing precise traveling wave solutions [1]. Fan introduced this arithmetic approach, which has since been refined by other academics [11].

The strategy is to find a not linear the improvement that can be applied to precisely solve the current issue [12]. In order to better understand the study in the following sections, we now provide some fundamental ideas and definitions related to fractional calculus theory.

1.1. Definition .[13]:Let $\boldsymbol{\psi}: [\mathbf{0}, \infty) \rightarrow \boldsymbol{R}$ become a function, subsequently its corresponding derivative that is fractionally equivalent to conformable which is $\boldsymbol{\alpha}$ th order is

$$T_\alpha \psi(\mu) = \lim_{\varepsilon \to 0} \frac{\psi(\mu + \varepsilon \mu^{1-\alpha}) - \psi(\mu)}{\varepsilon}, \quad (\alpha \in (0,1), \mu > 0). \tag{1}$$

If $l > 0$ as well as the function ψ has become α-differentiable within $(0,1)$.

The adaptable derivative derived at 0 is defined if the function ψ is α-differentiable in $(0,1)$ over $l > 0$ as well as if $\lim_{\mu \to 0^+} \psi^{(n)}(\mu)$ is present

$$\psi^n(0) = \lim_{\mu \to 0^+} \psi^{(n)}(\mu). \tag{2}$$

Furthermore, the definition of the conformable integral component of function ψ is

$$I_\alpha^l \psi(\mu) = \int_l^\mu \frac{\psi(t)}{t^{1-\alpha}} dt, \quad l \geq 0 \text{ and } \alpha \in (0,1]. \tag{3}$$

1.1.Theorem.[14]**:** Assume that for $\alpha \in (0,1]$, the functions u as well as v are α-differentiable beginning at any given point $\mu > 0$. Next, there is the characteristics listed below:

i) $D_\alpha(au + bv) = aD_\alpha(u) + bD_\alpha(v)$ for all $a, b \in R$.
ii) $D_\alpha(\mu^n) = n\mu^{n-\alpha}$ for all $n \in R$.
iii) $D_\alpha(k) = 0$ for all $v(\mu) = k$ (constant function).
iv) $D_\alpha(uv) = uD_\alpha(v) + vD_\alpha(u)$.
v) $D_\alpha\left(\frac{u}{v}\right) = \frac{vD_\alpha(u) - uD_\alpha(v)}{v^2}$,
vi) Additionally, if the function v is differentiable, then $D_\alpha(v)(\mu) = \mu^{1-\alpha}\frac{dv}{d\mu}$.

2. An explanation of the HB Analysis and Method

To illustrate the basic idea underlying our method, consider a nonlinear conformable fractional equation

$$F\left(u, u_\mu, u_t, D_t^\alpha u, D_\mu^\alpha u, D_{tt}^{2\alpha} u, D_{\mu\mu}^{2\alpha} u, \dots\right), \quad (0 < \alpha < 1). \tag{4}$$

In this case, F is a representation of any polynomials of a function as well as its incomplete fractional derivatives. The phases of the recommended approach are as follows:

Step 1: Considering the transformation

$$u(\mu, t) = U(\xi), \quad \xi = k\frac{\mu^\alpha}{\alpha} + c\frac{t^\alpha}{\alpha}, \tag{5}$$

in which k and c are unidentified integer constants. The nonlinear inverse FPDE in eq. (4) had been converted into the nonlinear normal differential equation.

$$H(U, U', U'', U''', \dots) = 0. \tag{6}$$

In this case, $U(\xi)$ determines H, as well as the prime represents its derived products with respect to ξ.

Step 2: Assume the solution to Equation (6) is as follows:

$$U(\xi) = \sum_{i=0}^{n} q_i \varphi^i, \tag{7}$$

in which n is an advantageous integer chosen by weighing the nonlinear term and the maximum order derivative period in equation (6) and $a_i (i = 0,1,2, \dots n)$ are constants to be computed later.

