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Pythagorean Fuzzy Ideals of ρ-Algebras

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ABSTRACT

This paper develops a Pythagorean fuzzy ideal framework for ρ-algebras. The approach combines the algebraic ideal condition with two grades satisfying the Pythagorean restriction. PFSs are used to represent the fuzzy data, while PFIs encode the ideal requirements and PFLSs connect them with ordinary ideals. The behavior of PFIs under homomorphic inverse images and Cartesian products is then examined. A PPFI notion is introduced through a coupled grade condition, which yields a direct characterization by prime level ideals. The resulting framework provides a concise way to organize ideal constructions in the Pythagorean setting of ρ-algebras.

1. Introduction

Algebraic systems built around implication-type binary operations form a family in which order and ideal conditions can be compared across several related structures. Classical BCK-algebras provide one of the early settings for ideal theory, while BCI-algebras and Q-algebras offer neighboring frameworks with closely related implication behavior . The investigation of pseudo Q-algebras and pseudo ideals extends this line toward structures in which the algebraic operation interacts with additional ideal constraints . These developments suggest a natural question: how can the same ideal information be represented when membership is not restricted to a binary yes-or-no description?

Fuzzy set theory provides a direct language for this purpose. In the classical model of Zadeh, membership is measured by a value in $[0,1]$, and subsequent work established a large mathematical framework for manipulating such grades . Intuitionistic fuzzy sets added a separate non-membership grade, allowing positive and negative information to be recorded simultaneously . Hesitant fuzzy sets and generalized orthopair models broadened this two-sided description of uncertainty and supplied further ways of representing incomplete information .

PFS models are particularly useful when two grades are required, while the admissible region is broader than the usual intuitionistic triangle. Their defining condition is expressed through the squares of the membership and non-membership grades, namely

$$\alpha_F(u)^2 + \beta_F(u)^2 \leq 1.$$

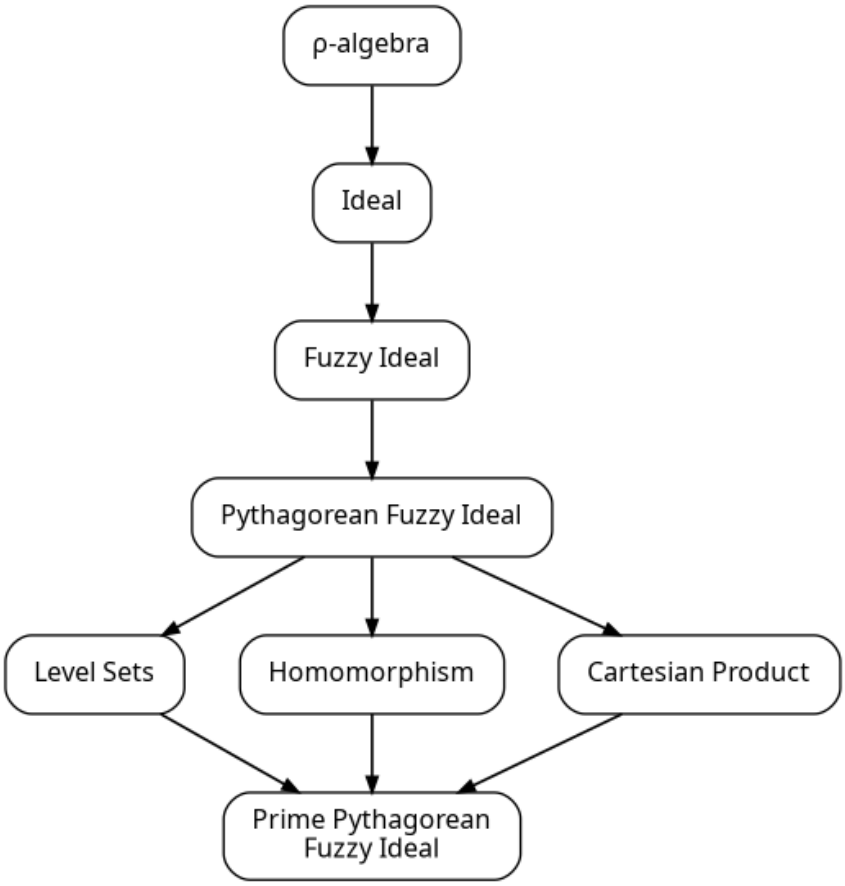
This model, introduced by Yager, has subsequently appeared in problems involving aggregation, information processing, and decision analysis . From the viewpoint of algebra, this formulation suggests replacing each classical membership statement by a pair of compatible grade inequalities.

