

A Review of the Application of Integral Transforms in Medical Modeling

Authors Names	ABSTRACT
^aRusul Ahmed Flayyih Nada Hadi Malik^b Suha Raisan Harbi Publication date: 11 / 10 /2026 Keywords: : integral transforms , Laplace transform , AL-Sadiq transform , AL-Sadiq- Emad – Eman transform ,ElZaki transform ,Differential equations .	Since integral transforms are among the most practical tools in mathematics, there has been a lot of interest in studying these procedures . Therefore, the aim of the current study is to apply well-known transforms such AL-Sadik - Emad – Eman (SEE), AL-Sadik transform, ElZaki transform and the Laplace transform on the pharmacokinetics problem . In the other hand, the results showed that the Ordinary Differential Equation could be absorbed by the data generated by the mathematical model and that this medical model could be solved using several well-known transforms.

1. Introduction

The Transform is an important part of the mathematical background required for physicists, engineers and mathematicians. The integral transform method provide easy and effective techniques for solving many problems in fields of medical, science and engineering.

The current paper aims to apply well-known transforms such the Laplace transform, AL-Sadiq transform, AL-Sadiq Emad Eman transform and ElZaki transform. Integral transforms are among the most useful tools in mathematics, and research on these processes has attracted a lot of interest [1]. The Laplace transform was defined in 1814 ,it is consider one of the more well known transforms .later many transforms derived from it [2] .the Sadik Transform was introduced in 2018 by the mathematician Sadik ali shaikh from Maulana Azad College[3] .it used in applied Mathematics to solve differential equation .and Sadik Emad-Eman Transform was introduced in2022 it is the a method that makes it easier to

solve ordinary differential equations in the medical model [2] .and El Zaki Transform was defined by Tariq M.Elzaki in 2011,and it is a modefide of the Sumudu and Laplace transforms ,used to solve differential equation [4]. In this work we will study these transforms and applying to solve a Problem of pharmacokinetics [5 , 6].

2. Preliminaries

This section introduces basic definitions and preliminaries. All of the transforms concepts are widely used and are found in references like [2, 3, 5, 6].

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Definition 2.1([7]) Laplace transform. Assume $f(t)$ be a function defined for all $t \geq 0$. the Laplace transform of $f(t)$, denoted $F(s)$ or $L\{f(t)\}$, defined as $F(s) = L\{f(t)\} = \int_0^\infty f(t) e^{-st} dt$, where s is a complex variable, $f(t)$ satisfies certain conditions.

Definition 2.2([4]) the Sadik Transform: let $f(t)$ be a function defined for time domain ≥ 0 , the Sadik transform denoted by $S\{f(t)\}$ is defined as $S\{f(t)\} = \frac{1}{\Gamma(\alpha)} \int_0^\infty f(t) e^{-tv^\beta} dt, t > 0$. Where α a complex variable, β real number.

Definition 2.3([2]) Sadik Emad-Eman Transform: let $f(t)$ be a function of exponential order defined: $G = \{f(t) : \exists R, s_1, s_2 > 0, |f(t)| < Re^{t|s_j|} \text{ if } t \in (-1)^j \times [0, \infty)\}$ For function in the set G , the constant R must be finite number. $SEE\{f(t)\} = SE(f) = \frac{1}{v^n} \int_0^\infty f(t) e^{-vt} dt, t > 0, n \in \mathbb{Z}, Re(v) > 0, t \geq 0, s_1 < v < s_2$

Definition 2.4. ([5]) 2.4 El Zaki Transform: let $f(t)$ be a function belonging to the set $F = \{f(t) : \exists R, r_1, r_2 > 0, |f(t)| < Re^{t|r_j|} \text{ if } t \in (-1)^j \times [0, \infty)\}$ the ElZaki transform is defined as $E\{f(t)\} = \int_0^\infty f(t) e^{-\frac{t}{\epsilon}} dt, t > 0$.

3. Illustrative Medical Examples

In this section, medical application based on mathematical model (differential equations) is solved using some transforms: Laplace, AL-Sadiq Emad, AL-Sadiq ElZaki transforms respectively.