Step 3: The algebraic system of equations can be obtained by substituting equation (7) for equation (6). Maple can be used to solve this system, and the solutions to Equation (6) are all exact solutions to Equation (4).

2.1Definition **:** Assume that $\psi: R \to R$, $t \to \psi(t)$ is a continuous function. The following is the expression for its Riemann-Liouville derivative of fractional order η:

$$D_a^{\eta}\psi(t) = \frac{1}{\Gamma(1-\eta)}\frac{d}{d\mu}\int_a^{\mu}\frac{\psi(t)}{(\mu-t)^{\eta}}dt\ ,\ \ 0<\eta<1, \tag{8}$$

$$D_a^{\eta}\psi(t) = \frac{1}{s-\eta}\frac{d^s}{d\mu^s}\int_a^{\mu}\frac{\psi(t)}{(\mu-t)^{\eta-s+1}}dt\ ,\ \ (s-1\le\eta\le s, s\in N). \tag{9}$$

From the definition given above, we obtain

$$D^{\eta}t^s = \frac{\Gamma(1+s)}{\Gamma(1+s-\eta)}t^{s-\eta}\ \ ,\ \ s>-1\ ,\ \ 0<\eta<1. \tag{10}$$

The following are some essential properties of the Riemann-Lowville derivative:

If $\psi: R\to R$ ongoing function, after which its fractional derivatives is expressed as an integral alongside regard to $(d\mu)^{\eta}$

$$D_{\mu}^{\eta}g(\mu) = \frac{1}{\Gamma(\eta)}\int_0^{\mu_1}(g-\tau)^{\eta-1}g(\tau)d(\tau) = \frac{1}{\Gamma(1+\eta)}\frac{d}{d\mu}\int_0^{\mu}g(\tau)\big(d(\tau)\big)^{\eta},\ \ (0<\eta<1).$$

The fractional derivatives' definitions for the standard linear combination for the standardized constants a as well as b and the functions $\psi(\mu)$ as well as $g(\mu)$ is as follows:

$$D_{\mu}^{\eta}\big(a\psi(\mu)+bg(\mu)\big) = aD_{\mu}^{\eta}\big(\psi(\mu)\big)+bD_{\mu}^{\eta}\big(g(\mu)\big),$$

for $g(\mu)=\mu^k$, the derivative of fractions is

$$D_{\mu}^{\eta}\big((\mu)^k\big) = \frac{\Gamma(1+k)}{\Gamma(1+k-\eta)}(\mu)^{k-\eta}.$$

***2.2.Definition*:**Consider the function $\psi:[0,\infty)\to R$. The ηth order fractional conformable derivative of that derivative is then

$$T_{\eta}\psi(\mu) = \lim_{\varepsilon\to 0}\frac{\psi(\mu+\varepsilon\mu^{1-\eta})-\psi(\mu)}{\varepsilon}, \tag{11}$$

where $\eta\in(0,1)$ and it applies to everyone $\mu>0$.

The definition of the conformable derived at zero is just as follows if the function ψ is η-differentiable in $(0,1)$ for $l>0$ and $\lim_{\mu\to 0^+}\psi^n(\mu)$ exists

$$\psi^n(0) = \lim_{\mu\to 0^+}\psi^n(\mu). \tag{12}$$

Additionally, the definition of a conformable integral of function ψ is:

$$I_{\eta}^{l}\psi(\mu) = \int_l^{\mu}\frac{\psi(t)}{t^{1-\eta}}dt,\ \ (l\ge 0,\eta\in(0,1]). \tag{13}$$

3. Prior Research on the Homogeneous Balance Method (HBM)

To find a single wave solutions for two distinct types of variant-based business problems, Wang introduced a systematic version of the (HBM) in 1995. This approach provided a solid framework for researching and resolving non-linear equations.[15] (Wang, 1995) and in 1996 Wang et al. used the solitary wave solution to solve the dispersive long wave equations in (2+1) dimensions, the coupled KdV equations, and the approximation equations of long water waves.[16]. In 2009, Moussa and Shikh used similarity reductions and the Becklund transformation to discover new precise answers with variable coefficients to the general Hirota-Satsuma paired KdV system. These solutions took the form of elliptical Jacobic functions, hyperbolic functions, and periodic functions.[17].