The purpose of this paper is to formulate that two-grade viewpoint for ideals of ρ-algebras. We use a uniform notation throughout the development and concentrate on constructions that lead directly to later results. The main questions are fourfold. First, which inequalities specify a PFI in the present algebraic setting? Second, when does a pair of grades produce ordinary ideals at fixed thresholds? Third, how are

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fuzzy ideals transported by homomorphisms and combined by Cartesian products? Finally, what is an appropriate prime condition that remains faithful to prime level ideals? Related studies on fuzzy ideals in algebraic systems show that this interaction between ideal closure and grade functions is a useful general method .

The paper is organized around these questions. Section 2 fixes the algebraic terminology. Section 3 introduces PFIs, derives their level-set characterization, and treats inverse images and products. Section 4 develops the PPFI notion and its relationship with prime level ideals. Section 5 closes with the main consequences and possible directions for continuation.



Main construction path used in the paper.

2. Preliminaries on ρ-Algebras

Only the notions required later are recalled. The symbols u, v, w denote generic elements, whereas a, b, c, d are used in finite illustrations.

Definition 2.1 [17]. A ρ-algebra is a triple $(\mathcal{A}, \diamond, 0)$ in which $\mathcal{A}$ is a nonempty set, $\diamond$ is a binary operation on $\mathcal{A}$, and $0 \in \mathcal{A}$ satisfies, for all $u, v, w \in \mathcal{A}$,

1. $(u \diamond v) \diamond w = (u \diamond w) \diamond (v \diamond w)$;

2. $u \diamond u = 0$;

3. $u \diamond v = 0$ and $v \diamond u = 0 \Rightarrow u = v$.

Remark 2.2. The first identity is the basic interaction between three elements and will be inherited componentwise when Cartesian products are considered below. The distinguished element 0 plays the role of the algebraic reference point in every ideal definition.

Definition 2.3 [17]. For a ρ-algebra $(\mathcal{A},\diamond,0)$, define a relation on $\mathcal{A}$ by $u \leq v \Leftrightarrow u \diamond v = 0$.

Definition 2.4 [1]. A nonempty subset $I \subseteq \mathcal{A}$ is an ideal of $(\mathcal{A},\diamond,0)$ when $0 \in I$ and $u \diamond v \in I$, $v \in I \Rightarrow u \in I$, for all $u, v \in \mathcal{A}$.

Definition 2.5 [17]. Let $(\mathcal{A},\diamond,0)$ and $(Y,\star,0')$ be ρ-algebras. A map $f: \mathcal{A} \to Y$ is a homomorphism if $f(u \diamond v) = f(u) \star f(v)$, $u, v \in \mathcal{A}$.

Definition 2.6 [13]. A Pythagorean fuzzy set (PFS) on $\mathcal{A}$ is a pair $F = (\alpha_F, \beta_F)$, where $\alpha_F, \beta_F: \mathcal{A} \to [0,1]$ satisfy $\alpha_F(u)^2 + \beta_F(u)^2 \leq 1$, $u \in \mathcal{A}$.

Definition 2.7 [13]. For a A PFS $F = (\alpha_F, \beta_F)$ and $\lambda, \kappa \in [0,1]$ with $\lambda^2 + \kappa^2 \leq 1$, its (λ,κ)-level set is $F_{(\lambda,\kappa)} = \{u \in \mathcal{A}: \alpha_F(u) \geq \lambda,\ \beta_F(u) \leq \kappa\}$.

Definition 2.8. An ideal P of $(\mathcal{A},\diamond,0)$ is prime when $u \diamond v \in P \Rightarrow u \in P$ or $v \in P$, for all $u, v \in \mathcal{A}$.

3. Pythagorean Fuzzy Ideals of ρ-Algebras

Definition and Basic Properties

The PFI conditions express the ordinary ideal requirement through the two grade functions simultaneously: membership is lower-bounded and non-membership is upper-bounded.

