

## Connected Topological Space (Review)

| Authors Names                                                                                                          | ABSTRACT                                                                                                                                                                                                                                                                                                                                        |
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| Hadeel Husham Kadhim<br>Publication date: 12 /10 /2026<br>Keywords: connected ,<br>locally connected, Global property. | This study compares two fundamental concepts in general topology :<br>Connectedness and local connectedness . Connectedness represents a global<br>property that prevents the topological space from fragmenting into widely<br>separated components , whereas local connectivity within very small<br>neighborhoods around each of its points. |

### 1. Introduction

Connected and locally connected topological spaces are fundamental concepts in topology that establish the nature of connectedness and cohesion within mathematical spaces whether considered globally or in neighborhood of each point .[2] presented connected and locally connected spaces. Willard. S. study definition of topological space . Monkres explained that connectedness is a “ global property “ encompassing the space as a whole whereas local connectedness is a “ local property ” describing the behavior of the space at each point and its neighborhoods. [3] is mentioned in the examples and counter examples.

### 2. Preliminaries

This section introduces definition of topological space and connected topological space .All these concepts are found in references [2,4]

**Definition(2.1)[4]** If is  $\mu \neq \emptyset$  . Then the collection  $\tau$  of subsets of  $\mu$  is called topology for  $\mu$  if  $\tau$  satisfies the following axioms

- i-  $\mu$  and  $\emptyset$  are member of  $\tau$
- ii- If  $A_1$  and  $A_2$  belong to  $\tau$  then the intersection of them is belong to it.
- iii- If  $A_\vartheta$  members of  $\tau$  such that  $\vartheta \in \Delta$  . Then  $\cup A_\vartheta \in \tau$  .

If  $\tau$  topology on  $X$  then  $(\mu , \tau)$  is said to be topological space . the member of  $\tau$  is called open sets  
the complement of open is called closed set

#### 2.1. Connected space

**Definition(2.1.1)[2]**A space  $\mu$  is called not connected if it can be expressed as the union of two disjoint open sets . other wise  $\mu$  is connected .

**Mental Image :** Imagine the space as a single magnet or continuous rope ;no matter how you examine it, represents a unified whole with no separating gaps

Figures(1) and (2) an explain of the concepts ( connected and disconnected )

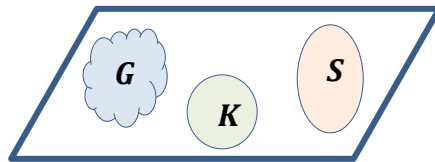


Figure (1)

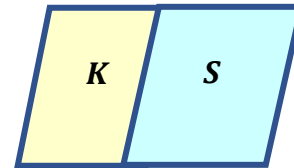


Figure (2)

### Examples and antonyms

**Example(2.1.1)** The usual topology  $(\mathbb{R}, \mathcal{U})$  is connected because there are no two disjoint open sets within it whose union equals the space itself .

**Example(2.1.2)** The discrete topological space is disconnected because there are two disjoint open sets within it whose union equals the space itself .

**Example(2.1.3)** Fungal networks ( globally connected space )

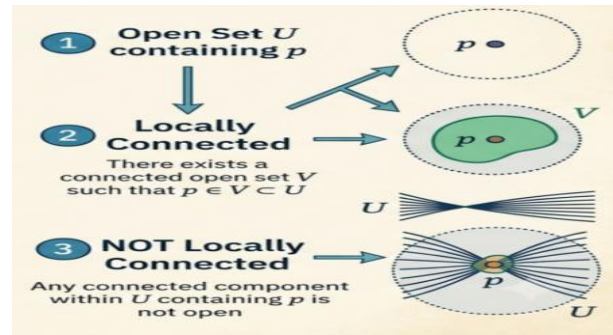
Consider the vast underground fungal networks known as mycelium . They represent a globally connected space because they form a single massive network extending over vast distances without interruption .

## 2.2 Locally connected

**Definition(2.2.1)[2]**A space  $\mu$  is called locally connected if for every point  $x \in X$  and every  $V$  open set such that  $x \in V$  there is connected open set  $K$  such that  $x \in K \subseteq V$  .

**Mental Image :** Imagine a group of islands . The space as a whole is disconnected ( because water us separates the islands ) , but if you stand on any island and observe the small area around your feet , you will find it to be locally connected land , free of gaps.

The following figure an explain of the concept locally connected



Finger explain the concept of locally connected

### Examples and antonyms

#### Example(2.2.1) Open and closed balls

Any open connected set in  $R^n$  ( such as the interior of a ball ) is locally connected

#### Example(2.2.2) The set of rational numbers with usual topology ( $Q, U$ )

It is not locally connected , Any open interval around a rational number contains irrational numbers , meaning that the only connected sets within are singletons and singletons are not open sets in the standard topology .

#### Example(2.2.3) Fungal networks ( globally locally connected space )

These are locally connected because if you examine a tiny sample from any part under a microscope . you will find the fungal filaments fused and seamlessly interconnected . Here ,the whole and the part exist harmony .

| The property                 | Connected                                                             | Locally connected                                                                                 |
|------------------------------|-----------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|
| The fundamental definition   | It cannot be partitioned into two disjoint open sets                  | Each element within it has a neighborhood base consisting of open and connected group             |
| Nature of the property       | Encompasses the space as a single ,complete whole ( Global property ) | Describes the behavior of the space at the points and their small neighborhoods ( Local property) |
| Connectedness and components | The space contains one large connected component (the space itself)   | The connected components of any open set within it are always open                                |

The following table shows the comparison between a connected and locally connected spaces.

### **3. Conclusions**

General connectivity ensures inclusivity ensures inclusivity and the comprehensive interconnection of systems , where as local connectivity ensures flexible reliability and continuity at the immediate level.

### **References**

- [1] Munkres, J.R. “ Topology second Edition ” Massachusetts Institute of Technology (2000)
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- [4] Willard. S. “General Topology” University of Alberta, Addison Wesley Publishing Company Reading Mass (1970).