In 2010, Rady and Kumar obtained new traveling-phase solutions for the BBM formula using the homogeneous equilibrium method and nonlinear diffusion-reaction (D-R) equations. Rational and periodic solutions were successfully obtained by the methods used to find polynomial solutions to traveling waves. Numerous physical, chemical, biological, and social issues involve these equations.[18][19]. In 2014, Zayed and Alurrf used the homogeneous balancing approach to find precise solutions to a number of nonlinear equations for evolution in computational physics, including the Laxity equation, the Sawada-Kotera formula, Ito's equation, the Caudrey-Dodd-Gibbon formula, and the Kaup-Kupershmidt equation. There are several uses for these equations in nonlinear optics along with quantum physics. It was shown that this approach was successful in generating these precise answers [20].

The Benny-Luke and Vakhnenko-Parkes equations were used by Ibrahim et al. in 2019 to demonstrate the research for solutions of nonlinear partial the application of differential equations using going waves. The repeated homogeneous balance method has been highlighted as an effective method for increasing the number of going wave solutions for partial differential equations that are not linear [1].

In 2021, Radha and Duraisamy made some observations about the same approach and talked about the kind of general exact solution for the Burgers and Gardner equations. are acquired through the homogeneous balance approach[21] .

4. Prior research using the modified homogeneous method

El-Wakil et al. (2007) proposed a modified version of the HBM in an attempt to find the travelling wave solution of the nonlinear evaluation equation. In this version, the HBM is successfully applied to the Riccati equation solution to generate multiple new travelling waves.[22].

Amer used (HBM) to solve the reduced non-ODE and the Riccati equation $\varphi' = a\varphi^2 + b\varphi + c$ in 2011. Several new precise traveling wave solutions for different nonlinear partial differential equations related to nonlinear dispersive dissipative equations were then given for the (KdV) and (MKdV) equations, and the outcomes were compared [23].

In order to solve the Riccati equation $\varphi' = a\varphi^2 + b\varphi + c$ and find new travelling wave solutions of the Maccari system equation, Mirzazadeh (2014) employed a modified version of (HBM) to construct exact traveling wave solutions of the Maccari system[24].

In the same year, Eslami et al. (2014) introduced a modified (HBM) method for solving the modified Zakharov–Kuznetsov equation. This revised approach relies on the Riccati equation $\varphi' = a\varphi^2 + b\varphi + c$, which has the following solution.

Case1: Let

$$\varphi = \sum_{i=0}^{m} b_i \tanh^i \xi,$$

or

$$\sum_{i=0}^{m} b_i \coth^i \xi.$$

Case2: Let

$$\varphi = \begin{cases} -\sqrt{-c}\tanh\sqrt{-\xi} & i\psi \quad c < 0, \\ \varphi = \dfrac{-1}{c} & i\psi \quad c = 0, \\ \sqrt{c}\tanh\sqrt{\xi} & i\psi \quad c > 0. \end{cases}$$

Case3: Let

$$\varphi = A_0 + \sum_{I=1}^{m} \sinh^{i-1}(A_i \sinh\omega + B_i \cosh\omega).$$

The exact traveling wave solutions of the nonlinear modified Zakharov-Kuznetsov equation then include soliton and periodic solutions [25].

The Riccati equation $\omega' = p\varphi^2 + \mu\varphi + k$ was applied by Taghizadah et al. in 2014 using the modified HB method. They then used the solution of the Joseph's Equation standard (TRLW) equation to find a variety of precise moving waveform solutions, including rational and periodic-like solutions [26].

