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Picture Fuzzy Bi-Ideal of a Ring

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ABSTRACT

In this paper , the notion of picture fuzzy bi-ideal of a ring are defined and study some of their properties and the related concepts of them

1. Introduction

In 1971, S.Lajos and F. szasz [1] introduce the notions of the concept bi-ideals in associative rings , while Chelvam and Ganesan [2] in the same year introduce the notions of the same concepts but in near ring(not a ring), in 1981,N.kuroki [3] define the concept of fuzzy bi- idealin semigroups and gave some properties of it. S.K.Datta in 1997in his thesis introduce a detailed explanation of bi- ideal and fuzzy bi – ideals in a rings , also he introduce in 2009 in [6 -7] the notions of anti fuzzy bi-ideals of a rings and the lower level sets of anti fuzzy bi – ideals , the concept intuitionistic fuzzy bi-ideals of a rings was introduced by A.Duggal and P.K. Sharma in 2019[8].

B.coung in 2014 study the concept of picture fuzzy set[9] and gave most of its properties , P.Panpho and P. Yiarayong in [10]also S. Dogra and M. Pal [11]study the picture fuzzy subrings and ideals. In this paper we introduce the notion of concepts of picture fuzzy bi-ideals of rings and some related examples and theorems of it .

2. Preliminaries

In this sectionr we stated the preliminaries of picture fuzzy subring and ideals and some concepts connected with them.

Definition 2.1. let R be a ring. A picture fuzzy set $A = \{< x, \mu_A(x), \eta_A(x), V_A(x) > | X \in R\}$ is said to be picture fuzzy subring (PFSR) of R if :

i) $\mu_A(x-y) \geq min\{\mu_A(x), \mu_A(y)\}, \eta_A(x-y) \geq min\{\eta_A(x), \eta_A(y)\}, V_A(x-y) \leq max\{V_A(x), \eta_A(y)\}$
ii) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}, \eta_A(xy) \geq \min\{\eta_A(x), \eta_A(y)\}, V_A(xy) \leq \max\{V_A(x), V_A(y)\}$

Definition 2.2. A picture fuzzy set $A = \{< x, \mu_A(x), \eta_A(x), V_A(x) > | X \in R\}$ of a ring R is said to be

i) Picture fuzzy left ideal of R (PFLI) if
   1) $\mu_A(x-y) \geq min\{\mu_A(x), \mu_A(y)\}$
      $\mu_A(xy) \geq \mu_A(y)$

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2) $\eta_A(x-y) \geq min\{\eta_A(x), \eta_A(y)\}$
   $\eta_A(xy) \geq \eta_A(y)$

3) $V_A(x-y) \leq max\{V_A(x), \eta_A(y)\}$
   $V_A(xy) \leq V_A(y)$,for all $x, y \in R$

ii) Picture fuzzy right ideal of R (PFRI) if

1) $\mu_A(x-y) \geq min\{\mu_A(x), \mu_A(y)\}$
   $\mu_A(xy) \geq \mu_A(x)$

2) $\eta_A(x-y) \geq min\{\eta_A(x), \eta_A(y)\}$
   $\eta_A(xy) \geq \eta_A(x)$

3) $V_A(x-y) \leq max\{V_A(x), \eta_A(y)\}$
   $V_A(xy) \leq V_A(x)$, for all $x, y \in R$

iii) Picture fuzzy ideal of R (PFI) if

1) $\mu_A(x-y) \geq min\{\mu_A(x), \mu_A(y)\}$
   $\mu_A(xy) \geq \max\{\mu_A(x), \mu_A(y)\}$

2) $\eta_A(x-y) \geq min\{\eta_A(x), \eta_A(y)\}$
   $\eta_A(xy) \geq \max\{\eta_A(x), \eta_A(y)\}$

3) $V_A(x-y) \leq max\{V_A(x), \eta_A(y)\}$
   $V_A(xy) \leq \min\{V_A(x), V_A(y)\}$