3.1 Problem of pharmacokinetics ([6],[7]) The evaluation of an oral dosage form, such as a capsule or tablet, is carried out by measuring the amount of drug present in the blood plasma at a specific time. Predicting the in-vivo performance of the drug in terms of its concentration in the blood stream is often challenging and involves several difficulties. So the described equation will be:

$$\frac{df(t)}{dt} + \alpha f(t) = \frac{\beta}{vol}, \quad t > 0, f(0) = 0$$

Where:

$f(t)$: drug concentration in the blood at any time t ,

α : elimination velocity constant,

β : infusion rate in $(\frac{mg}{min})$,

Vol: volume in which drug is distributed.

- Applying Laplace transform for both sides with $f(0) = 0$, $L\{f(t)\} = F(s)$:

$$L\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0) = sF(s),$$

$$sF(s) + \alpha F(s) = \frac{\beta}{vol} \cdot \frac{1}{s} = F(s)(s + \alpha) = \frac{\beta}{vol} \cdot \frac{1}{s}$$

$$\rightarrow F(s) = \frac{\beta}{vol} \cdot \frac{1}{s(s + \alpha)}$$

$$\text{Since } \frac{1}{s(s + \alpha)} = \frac{A}{s} + \frac{B}{s + \alpha} \rightarrow \frac{A(s + \alpha) + Bs}{s(s + \alpha)} = \frac{1}{s(s + \alpha)}$$

Which gives:

$$As + A\alpha + Bs = 1 \rightarrow (A+B)s + B\alpha = 1,$$

$$\text{so: } A = \frac{1}{\alpha} \quad \text{and} \quad B = -\frac{1}{\alpha}.$$

Thus $\frac{1}{s(s+\alpha)} = \frac{1}{\alpha s} - \frac{1}{\alpha(s+\alpha)}$, using the inverse of Laplace transform:

$$L^{-1}\left\{\frac{B}{\text{vol}} \cdot \frac{1}{s(s+\alpha)}\right\} = \frac{B}{\text{vol}} \cdot \frac{1}{s} \{1 - e^{\alpha t}\}.$$

• **Applying Al Sadiq -Emad -Eman Transform with $f(0)=0$, $SEE(f(t))=SE(F)$**

$$SE\left[\frac{df(t)}{dt}\right] + SE[\alpha f(t)] = SE\left[\frac{\beta}{\text{vol}}\right], \quad t > 0$$

Let $f(0)=0$, $SEE f(t)=SE(f)$.

$$v SE(f) + \alpha SE(f) = \frac{\beta}{\text{vol}} \cdot \frac{1}{v^{n+1}} = SE(f)(v + \alpha) = \frac{\gamma}{\text{vol}} \cdot \frac{1}{v^{n+1}}$$

$$\rightarrow SE(f) = \frac{\beta}{\text{vol}} \cdot \frac{1}{v^{n+1}(v + \alpha)}$$

$$\text{Since } \frac{1}{v^n v(v+\alpha)} = \frac{A}{v} + \frac{B}{v+\alpha} \rightarrow \frac{A(v+\alpha)+Bv}{v(v+\alpha)} = \frac{1}{v(v+\alpha)}$$

$$Av + A\alpha + Bv = 1 \rightarrow (A+B)v + B\alpha = 1$$

$$A\alpha = 1 \rightarrow A = \frac{1}{\alpha} \quad \text{and} \quad A+B=0 \rightarrow B + \frac{1}{\alpha} = 0 \rightarrow B = -\frac{1}{\alpha}$$

$$\text{Then } \frac{1}{v^n v(v+\alpha)} = \frac{1}{v^n} \left[\frac{1}{\alpha v} - \frac{1}{\alpha(v+\alpha)} \right] = \frac{1}{\alpha} \left[\frac{1}{v^{n+1}} - \frac{1}{v^n(v+\alpha)} \right].$$

Take inverse of Al sadiq transformation

$$L^{-1}\left\{\frac{B}{\text{vol}} \cdot \frac{1}{v^n v(v+\alpha)}\right\} = \frac{B}{\text{vol}} \cdot \frac{1}{\alpha} \{1 - e^{\alpha t}\}.$$

