Two Methods for Constructing Minimal (b,t)-Blocking Sets in PG(2,5) with Associated Complete Arcs and Projective Codes

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Publication date: 8/11/2026

Keywords: *Construction; Minimal (b,t)-blocking sets; Complete(k,n)-arcs; Projective linear cods .*

ABSTRACT

Minimal (b,t)-blocking sets constitute a significant class of combinatorial configurations in finite projective geometry because of their close connections with complete arcs and projective linear codes. This study presents a systematic construction and comparison of minimal (b,t)-blocking sets in the projective plane PG(2,5), for $1 \leq t \leq 5$, using two distinct approaches: the Rédei-type method and the conic-type method. For each construction, both the blocking property and minimality of the resulting sets are verified. The Rédei-type method produces blocking sets of cardinalities 10,15,20,25 and 30, whereas the conic-type method yields sets of cardinalities 9,16,20,25 and 30, corresponding respectively to t=1,2,3,4 and 5. For n=5,4,3 and 2, respectively, the Rédei-type construction generates complete (k,n)-arcs of sizes k=21,16,11 and 6. These arcs give rise to the projective linear codes $[21,3,16]_5$, $[16,3,12]_5$, $[11,3,8]_5$, and $[6,3,4]_5$, respectively. Similarly, the conic-type construction produces complete (k,n)-arcs of sizes k=22,15,11 and 6, corresponding respectively to n=5,4,3 and 2, from which the projective linear codes $[22,3,17]_5$, $[15,3,11]_5$, $[11,3,8]_5$, and $[6,3,4]_5$ are derived. The obtained results demonstrate the effectiveness of both construction methods and establish a clear structural connection among minimal blocking sets, complete arcs, and projective linear codes in PG(2,5). Future studies may extend this investigation to minimal (b,t)-blocking sets in PG(2,5) for q=7,9,11,13, as well as for other prime-power values, together with the complete arcs and projective linear codes arising from these configurations.

1. Introduction

In the projective plane PG(2,q) define over the Galois field GF(q),the total number of points q^2+q +1. Moreover, every line of the plane contain$q$ +1 points.

The projective plane PG (2,q) consists of exactlyq^2+q+1 lines, and each point is incident with precisely q+1 lines. The point A (a_1 , a_2 ,a_3) is on theline B [b_1,b_2 ,b_3] if and only ifa_1b_1+a_2b_2+a_3b_3= 0.

Definition(1.1): [3],[7]

A blocking set B in the projective plane PG (2,q) is a collection of b distinct points such that every line of the plane meets B in at least t points. The blocking set is called trivial whenever it contains all the points of some line.

Lemma (1.2):[3],[5]

A (b ,1)-blocking set B in PG(2,q) is called minimal provided that, for every point T ∈ B, there exists a line $\mathscr{L}$ satisfying B ∩$\mathscr{L}$ = {T}.

Theorem (1.3):[2],[4],[11]

A (b ,t)- blocking set B of PG(2,q) is minimal exactly when each point of B lies on `a t-secant line of B.

Theorem(1.4):[5],[18]

1. Let β_2 be a double blocking set in PG (2,q) then β_2 has at least 3q points if q<9. Also $|\beta_2| \geq \frac{5q+7}{2}$ if q = 11,13,17 or 19.

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2. Let β_3 be a trible blocking set in PG(2,q), then β_3 contains at least 4q points whenever q = 5,7,or 9.In the special case q = 8, the set β_3 must contain at least 31 points. Moreover, for q = 11,13, or 17 then$|\beta_3| \geq \frac{7q+9}{2}$.

Definition(1.5): [1],[2],[4]

A(K, n)-arc K $\subset$ PG (2,q) is a collection of K points with the property that at least one line contains exactly n of its points, and no line contains more than n points of K. An arc is considered complete if the insertion of any further point necessarily destroys the property that characterizes it.

Definition(1.6):[4],[6]

For a line $\mathscr{L}$ in PG (2,q), it is called an i-secant of the (k,n)-arc whenever |L ∩ K|= i

1. A 0-secant is referred to as an external line.
2. A 1-secant is called unisecant line.
3. A 2-secant is known as a bisecant line.

Definition (1.7): [4],[5] A point P$\notin$ K is said to possess index i if exactly i distinct n–secant lines of the set K pass through it. The number of points having index i is denoted by C_{i}.