Definition 2.3. If A is a PFS of a ring R then (α, β, λ)-*cut* of A is a crisp subset $C_{(\alpha,\beta,\lambda)}(A)$ of the *PFS* A is given by

$$C_{(\alpha,\beta,\lambda)}(A) = \{x \in R | \mu_A(x) \geq \alpha, \eta_A(y) \geq \beta, V_A(x) \leq \lambda\}$$

where $\alpha, \beta, \lambda \in [0,1]$ and $\alpha + \beta + \lambda \leq 1$

Proposition 2.4. Let A be a *PFS* of a ring R. Then A is *PFSR(PFI)* of R if and only if $C_{(\alpha,\beta,\lambda)}(A)$ is subring (ideal) of R, for all $\alpha, \beta, \lambda \in [0,1]$ with $\alpha + \beta + \lambda \leq 1$

Definition 2.5. A subring Sof a ring R is called a bi-ideal of R if $SRS \subseteq S$ holds, where SRS is the additive of all elements of the form $srs, s \in S$ and $r \in R$

Definition2.6. A non-empty fuzzy subset μ of a ring R (i.e. $\mu(x) \neq 0$ for some $x \in R$) is called an fuzzy bi-ideal of R if :

i) $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$
ii) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$ and
iii) $\mu(xyz) \geq \min\{\mu(x), \mu(z)\}$, for all $x, y, z \in R$

3. Picture fuzzy bi-ideal of a ring

Definition 3.1. A picture fuzzy set $A = \{< x, \mu_A(x), \eta_A(x), V_A(x) > | X \in R\}$ of a ring R is said to be picture fuzzy bi-ideal of R (PFBI) if R if :

i) $\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\}$

$\eta_A(x-y) \geq \min\{\eta_A(x), \eta_A(y)\}$

$V_A(x-y) \leq \max\{V_A(x), V_A(y)\}, \ \forall x, y \in R.$

ii) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$

$\eta_A(xy) \geq \min\{\eta_A(x), \eta_A(y)\}$

$V_A(xy) \leq \max\{V_A(x), V_A(y)\}, \ \forall x, y \in R.$

iii) $\mu_A(xry) \geq \min\{\mu_A(x), \mu_A(y)\}$

$\eta_A(xry) \geq \min\{\eta_A(x), \eta_A(y)\}$

$V_A(xry) \leq \max\{V_A(x), V_A(y)\}, \ \forall r, x, y \in R.$

Example 3.2. Let R be the ring of all 2×2 matrices over the ring of integers with respect to the matrix addition and multiplication. Let $A = \{< x, \mu_A(x), \eta_A(x), V_A(x) > | X \in R\}$ be a picture fuzzy subset of R defined as follows:

$$\mu_A\left[\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right] = \begin{cases} 0.6 & ,if\ a=b=c=d=0 \\ 0.2 & ,if\ a\neq 0, even\ and\ b=c=d=0 \\ 0.3 & ,if\ a\neq 0, odd\ and\ b=c=d=0 \\ 0, & if\ all\ other\ cases \end{cases}$$

$$\eta_A\left[\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right] = \begin{cases} 0.2 & ,if\ a=b=c=d=0 \\ 0.4 & ,if\ a\neq 0, even\ and\ b=c=d=0 \\ 0.3 & ,if\ a\neq 0, odd\ and\ b=c=d=0 \\ 0 & ,if\ all\ other\ cases \end{cases}$$

$$V_A\left[\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right] = \begin{cases} 0 & ,if\ a=b=c=d=0 \\ 0.3 & ,if\ a\neq 0, even\ and\ b=c=d=0 \\ 0.3 & ,if\ a\neq 0, odd\ and\ b=c=d=0 \\ 0.5 & ,if\ all\ other\ cases \end{cases}$$

Then A is a picture fuzzy bi-ideal of R.

Theorem 3.3. Every PFLI(PFRI) of a ring is PFBI of R.