In order to solve the nonlinear PDEs, the Kortewage de Veris equation, the scalar Boussinesq equation, and the Kaup–Boussinesq equations, Nguyen (2015) employed a different equation of modification of the homogeneous balance method, where $\varphi = 2\log\theta$, where $\theta = \exp[\frac{\alpha(t)\mu+\beta(t)}{2a}]$. The exact solutions were obtained [27].

The outcome of Abdelsalam's 2017 proposal to derive novel precise traveling wave answers to partial differential equations (PDEs) of shallow-water-based equations using the extended homogeneous balancing approach is as follows.

Case1: Let

$$\omega = \sum_{i=0}^{m} b_i \tanh^i \xi,$$

or

$$\sum_{i=0}^{m} b_i \coth^i \xi.$$

Case2: Let

$$\omega = \begin{cases} -\sqrt{-k}\tanh\sqrt{-k}\xi & i\psi \quad k < 0, \\ \dfrac{-1}{k} & i\psi \quad k = 0, \\ \sqrt{k}\tan\sqrt{k} & i\psi \quad k > 0. \end{cases}$$

Case3: Let

$$\omega = A_0 + \sum_{I=1}^{m} \sinh^{i-1}(A_i \sinh\omega + B_i \cosh\omega).$$

Case4: Let

$$\omega = \sum_{I=1}^{m} A_i \tanh^i(p_1)\xi.$$

Abdelsalam (2017) states that we think there are precise solutions to the fourth request Boussinesq equation as well as the nonlinear broader shallow-water equation [28].

In 2021, Ibrahim et al. applied the Riccati equation $\omega' = \ln(\rho)(\mu\varphi^2 + \nu\varphi + \lambda)$ using a modified HB method. They then obtained numerous exact going-wave solutions by integrating exponential, hyperbolic, and trigonometric as well as rational functions, using the answer derived from the Klein-Gordon equation alongside Kerr's leadership law of non-linearity [29].

5. Benefits

The suggested analytical approach of HBM would significantly aid in overcoming the challenges of current approaches, including VIM, ADM, and others. For instance, when resolving FPDEs, the suggested approach will assist us in reducing the multifaceted nature of the unsure nonlocal derivative. The proposed methods would pave the way towards setting reliable mathematical models of nonlinear form or of high-order to describe the complex real problems in a way that makes the model more realistic.

6. Restrictions

The study focuses on the application and development of FPDEs to five examples of problems involving FPDEs under the conformable definitions of fractional. Experimental work would be carried out on several examples of nonlinear physical and engineering problems. The experimental work on examples of FPDEs in this thesis will be conducted by using Maple software.

7. Conclusions

Homogeneous balance method (HB) was used as a basic tool for finding new exact solution for various types of fractional nonlinear partial deferential equation. Numerous nonlinear FPDEs were successfully solved using the homogeneous equilibrium method. First, the general traveling equation as well as the fractional partial differential equation are solved using the homogeneous equilibrium method wave solution, which are hyperbolic, trigonometric, and rational solutions, that have successfully obtained. The solitary wave solution is derived from these functions and Some solutions of HB method are the solution obtained by the $\frac{G'}{G}$ -expansion method and F – expansion method under special conditions. Were taken into consideration to demonstrate how effective and potent these modified methods are for determining the precise solutions for a variety of nonlinear PDE classes.

References

[1] I. A. Ibrahim, W. M. Taha, and M. S. M. Noorani, “Homogenous balance method for solving exact solutions of the nonlinear Benny -luke equation and Vakhnenko-Parkes equation,” pp. 1–5, 2019.

[2] M. K. Elboree, “Hyperbolic and trigonometric solutions for some nonlinear evolution equations,” *Commun. Nonlinear Sci. Numer. Simul.*, vol. 17, no. 11, pp. 4085–4096, Nov. 2012, doi: 10.1016/j.cnsns.2012.03.029.

[3] B. Kafash, A. Delavarkhalafi, and S. M. Karbassi, “Application of variational iteration method for Hamilton–Jacobi–Bellman equations,” *Appl. Math. Model.*, vol. 37, no. 6, pp. 3917–3928, Mar. 2013, doi: 10.1016/j.apm.2012.08.013.