Remark(1.8) :[1],[3],[4] The (K, n)_arc K is called complete whenever $C_0 = 0$. Equivalently, every point in PG (2,q) belongs to at least one n-secant line associated with K.

Definition(1.9): [6],[9] Let Q (2,q) represent the family of all quadratic curves in PG (2,q). These curves correspond to the algebraic variety V(F), where:

F= $a_{11}X_1^2 + a_{22}X_2^2 + a_{33}X_3^2 + a_{12}X_1X_2 + a_{13}X_1X_3 + a_{23}X_2X_3$……..(1)

If the variety V(F) is nonsingular, then the associated quadric represents a conic**.**

In other words, if the matrix A=$\begin{bmatrix} a_{11} & \frac{a_{12}}{2} & \frac{a_{13}}{2} \\ \frac{a_{12}}{2} & a_{22} & \frac{a_{23}}{2} \\ \frac{a_{13}}{2} & \frac{a_{23}}{2} & a_{33} \end{bmatrix}$ is nonsingular, then the quadric represents a conic.

1.10 On the Relationship Between (,n)- Arcs and (b,t)- Blocking Sets.[6],[10],[13]

In the finite projective plane PG(2,q), a strong combinatorial relationship exists between (b ,t)- blocking sets and (k ,n)- arc. These two configurations are complementary since their parameters fulfill the relation: n + t = q + 1 and k + b = q^2+q+1.

Definition(1.11): [6],[8],[12] Let V(n,q) denote the vector space consisting of all ordered vectors of length n over the field GF(q) . A line code C with length n and dimension k can be regarded as a k-dimensional subspace of V(n,q). The vectors in C are referred to as codewords. To measure how different two codewords are, we use the Hamming distance, which simply counts the number of coordinate positions where the two codewords are not the same. The minimum distance of the code is the smallest Hamming distance between any two distance codewords. If this minimum distance equals d, then the code is referred to as an [n ,3 ,d] code.

Theorem(1.12): [2],[7] A projective [n,3,d] code exists if and only if there is an (n,n-d)-arc in PG (2,q).

2- The Projective Plane PG (2,5):[3],[10]

The Projective Plane PG (2,5) contains a total of 31 points together with 31 lines. Every line passes through exactly six points, while each point belongs to exactly six distinct lines. The points and lines of PG (2,5) are represented by p_i and l_i, respectively for i = 1,2,3,….,30,31. A detailed listing of all points and lines appears in table (2.1).

Table (2.1)

i	P_i			L_i					
1	1	0	0	2	7	12	17	22	27
2	0	1	0	1	7	8	9	10	11
3	1	1	0	6	7	16	20	24	28
4	2	1	0	4	7	14	21	23	30
5	3	1	0	5	7	15	18	26	29
6	4	1	0	3	7	13	19	25	31
7	0	0	1	1	2	3	4	5	6
8	1	0	1	2	11	16	21	26	31
9	2	0	1	2	9	14	19	24	29
10	3	0	1	2	10	15	20	25	30
11	4	0	1	2	8	13	18	23	28
12	0	1	1	1	27	28	29	30	31
13	1	1	1	6	11	15	19	23	27
14	2	1	1	4	9	16	18	25	27
15	3	1	1	5	10	13	21	24	27
16	4	1	1	3	8	14	20	26	27
17	0	2	1	1	17	18	19	20	21
18	1	2	1	5	11	14	17	25	28
19	2	2	1	6	9	13	17	26	30
20	3	2	1	3	10	16	17	23	29
21	4	2	1	4	8	15	17	24	31
22	0	3	1	1	22	23	24	25	26
23	1	3	1	4	11	13	20	22	29
24	2	3	1	3	9	15	21	22	28
25	3	3	1	6	10	14	18	22	31
26	4	3	1	5	8	16	19	22	30
27	0	4	1	1	12	13	14	15	16
28	1	4	1	3	11	12	18	24	30
29	2	4	1	5	9	12	20	23	31
30	3	4	1	4	10	12	19	26	28
31	4	4	1	6	8	12	21	25	29

2.1 The Conics in PG (2,5):[1],[5],[6]

Let A= [1,2,7,13] represent the set of reference and unit points in PG (2,5), where these points correspond to the homogeneous coordinates :

1 = (1,0,0), 2 = (0,1,0), 7 = (0,0,1), 13 = (1,1,1)

This set A forms a (4,2)-arc because no three of its points lie on the same line, which allows for the construction of conics based on these points.