Proof. In view of (3.1),we need only to prove the condition (iii)

Let A be PFLI of the ring R. And $x, y \in R$, $a \in A$

$$\mu_A(xay) \geq \mu_A\big((xa)y\big) \geq \mu_A(y)$$
$$\geq min\{\mu_A(x), \mu_A(y)\}$$

$$\eta_A(xay) \geq \eta_A\big((xa)y\big) \geq \eta_A(y)$$
$$\geq min\{\eta_A(x), \eta_A(y)\}$$

$$V_A(xay) \leq V_A\big((xa)y\big) \leq V_A(y)$$
$$\leq max\{V_A(x), V_A(y)\}$$

Hence A is *PFBI* of R.

The right case is proved in the same way.

Theorem 3.4. Let $A = (\mu_A, \eta_A, V_A)$ be PFBI of a field F. Then A is of the form

$$\mu_A(x) = \begin{cases} \mu_A(1) & ,if\ x \neq 0 \\ \mu_A(0) & ,if\ x = 0 \end{cases}$$

$$\eta_A(x) = \begin{cases} \eta_A(1) & ,if\ x \neq 0 \\ \eta_A(0) & ,if\ x = 0 \end{cases}$$

And $V_A(x) = \begin{cases} V_A(1) & ,if\ x \neq 0 \\ V_A(0) & ,if\ x = 0 \end{cases}$

Where $\mu_A(1) \leq \mu_A(0), \eta_A(1) \leq \eta_A(0), V_A(1) \geq V_A(0)$

Proof. Let $A = \{< x, \mu_A(x), \eta_A(x), V_A(x) > | x \in F\}$ be a PFBI of a field F, and Let $0 \neq x \in F$ be any element. Then

$$\mu_A(x) = \mu_A(1.\times .1\ ) \geq min\{\mu_A(1), \mu_A(1)\} = \mu_A(1) = \mu_A(1.1)$$

$$\mu_A\{(xx^{-1})(x^{-1}x)\} = \mu_A\{x(x^{-1}x^{-1})x\} \geq min\{\mu_A(x), \mu_A(x)\} = \mu_A(x)$$

i.e. $\mu_A(x) = \mu_A(1)$

$$\eta_A(x) = \eta_A(1.\times .1\ ) \geq min\{\eta_A(1), \eta_A(1)\}$$
$$= \eta_A(1) = \eta_A(1.1)$$

$$\eta_A\{(xx^{-1})(x^{-1}x)\} = \eta_A\{x(x^{-1}x^{-1})x\} \geq min\{\eta_A(x), \eta_A(x)\} = \eta_A(x)$$

i.e. $\eta_A(x) = \eta_A(1)$

and $V_A(x) = V_A(1.\times .1) \leq max\{V_A(1), \eta_A(1)\} = V_A(1) = V_A(1.1)$

$V_A\{(xx^{-1})(x^{-1}x)\} = V_A\{x(x^{-1}x^{-1})x\} \leq max\{V_A(x), V_A(x)\} = V_A(x)$

i.e. $V_A(x) = V_A(1)$

Corollary 3.5. If A is PFBI of a field F such that $\mu_A(0) = \mu_A(1)$, $\eta_A(0) = \eta_A(1)$ and $V_A(0) = V_A(1)$, then A is constant .

Theorem 3.6. If A, B be two PFBI'S of a ring , then A ∩ B is PFBI of ring R.

Proof. Let $A = (\mu_A, \eta_A, V_A)$ and $B = (\mu_B, \eta_B, V_B)$ be two PFBI'S of a ring R.