• **Applying Elzaki transform for $E(f(t))=E(F)$, $f(0)=0$**

Taking E for every side

$$E\left[\frac{df(t)}{dt}\right] + E[\alpha f(t)] = E\left[\frac{\beta}{\text{vol}}\right],$$

$$\frac{1}{\epsilon} E(F) + \alpha E(F) = \frac{\beta}{\text{vol}} \cdot \epsilon^2 = E(F) \left(\frac{1}{\epsilon} + \alpha\right) = \frac{\gamma}{\text{vol}} \cdot \epsilon^2$$

$$\rightarrow E(F) = \frac{\beta}{\text{vol}} \cdot \frac{\epsilon^3}{(1+\alpha\epsilon)}, \text{ since } E[1] = \epsilon^2 \text{ and } E\{e^{-\alpha t}\} = \frac{\epsilon^2}{1+\alpha\epsilon}$$

$$\frac{\epsilon^3}{(1+\alpha\epsilon)} = \frac{1}{\alpha} \left(\epsilon^2 - \frac{\epsilon^2}{1+\alpha\epsilon} \right) \rightarrow E(F) = \frac{\beta}{\text{vol}} \frac{1}{\alpha} \left(\epsilon^2 - \frac{\epsilon^2}{1+\alpha\epsilon} \right)$$

Taking inverse of Al Zaki transform

$$\text{Take inverse of Al zaki transformation} = E^{-1} \left\{ \frac{\beta}{\text{vol}} \frac{1}{\alpha} \left(\epsilon^2 - \frac{\epsilon^2}{1+\alpha\epsilon} \right) \right\} = \frac{B}{\text{vol}} \cdot \frac{1}{\alpha} \{1 - e^{\alpha t}\}.$$

Table 1 of Previous Famous Transforms for some Basic Functions

The following table shows the previous famous transforms for some elementary functions.

$f(x)$	$L(f(x))$	$S(f(x))$	$SEE(f(x))$	$E(f(x))$
1	$\frac{1}{v}$	$\frac{1}{v^{\beta+\alpha}}$	$\frac{1}{v^{n+1}}$	v^2
x	$\frac{1}{v^2}$	$\frac{1}{v^{2\alpha+\beta}}$	$\frac{1}{v^{n+2}}$	v^3
x^2	$\frac{2!}{v^3}$	$\frac{2!}{v^{3\alpha+\beta}}$	$\frac{2}{v^{n+3}}$	$2! v^4$
e^{ax}	$\frac{1}{v-a}$	$\frac{1}{v^\beta (v^\alpha - a)}$	$\frac{1}{v^n (v - a)}$	$\frac{v^2}{1 - av}$
Sin ax	$\frac{a}{(v^2 + a^2)}$	$\frac{a}{v^\beta (v^{2\alpha} + a^2)}$	$\frac{a}{v^n (v^2 + a^2)}$	$\frac{av^3}{1 + a^2 v^2}$
Cos ax	$\frac{v}{v^2 + a^2}$	$\frac{v^\alpha}{v^\beta (v^{2\alpha} + a^2)}$	$\frac{1}{v^{n-1} (v^2 + a^2)}$	$\frac{v^2}{1 + a^2 v^2}$
Sinh ax	$\frac{a}{v^2 - a^2}$	$\frac{a}{v^\beta (v^{2\alpha} - a^2)}$	$\frac{a}{v^n (v^2 - a^2)}$	$\frac{av^3}{1 - a^2 v^2}$
Cosh ax	$\frac{v}{v^2 - a^2}$	$\frac{v^\alpha}{v^\beta (v^{2\alpha} - a^2)}$	$\frac{1}{v^{n-1} (v^2 - a^2)}$	$\frac{v^2}{1 - a^2 v^2}$

4. Conclusions

It is clear that the use of differential equations is more crucial in the medical field for the analysis, diagnosis, interpretation, and treatment of different illnesses. Ordinary differential equations is a mathematical model that solved using transforms yield more accurate results than classical model. Additionally, it has been noted that implementation of computational models enhances the efficiency of mathematical discoveries in the medical sector. Future research on the use of integral transforms based on differential equations in the medical area will use this publication as a guide.

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