Definition 3.1. Let $(\mathcal{A},\diamond,0)$ be a ρ-algebra. A PFS $F = (\alpha_F, \beta_F)$ is a **Pythagorean fuzzy ideal** if, for all $u, v \in \mathcal{A}$,

1. $\alpha_F(0) \geq \alpha_F(u)$, $\beta_F(0) \leq \beta_F(u)$;

2. $\alpha_F(u) \geq \min\{\alpha_F(u \diamond v), \alpha_F(v)\}$;

3. $\beta_F(u) \leq \max\{\beta_F(u \diamond v), \beta_F(v)\}$.

Example 3.2. Consider the four-element set $\mathcal{A} = \{0, a, b, c\}$ with the operation

$\diamond$	0	a	b	c
0	0	0	0	0
a	a	0	0	0
b	b	b	0	0
c	c	c	c	0

which satisfies the defining identities of a ρ-algebra. Define $F = (\alpha_F, \beta_F)$ by

u	0	a	b	c
$\alpha_F(u)$	0.95	0.75	0.55	0.35
$\beta_F(u)$	0.10	0.25	0.45	0.60

For each $u \in \mathcal{A}$, $\alpha_F(u)^2 + \beta_F(u)^2 \leq 1$. The first condition follows from the largest membership and smallest non-membership grades occurring at 0. Direct substitution in the defining table verifies the remaining two inequalities. Hence F is a Pythagorean fuzzy ideal of $\mathcal{A}$.

Remark 3.3. This finite example provides a direct check of the PFI conditions; the theorem below gives the corresponding general interpretation through threshold sets.

Lemma 3.4. Let $F = (\alpha_F, \beta_F)$ be a PFS on $\mathcal{A}$. For $u, v \in \mathcal{A}$, define $\lambda = \min\{\alpha_F(u), \alpha_F(v)\}$, $\kappa = \max\{\beta_F(u), \beta_F(v)\}$. Then $\lambda^2 + \kappa^2 \leq 1$.

Proof. The value κ is either $\beta_F(u)$ or $\beta_F(v)$. Suppose first that $\kappa = \beta_F(u)$. Since $\lambda \leq \alpha_F(u)$, we obtain

$$\lambda^2 + \kappa^2 \leq \alpha_F(u)^2 + \beta_F(u)^2 \leq 1.$$

When $\kappa = \beta_F(v)$, the same argument with v gives the conclusion. □

Theorem 3.5. Let $F = (\alpha_F, \beta_F)$ be a PFS on a ρ-algebra $(\mathcal{A}, \diamond, 0)$. Then F is a PFI if and only if every nonempty PFLS $F_{(\lambda,\kappa)}$ is an ideal of $\mathcal{A}$.

Proof. Assume first that F is a Pythagorean fuzzy ideal and let $F_{(\lambda,\kappa)} \neq \varnothing$. Choose $v \in F_{(\lambda,\kappa)}$. By the first condition of the definition,

$$\alpha_F(0) \geq \alpha_F(v) \geq \lambda, \qquad \beta_F(0) \leq \beta_F(v) \leq \kappa,$$

so $0 \in F_{(\lambda,\kappa)}$.

Next suppose that $u \diamond v$ and v belong to the same level set. Then

$$\alpha_F(u \diamond v), \alpha_F(v) \geq \lambda, \qquad \beta_F(u \diamond v), \beta_F(v) \leq \kappa.$$

The defining inequalities therefore give

$$\alpha_F(u) \geq \min\{\alpha_F(u \diamond v), \alpha_F(v)\} \geq \lambda,$$

$$\beta_F(u) \leq \max\{\beta_F(u \diamond v), \beta_F(v)\} \leq \kappa.$$

Hence $u \in F_{(\lambda,\kappa)}$, proving that the PFLS is an ideal.