Let $x, y \in A \cap B$ be any element. Then

$\mu_{A\cap B}(x-y) = min\{\mu_A(x-y), \mu_B(x-y)$

$\geq min\{min\{\mu_A(x), \mu_A(y)\}, min\{\mu_B(x), \mu_B(y)\}\}$

$= min\{min\{\mu_A(x), \mu_B(x)\}, min\{\mu_A(y), \mu_B(y)\}\}$

$= min\{\mu_{A\cap B}(x), \mu_{A\cap B}(y)\}$

Thus $\mu_{A\cap B}(x-y) \geq \min\{\mu_{A\cap B}(x), \mu_{A\cap B}(y)\}$

by the same way, We can prove

$\eta_{A\cap B}(x-y) \geq \min\{\eta_{A\cap B}(x), \eta_{A\cap B}(y)\}$

Also we can show that

$V_{A\cap B}(x-y) = max\{V_A(x-y), V_B(x-y)$

$\leq max\{max\{V_A(x), V_A(y)\}, max\{V_B(x), V_B(y)\}\}$

$= max\{min\{V_A(x), V_B(x)\}, max\{V_A(y), V_B(y)\}\}$

$= max\{\mu_{A\cap B}(x), \mu_{A\cap B}(y)\}$

Thus $V_{A\cap B}(x-y) \leq max\{V_{A\cap B}(x), V_{A\cap B}(y)\}$

Also, $\mu_{A\cap B}(xy) = min\{\mu_A(xy), \mu_B(xy)$

$\geq min\{min\{\mu_A(x), \mu_A(y)\}, min\{\mu_B(x), \mu_B(y)\}\}$

$= min\{min\{\mu_A(x), \mu_B(x)\}, min\{\mu_A(y), \mu_B(y)\}\}$

$= min\{\mu_{A\cap B}(x), \mu_{A\cap B}(y)\}$

Thus $\mu_{A\cap B}(xy) \geq min\{\mu_{A\cap B}(x), \mu_{A\cap B}(y)\}$

By the same way

$$\eta_{A\cap B}(xy) \geq min\{\eta_{A\cap B}(x), \eta_{A\cap B}(y)\}$$

$$V_{A\cap B}(xy) \leq max\{V_{A\cap B}(x), V_{A\cap B}(y)\}$$

Next; let $x, y \in A \cap B$ and $r \in R$ be any element, then

$$\begin{aligned}\mu_{A\cap B}(xry) &= min\{\mu_A(xry), \mu_B(xry)\\ &\geq min\{min\{\mu_A(x), \mu_A(y)\}, min\{\mu_B(x), \mu_B(y)\}\}\\ &= min\{min\{\mu_A(x), \mu_B(x)\}, min\{\mu_A(y), \mu_B(y)\}\}\\ &= min\{\mu_{A\cap B}(x), \mu_{A\cap B}(y)\}\end{aligned}$$

Thus $\mu_{A\cap B}(xry) \geq min\{\mu_{A\cap B}(x), \mu_{A\cap B}(y)\}$

Similarly, we can show that

$\eta_{A\cap B}(xry) \geq min\{\eta_{A\cap B}(x), \eta_{A\cap B}(y)\}$ and $V_{A\cap B}(xry) \leq max\{V_{A\cap B}(x), V_{A\cap B}(y)\}$

Hence $A \cap B$ is *PFBI* of ring R.

Corollary 3.7. Intersection of an arbitrary family of PFBI'S of a ring R is again a PFBI of R.

Theorem 3.8. Let A be PFLI and B be PFRI of a ring R then A ∩ B is PFBI of a ring R.

Proof. Let $A = (\mu_A, \eta_A, V_A)$ and $B = (\mu_B, \eta_B, V_B)$ be two PFBI'S of a ring R. Let $x, y \in A \cap B$ be any element. Then

$$\begin{aligned}\mu_{A\cap B}(x-y) &= min\{\mu_A(x-y), \mu_B(x-y)\\ &\geq min\{min\{\mu_A(x), \mu_A(y)\}, min\{\mu_B(x), \mu_B(y)\}\}\\ &= min\{min\{\mu_A(x), \mu_B(x)\}, min\{\mu_A(y), \mu_B(y)\}\}\\ &= min\{\mu_{A\cap B}(x), \mu_{A\cap B}(y)\}\end{aligned}$$

Thus $\mu_{A\cap B}(x-y) \geq min\{\mu_{A\cap B}(x), \mu_{A\cap B}(y)\}$

Similarly, we can show that

$$\eta_{A\cap B}(x-y) \geq min\{\eta_{A\cap B}(x), \eta_{A\cap B}(y)\} \text{ and } V_{A\cap B}(x-y) \leq max\{V_{A\cap B}(x), V_{A\cap B}(y)\}$$

Also $$\mu_{A\cap B}(xy) = min\{\mu_A(xy), \mu_B(xy)\}\text{……..(1)}$$

Since A is PFLI and B is PFRI of the ring R.