[4] M. Krisnangkura, S. Chinviriyasit, and W. Chinviriyasit, “Analytic study of the generalized Burger’s–Huxley equation by hyperbolic tangent method,” *Appl. Math. Comput.*, vol. 218, no. 22, pp. 10843–10847, Jul. 2012, doi: 10.1016/j.amc.2012.04.044.

[5] Q. Shi, Q. Xiao, and X. Liu, “Extended wave solutions for a nonlinear Klein–Gordon–Zakharov system,” *Appl. Math. Comput.*, vol. 218, no. 19, pp. 9922–9929, Jun. 2012, doi: 10.1016/j.amc.2012.03.079.

[6] A. M. Wazwaz, “The extended tanh method for new solitons solutions for many forms of the fifth-order KdV equations,” *Appl. Math. Comput.*, vol. 14, no. 2, pp. 1002–1014, 2007, doi: 10.1016/j.amc.2006.07.002.

[7] S. Bibi and S. T. Mohyud-Din, “Traveling wave solutions of KdVs using sine–cosine method,” *J. Assoc. Arab Univ. Basic Appl. Sci.*, vol. 15, no. 1, pp. 90–93, Apr. 2014, doi: 10.1016/j.jaubas.2013.03.006.

[8] M. K. Elboree, “The Jacobi elliptic function method and its application for two component BKP hierarchy equations,” *Comput. Math. with Appl.*, vol. 62, no. 12, pp. 4402–4414, Dec. 2011, doi: 10.1016/j.camwa.2011.10.015.

[9] M. Shakeel and S. T. Mohyud-Din, “New (G ′/ G)-expansion method and its application to the Zakharov-Kuznetsov–Benjamin-Bona-Mahony (ZK–BBM) equation,” *J. Assoc. Arab Univ. Basic Appl. Sci.*, vol. 18, no. 1, pp. 66–81, Oct. 2015, doi: 10.1016/j.jaubas.2014.02.007.

[10] D. Jawad Hashim *et al.*, “New Series Approach Implementation for Solving Fuzzy Fractional Two-Point Boundary Value Problems Applications,” *Math. Probl. Eng.*, vol. 2022, pp. 1–13, 2022, doi: 10.1155/2022/7666571.

[11] J. H. Choi, H. Kim, and R. Sakthivel, “Exact solution of the Wick-type stochastic fractional coupled KdV