Conversely, assume that every nonempty PFLS of F is an ideal. Fix $u \in \mathcal{A}$. Because F satisfies the Pythagorean condition, $F_{(\alpha_F(u),\beta_F(u))}$ is admissible and contains u. It is therefore nonempty, and by assumption it contains 0. Consequently,

$$\alpha_F(0) \geq \alpha_F(u), \qquad \beta_F(0) \leq \beta_F(u).$$

For arbitrary $u, v \in \mathcal{A}$, set

$$\lambda = \min\{\alpha_F(u \diamond v), \alpha_F(v)\}, \qquad \kappa = \max\{\beta_F(u \diamond v), \beta_F(v)\}.$$

The lemma gives $\lambda^2 + \kappa^2 \leq 1$. Moreover, $u \diamond v, v \in F_{(\lambda,\kappa)}$. Since the PFLS is an ideal, $u \in F_{(\lambda,\kappa)}$. Therefore

$$\alpha_F(u) \geq \min\{\alpha_F(u \diamond v), \alpha_F(v)\},$$

and

$$\beta_F(u) \leq \max\{\beta_F(u \diamond v), \beta_F(v)\}.$$

Thus F is a Pythagorean fuzzy ideal. □

Corollary 3.6. If F is a PFI, then every nonempty PFLS $F_{(\lambda,\kappa)}$ is closed under the ideal implication condition. Moreover, for fixed $u, v \in \mathcal{A}$, the threshold parameters used in the proof of the theorem are admissible automatically.

Proof. The first statement is exactly the forward implication of the theorem. The second follows from the lemma. □

Definition 3.7. Let $f: \mathcal{A} \to Y$ and let $G = (\alpha_G, \beta_G)$ be a PFS on Y. Its inverse image is again a two-grade fuzzy set $f^{-1}(G) = \left(\alpha_{f^{-1}(G)}, \beta_{f^{-1}(G)}\right)$, where, for every $u \in \mathcal{A}$, $\alpha_{f^{-1}(G)}(u) = \alpha_G\big(f(u)\big)$, $\beta_{f^{-1}(G)}(u) = \beta_G\big(f(u)\big)$.

Theorem 3.8. Let $f: (\mathcal{A}, \diamond, 0) \to (Y, \star, 0')$ be a homomorphism of ρ-algebras. If G is a PFI of Y, then $f^{-1}(G)$ is a PFI of $\mathcal{A}$.

Proof. Put $F = f^{-1}(G)$. For every $u \in \mathcal{A}$,

$$\alpha_F(u)^2 + \beta_F(u)^2 = \alpha_G\big(f(u)\big)^2 + \beta_G\big(f(u)\big)^2 \leq 1.$$

Since $f(0) = 0'$, the first ideal condition follows from that of G:

$$\alpha_F(0) = \alpha_G(0') \geq \alpha_G\big(f(u)\big) = \alpha_F(u),$$

$$\beta_F(0) = \beta_G(0') \leq \beta_G\big(f(u)\big) = \beta_F(u).$$

For the remaining conditions, let $u, v \in \mathcal{A}$. Then

$$\alpha_{F\times G}(u_1, u_2) \geq \min\{\min\{\alpha_F(u_1 \diamond v_1), \alpha_F(v_1)\}, \min\{\alpha_G(u_2 \star v_2), \alpha_G(v_2)\}\}.$$

$$\alpha_{F\times G}(u_1, u_2) \geq \min\{\alpha_{F\times G}\big((u_1, u_2) \diamond_\times (v_1, v_2)\big), \alpha_{F\times G}(v_1, v_2)\}.$$

Similarly,

$$\beta_F(u) \leq \max\{\beta_F(u \diamond v), \beta_F(v)\}.$$

Hence F satisfies all PFI conditions. □

Proposition 3.9. Let $f: \mathcal{A} \to Y$ be any map and G a PFS on Y. For every admissible pair (λ, κ), $(f^{-1}(G))_{(\lambda,\kappa)} = f^{-1}\big(G_{(\lambda,\kappa)}\big)$.

Proof. For $u \in \mathcal{A}$,

$$\alpha_{F\times G}(u_1,u_2) \geq \min\{\min\{\alpha_F(u_1 \diamond v_1), \alpha_F(v_1)\}, \min\{\alpha_G(u_2 \star v_2), \alpha_G(v_2)\}\}.$$

$$\alpha_{F\times G}(u_1,u_2) \geq \min\{\alpha_{F\times G}\big((u_1,u_2) \diamond_\times (v_1,v_2)\big), \alpha_{F\times G}(v_1,v_2)\}.$$

Definition 3.10. For ρ-algebras $(\mathcal{A},\diamond,0)$ and $(Y,\star,0')$, define the componentwise operation on $\mathcal{A} \times Y$ by $(u_1,u_2) \diamond_\times (v_1,v_2) = (u_1 \diamond v_1,\ u_2 \star v_2)$. The distinguished element is $(0,0')$.