Therefore, we have $\mu_A(xy) \geq \mu_A(y)$ and $\mu_B(xy) \geq \mu_B(x)$

$$min\{\mu_A(xy), \mu_B(xy)\} \geq min\{\mu_A(x), \mu_B(y)\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

As $A \cap B \subseteq A$ and $A \cap B \subseteq B$

So $\mu_{A\cap B}(x) \leq \mu_A(x)$ and $\mu_{A\cap B}(y) \leq \mu_B(y)$

Which implies that $\min\{\mu_A(x), \mu_B(y)\} \geq \min\{\mu_{A\cap B}(x), \mu_{A\cap B}(y)\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3)$

Therefore from (1),(2) and (3) we have

$$\mu_{A\cap B}(xy) = min\{\mu_A(xy), \mu_B(xy)\}$$
$$\geq min\{min\{\mu_A(x), \mu_B(y)\}$$
$$\geq min\{\mu_{A\cap B}(x), \mu_{A\cap B}(y)\}$$

Thus, we have $\mu_{A\cap B}(xy) \geq min\{\mu_{A\cap B}(x), \mu_{A\cap B}(y)\}$

Similarly , we can show that

$\eta_{A\cap B}(xy) \geq min\{\eta_{A\cap B}(x), \eta_{A\cap B}(y)\}$ and $V_{A\cap B}(xy) \leq max\{V_{A\cap B}(x), V_{A\cap B}(y)\}$

Further, let $x, y \in A \cap B$ and $r \in R$, then

$$\mu_{A\cap B}(xry) = min\{\mu_A(xry), \mu_B(xry)\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

But $\mu_A(xry) \geq \mu_A\big((xr)y\big) \geq \mu_A(y)$ and $\mu_B(xry) \geq \mu_B\big(x(ry)\big) \geq \mu_B(y)$ implies that

$$min\{\mu_A(xry), \mu_B(xry)\} \geq min\{\mu_A(y), \mu_B(x)\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

As $A \cap B \subseteq A$ and $A \cap B \subseteq B$.

So $\mu_{A\cap B}(y) \leq \mu_A(y)$ and $\mu_{A\cap B}(x) \leq \mu_B(x)$

Which implies that $min\{\mu_A(y), \mu_B(x)\} \geq min\{\mu_{A\cap B}(y), \mu_{A\cap B}(x)\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$

From (4), (5) and (6) we get

$$\mu_{A\cap B}(xry) \geq min\{\mu_{A\cap B}(y), \mu_{A\cap B}(x)\}$$

Similarly , we can show that

$\eta_{A\cap B}(xry) \geq min\{\eta_{A\cap B}(y), \eta_{A\cap B}(x)\}$ and $V_{A\cap B}(xry) \leq max\{V_{A\cap B}(y), V_{A\cap B}(x)\}$

Hence $A \cap B$ is *PFBI* of ring R.

Theorem 3.9. Let A be *PFS* of a ring R, then A is *PFBI* of R if and only if $C_{\alpha,\beta,\lambda}(A)$ is bi-ideal of R, for all $\alpha,\beta,\lambda \in [0,1]$ with $\alpha+\beta+\lambda \leq 1$, where $\mu_A(0) \geq \alpha, \eta_A(0) \geq \beta, V_A(0) \leq \lambda$.

Proof. Firstly, let A be *PFBI* of a ring R. Then by definition of (α,β,λ)-*cut* of A,

we have $C_{(\alpha,\beta,\lambda)}(A) = \{x \in R | \mu_A(x) \geq \alpha, \eta_A(x) \geq \beta, V_A(x) \leq \lambda\}$

Since $\mu_A(0) \geq \alpha, \eta_A(0) \geq \beta, V_A(0) \leq \lambda$this implies $C_{\alpha,\beta,\lambda}(A) \neq \phi$

Let $x, y \in C_{\alpha,\beta,\lambda}(A)$ be any element, then

$\mu_A(x) \geq \alpha, \mu_A(y) \geq \alpha$, $\eta_A(x) \geq \beta$, $\eta_A(y) \geq \beta$ and $V_A(x) \leq \lambda, V_A(y) \leq \lambda$