equations," *J. Math. Chem.*, vol. 52, no. 10, pp. 2482–2493, Nov. 2014, doi: 10.1007/s10910-014-0406-1.
[12] M. A. Tashtoush, I. A. Ibrahim, W. M. Taha, M. H. Dawi, A. F. Jameel, and E. A. Az-Zo'bi, "Various Closed-Form Solitonic Wave Solutions of Conformable Higher-Dimensional Fokas Model in Fluids and Plasma Physics," *Iraqi J. Comput. Sci. Math.*, vol. 5, no. 3, 2024, doi: 10.52866/ijcsm.2024.05.03.027.
[13] R. Khalil, M. Al Horani, A. Yousef, and M. Sababheh, "A new definition of fractional derivative," *J. Comput. Appl. Math.*, vol. 264, pp. 65–70, 2014, doi: 10.1016/j.cam.2014.01.002.
[14] A. Ali, M. A. Iqbal, Q. M. UL-Hassan, J. Ahmad, and S. T. Mohyud-Din, "An efficient technique for higher order fractional differential equation," *Springerplus*, vol. 5, no. 1, p. 281, 2016, doi: 10.1186/s40064-016-1905-2.
[15] M. Wang, "Solitary wave solutions for variant Boussinesq equations," *Phys. Lett. A*, vol. 199, no. 3–4, pp. 169–172, 1995, doi: 10.1016/0375-9601(95)00092-H.
[16] M. Wang, Y. Zhou, and Z. Li, "Application of a homogeneous balance method to exact solutions of nonlinear equations in mathematical physics," *Phys. Lett. A*, vol. 216, no. 1–5, pp. 67–75, 1996, doi: 10.1016/0375-9601(96)00283-6.
[17] M. H. M. Moussa and R. M. El Shikh, "Two Applications of the Homogeneous Balance Method for Solving the Generalized Hirota-Satsuma Coupled KdV System with Variable Coefficients B ¨ acklund transformation," *Int. J. Nonlinear Sci.*, vol. 7, no. 1, pp. 29–38, 2009.
[18] A. S. A. Rady, E. S. Osman, and M. Khalfallah, "The homogeneous balance method and its application to the Benjamin – Bona – Mahoney (BBM) equation," vol. 217, pp. 1385–1390, 2010, doi: 10.1016/j.amc.2009.05.027.
[19] R. Kumar, R. S. Kaushal, and A. Prasad, "Solitary wave solutions of selective nonlinear diffusion-reaction equations using homogeneous balance method," *Indian Acad. Sci.*, vol. 75, no. 4, pp. 607–616, 2010.
[20] E. M. E. Zayed and K. A. E. Alurrf, "THE HOMOGENEOUS BALANCE METHOD AND ITS APPLICATIONS FOR FINDING THE EXACT SOLUTIONS FOR NONLINEAR EVOLUTION EQUATIONS," *Ital. J. pure Appl. Math. – n*, vol. 33, pp. 307–318, 2014.
[21] B. Radha and C. Duraisamy, "RETRACTED ARTICLE: The homogeneous balance method and its applications for finding the exact solutions for nonlinear equations," *J. Ambient Intell. Humaniz. Comput.*, vol. 12, no. 6, pp. 6591–6597, Jun. 2021, doi: 10.1007/s12652-020-02278-3.
[22] S. A. El-Wakil, E. M. Abulwafa, A. Elhanbaly, and M. A. Abdou, "The extended homogeneous balance method and its applications for a class of nonlinear evolution equations," *Chaos, Solitons & Fractals*, vol. 33, no. 5, pp. 1512–1522, Aug. 2007, doi: 10.1016/j.chaos.2006.03.010.
[23] S. Amer, "Homogeneous Balance Method for Solving Nonlinear Dispersive Dissipative Equation," *Al-Mustansiriyah J. Sci*, vol. 22, no. 6, 2011.
[24] M. Mirzazadeh, "The extended homogeneous balance method and exact 1-soliton so- lutions of the Maccari system," *Comput. Methods Differ. Equations*, vol. 2, no. 2, pp. 83–90, 2014.
[25] M. Eslami, B. Fathi Vajargah, and M. Mirzazadeh, "Exact solutions of modified Zakharov-Kuznetsov equation by the homogeneous balance method," Faculty of Engineering, Ain Shams University, 2014. doi: 10.1016/j.asej.2013.06.005.
[26] N. Taghizadeh, "Exact travelling wave solutions of Joseph-Egri (TRLW) equation by the extended homogeneous balance method Exact travelling wave solutions of Joseph-Egri (TRLW) equation by the extended homogeneous balance method," *Int. J. Appl. Math. Comput.*, vol. 4(1) 2012, pp. 96–104, 2014.
[27] L. T. K. Nguyen, "Modified homogeneous balance method: Applications and new solutions," *Chaos, Solitons & Fractals*, vol. 73, pp. 148–155, 2015, doi: 10.1016/j.chaos.2015.01.017.
[28] U. M. Abdelsalam, "Traveling wave solutions for shallow water equations," Elsevier B.V., 2017. doi: 10.1016/j.joes.2017.02.002.
[29] I. A. Ibrahim, R. A. Hameed, W. M. Taha, and M. A. Rasheed, "New Path of Popularized Homogeneous Balance Method and Travelling Wave Solutions of a Nonlinear Klein-Gordon Equation," *Iraqi J. Sci.*, pp. 2656–2666, 2022, doi: 10.24996/ijs.2022.63.6.31.