Remark 3.11. The product operation is chosen componentwise so that the defining identity of a ρ-algebra is inherited from the corresponding identity in each factor. Thus the product construction does not introduce a new algebraic law; it combines the existing ones in parallel.

Definition 3.12. Let $F = (\alpha_F, \beta_F)$ and $G = (\alpha_G, \beta_G)$ be PFIs of $\mathcal{A}$ and Y. Their Cartesian product is $F \times G = (\alpha_{F\times G}, \beta_{F\times G})$, where $\alpha_{F\times G}(u,v) = \min\{\alpha_F(u), \alpha_G(v)\}$, $\beta_{F\times G}(u,v) = \max\{\beta_F(u), \beta_G(v)\}$.

Theorem 3.13. If F and G are PFIs of $(\mathcal{A},\diamond,0)$ and $(Y,\star,0')$, respectively, then $F \times G$ is a PFI of $\big(\mathcal{A} \times Y, \diamond_\times, (0,0')\big)$.

Proof. At the distinguished element,

$$\alpha_{F\times G}(0,0') = \min\{\alpha_F(0), \alpha_G(0')\} \geq \min\{\alpha_F(u), \alpha_G(v)\} = \alpha_{F\times G}(u,v),$$

and

$$\beta_{F\times G}(0,0') = \max\{\beta_F(0), \beta_G(0')\} \leq \max\{\beta_F(u), \beta_G(v)\} = \beta_{F\times G}(u,v).$$

Let $(u_1,u_2),(v_1,v_2) \in \mathcal{A} \times Y$. The ideal properties in the factors give

$$\alpha_F(u_1) \geq \min\{\alpha_F(u_1 \diamond v_1), \alpha_F(v_1)\},$$

$$\alpha_G(u_2) \geq \min\{\alpha_G(u_2 \star v_2), \alpha_G(v_2)\}.$$

Taking the minimum and using the definition of $\alpha_{F\times G}$ yields

$$\alpha_{F\times G}(u_1,u_2) \geq \min\{\min\{\alpha_F(u_1 \diamond v_1), \alpha_F(v_1)\}, \min\{\alpha_G(u_2 \star v_2), \alpha_G(v_2)\}\}.$$

$$\alpha_{F\times G}(u_1,u_2) \geq \min\{\alpha_{F\times G}\big((u_1,u_2) \diamond_\times (v_1,v_2)\big), \alpha_{F\times G}(v_1,v_2)\}.$$

The non-membership inequality follows in the dual way by taking maxima. Hence $F \times G$ is a PFI. □

Proposition 3.14. For every admissible pair (λ,κ), the PFLS of the Cartesian product satisfies $(F \times G)_{(\lambda,\kappa)} = F_{(\lambda,\kappa)} \times G_{(\lambda,\kappa)}$.

Proof. By definition, $(u,v) \in (F \times G)_{(\lambda,\kappa)}$ precisely when

$$\min\{\alpha_F(u), \alpha_G(v)\} \geq \lambda$$

and

$$\max\{\beta_F(u), \beta_G(v)\} \leq \kappa.$$

These two conditions are equivalent to $\alpha_F(u), \alpha_G(v) \geq \lambda$ and $\beta_F(u), \beta_G(v) \leq \kappa$, which is exactly the condition that $u \in F_{(\lambda,\kappa)}$ and $v \in G_{(\lambda,\kappa)}$. □

4. Prime Pythagorean Fuzzy Ideals

Definition and Level-Set Characterization

The PPFI construction is chosen so that the ordinary prime-ideal condition is recovered at each admissible threshold. For this reason, the two grade inequalities are coupled rather than treated as unrelated alternatives.

Definition 4.1. Let $(\mathcal{A}, \diamond, 0)$ be a ρ-algebra. A Pythagorean fuzzy ideal $F = (\alpha_F, \beta_F)$ is called a **prime Pythagorean fuzzy ideal** when, for all $u, v \in \mathcal{A}$, at least one of the following two alternatives holds: $\alpha_F(u \diamond v) \leq \alpha_F(u), \quad \beta_F(u \diamond v) \geq \beta_F(u)$, or $\alpha_F(u \diamond v) \leq \alpha_F(v), \quad \beta_F(u \diamond v) \geq \beta_F(v)$.