This mean that $min\{\mu_A(x), \mu_A(y)\} \geq \alpha$, $min\{\eta_A(x), \eta_A(y)\} \geq \beta$ and $max\{V_A(x), V_A(y)\} \leq \lambda$

Now

$$\mu_A(x-y) \geq min\{\mu_A(x), \mu_A(y)\} \geq \alpha$$

$$\eta_A(x-y) \geq min\{\eta_A(x), \eta_A(y)\} \geq \beta \text{ and } V_A(x-y) \leq max\{\mu_A(x), \mu_A(y)\} \leq \lambda$$

Also

$$\mu_A(xy) \geq min\{\mu_A(x), \mu_A(y)\} \geq \alpha \quad , \quad \eta_A(xy) \geq min\{\eta_A(x), \eta_A(y)\} \geq \beta$$

And $V_A(xy) \leq max\{\mu_A(x), \mu_A(y)\} \leq \lambda$

This implies that $x-y \in C_{\alpha,\beta,\lambda}(A)$ *and* $xy \in C_{\alpha,\beta,\lambda}(A)$

Thus $C_{\alpha,\beta,\lambda}(A)$ is a subring of R.

Next, let $x, y \in C_{\alpha,\beta,\lambda}(A)$ and $r \in R$ be any element.

Then

$$\mu_A(xry) \geq min\{\mu_A(x), \mu_A(y)\} \geq \alpha \ , \ \eta_A(xry) \geq min\{\eta_A(x), \eta_A(y)\} \geq \beta$$

And $V_A(xry) \leq max\{\mu_A(x), \mu_A(y)\} \leq \lambda$

Hence xry $\in C_{\alpha,\beta,\lambda}(A)$.

Thus $C_{\alpha,\beta,\lambda}(A)R\ C_{\alpha,\beta,\lambda}(A) \subseteq C_{\alpha,\beta,\lambda}(A)$

Hence $C_{\alpha,\beta,\lambda}(A)$ is bi-ideal of R.

Conversely, let $C_{\alpha,\beta,\lambda}(A)$ be bi-ideal of R, for all $\alpha,\beta,\lambda \in [0,1]$ with $\alpha+\beta+\lambda \leq 1$, where $\mu_A(0) \geq \alpha, \eta_A(0) \geq \beta, V_A(0) \leq \lambda$.

Then $C_{\alpha,\beta,\lambda}(A)$ is subring of R.

And $C_{\alpha,\beta,\lambda}(A)R\ C_{\alpha,\beta,\lambda}(A) \subseteq C_{\alpha,\beta,\lambda}(A)$.

This implies that A is picture fuzzy subring of R (by Theorem(2.5))

Let $x, r, y \in R$ and such that $\mu_A(xry) < min\{\mu_A(x), \mu_A(y)\}$

choose $\alpha \in [0,1]$ such that $\mu_A(xry) < \alpha < min\{\mu_A(x), \mu_A(y)\}$

This Implies that $\mu_A(x) \geq \alpha, \mu_A(y) \geq \alpha$

Hence $\eta_A(x) \geq 1 - \mu_A(x) \geq 1 - \alpha$

$\mathrm{V_A(x)} \leq 1 - \mu_A(\mathrm{x}) \leq 1 - \alpha$

And $V_A(x) \leq 1 - \mu_A(x) \leq 1 - \alpha$

Thus $x, y \in C_{\alpha,1-\alpha,1-\alpha}(A)$ and $xry \in C_{\alpha,1-\alpha,1-\alpha}(A)$ so $\mu_A(xry) \geq \alpha$,

Which is a contradiction.

So $\mu_A(xry) \geq min\{\mu_A(x), \mu_A(y)\}$

Similarly, we have

$\eta_A(xry) \geq min\{\eta_A(x), \eta_A(y)\}$ and $\mathrm{V_A(xry)} \leq \max\{\mathrm{V_A(x)}, \mathrm{V_A(y)}\}$

Hence A is *PFBI* of the ring R.

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