Remark 4.2. The coupled formulation is stronger than imposing only the two separate bounds $\alpha_F(u \diamond v) \leq \max\{\alpha_F(u), \alpha_F(v)\}, \quad \beta_F(u \diamond v) \geq \min\{\beta_F(u), \beta_F(v)\}$. The advantage of coupling is that the same element is responsible for both threshold conditions. This is exactly what is required when a fuzzy level set is to become a prime ordinary ideal.

Example 4.3. Let $\mathcal{A} = \{0, a, b, c\}$ with the operation from Example 3.2 and define constant grades by $\alpha_F(u) = 0.8, \quad \beta_F(u) = 0.2, \quad u \in \mathcal{A}$. The Pythagorean condition is immediate since $0.8^2 + 0.2^2 = 0.68 \leq 1$. For every pair of elements, both alternatives in the prime definition hold with equality. Hence F is a PPFI.

Theorem 4.4. Let $F = (\alpha_F, \beta_F)$ be a PFI of a ρ-algebra $\mathcal{A}$. Then F is a PPFI if and only if every nonempty PFLS $F_{(\lambda,\kappa)}$ is a prime ideal of $\mathcal{A}$.

Proof. Assume first that F is prime in the sense of Definition 4.1 and let $F_{(\lambda,\kappa)} \neq \varnothing$. Take $u, v \in \mathcal{A}$ with $u \diamond v \in F_{(\lambda,\kappa)}$. Then

$$\alpha_F(u \diamond v) \geq \lambda, \qquad \beta_F(u \diamond v) \leq \kappa.$$

By the prime condition, either

$$\alpha_F(u \diamond v) \leq \alpha_F(u), \qquad \beta_F(u \diamond v) \geq \beta_F(u),$$

or the analogous pair of inequalities holds for v. In the first case,

$$\alpha_F(u) \geq \alpha_F(u \diamond v) \geq \lambda, \qquad \beta_F(u) \leq \beta_F(u \diamond v) \leq \kappa,$$

so $u \in F_{(\lambda,\kappa)}$. The second case gives $v \in F_{(\lambda,\kappa)}$. Hence the level set is prime.

Conversely, suppose every nonempty PFLS is prime. Fix $u, v \in \mathcal{A}$ and set

$$\lambda = \alpha_F(u \diamond v), \qquad \kappa = \beta_F(u \diamond v).$$

The Pythagorean condition makes (λ, κ) admissible, and $u \diamond v \in F_{(\lambda,\kappa)}$. Since this PFLS is prime, either $u \in F_{(\lambda,\kappa)}$ or $v \in F_{(\lambda,\kappa)}$.

If u belongs to the level set, then

$$\alpha_F(u \diamond v) \leq \alpha_F(u), \qquad \beta_F(u \diamond v) \geq \beta_F(u).$$

If v belongs instead, then the corresponding pair of inequalities holds for v. Thus one of the two alternatives in Definition 4.1 is satisfied, so F is prime. □

Corollary 4.5. A prime Pythagorean fuzzy ideal has prime ordinary ideals at every nonempty admissible threshold, and these threshold ideals are compatible with the Cartesian product construction through Proposition 3.14.

Proof. The first statement follows from Theorem 4.4. The second follows from the level-set identity for Cartesian products and the fact that the componentwise operation is defined independently on the two factors. □

5. Conclusions

A focused framework for Pythagorean fuzzy ideals in ρ-algebras has been developed using the uniform notation u, v, w for generic elements and α, β-type grades for the fuzzy data. The main results connect the PFI definition with ordinary level ideals, establish preservation under homomorphic inverse images, describe a Cartesian-product construction, and provide a PPFI formulation compatible with prime level ideals. The additional level-set identities make the interaction among these constructions explicit.

Several directions remain open. One may investigate generated Pythagorean fuzzy ideals, quotient constructions, additional prime-type conditions, and other fuzzy models with orthopair constraints. The same strategy may also be adapted to related implication algebras whose ideal theory admits a compatible level-set description.

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