2.1.1 Conics in PG (2,5) Determined by The Reference and Unit point.[5],[11],[12]

A conic in a projective plane may be written in the general quadratic form:

$$a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3 = 0 \dots\dots\dots (1)$$

By inserting the points of the set A into equation (1),we obtain:

$a_{12}+a_{13}+a_{23}=0$ and $a_{11} = a_{22} = a_{33} = 0$, so (1) become :

$a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3 = 0$……..(2)

If $a_{12} = 0$, the resulting conic becomes degenerate . Hence, to avoid degeneration, it is necessary that $a_{12} \neq 0$, and similarly $a_{13} \neq 0$ and $a_{23} \neq 0$.

Upon dividing equation (2) by a_{12}, the resulting form is :

$x_1x_2+\alpha x_1x_3 +\beta x_2x_3 = 0$ … ….(3)

Where $\alpha = \frac{a_{13}}{a_{12}}$, $\beta = \frac{a_{23}}{a_{12}}$, then $\beta = -(1+\alpha)$ since $1 + \alpha + \beta = 0 \pmod 5$

Therefore, equation (3) can be rewritten in the equivalent form :

$x_1x_2+ \alpha x_1x_3 - (1+\alpha)x_2x_3 = 0$ …… (4)

The conic remains non-degenerate whenever α is neither 0 nor 4. Hence, valid values of α are α = 1,2, and 3.

2.1.2 Determination of Conic Equations and Associated points Sets in PG (2,5) Based on Reference and Unit Elements.[1],[17],[18]

Each value of α determines a unique conic passing through eight points, including the four reference and unit points α

1. For α = 1, the resulting conic C_1 has the equation : $x_1x_2+x_1x_3 + 3x_2x_3 = 0$
   The set of points that lie on C_1 consists of :1,2,7,13,20,26.
2. For α = 2, the resulting conic C_2 has the equation : $x_1x_2+2\,x_1x_3 + 2x_2x_3 = 0$
   The set of points that lie on C_2 consists of :1,2,7,13,21,29.
3. For α = 3, the resulting conic C_3 has the equation : $x_1x_2+ 3\,x_1x_3 + x_2x_3 = 0$
   The set of points that lie on C_3 consists of: 1,2,7,3,24,32,30.
   In total, five distinct conics emerge from the construction. Among them, two are degenerate, whereas the rest (C_1 through C_3) are non- degenerate.

3. Construction Methods of minimal (b,t)- blocking sets in PG (2,5).[4]

In this chapter, we present the construction of minimal(b,t)-blocking sets via two approaches, namely the Redei-type and the conic-type constructions. These constructions facilitate the derivation of complete(k,n)-arcs and linear projective cods associated with such sets.

3.1The First Method for Constructing Minimal(b,t)-Blocking Sets of Redei-Type.[2],[3],[14]

Within PG(2,5), suppose that the set B contains a line $\mathscr{L}$ excluding a point q, together with a collection of p points selected on p distinct lines passing through q, where these points are not all collinear. Then B forms a minimal (b,t)-blocking set. Blocking sets obtained by this construction are usually known as Redei-type. These sets were studied by [Bruen, A.A. and Thas, J.A. (1977)] and in [Blockhuis, A.A. and Brouwer, E. and S.Z. onyi, T. (1995)].

3.1.1 Construction of a Minimal (b,1)-Blocking Set.

Let PG (2,5) = π. Consider the line L_7= {1,2,3,4,5,6}, and remove the point 6. Next of five distinct lines through the point 6, provided that these selected points are not collinear, which are 7,12,13,15,18. Then β_1= {1,2,3,4,5,7,12,13,15,18} is a minimal (10,1)-blocking set of Redei-type with size 2q in PG (2,5) (see figure1).

Now define K_5= $\pi \setminus \beta_1$={6,8,9,10,11,14,16,17,19,20,21,22,23,24,25,26,27,28,29,30,31}

Thus, K_5 forms a complete (21,5)_arc as listed in table (2.2).

Furthermore, by theorem (1.12), a projective line code with parameters [21,3,16] is obtained.

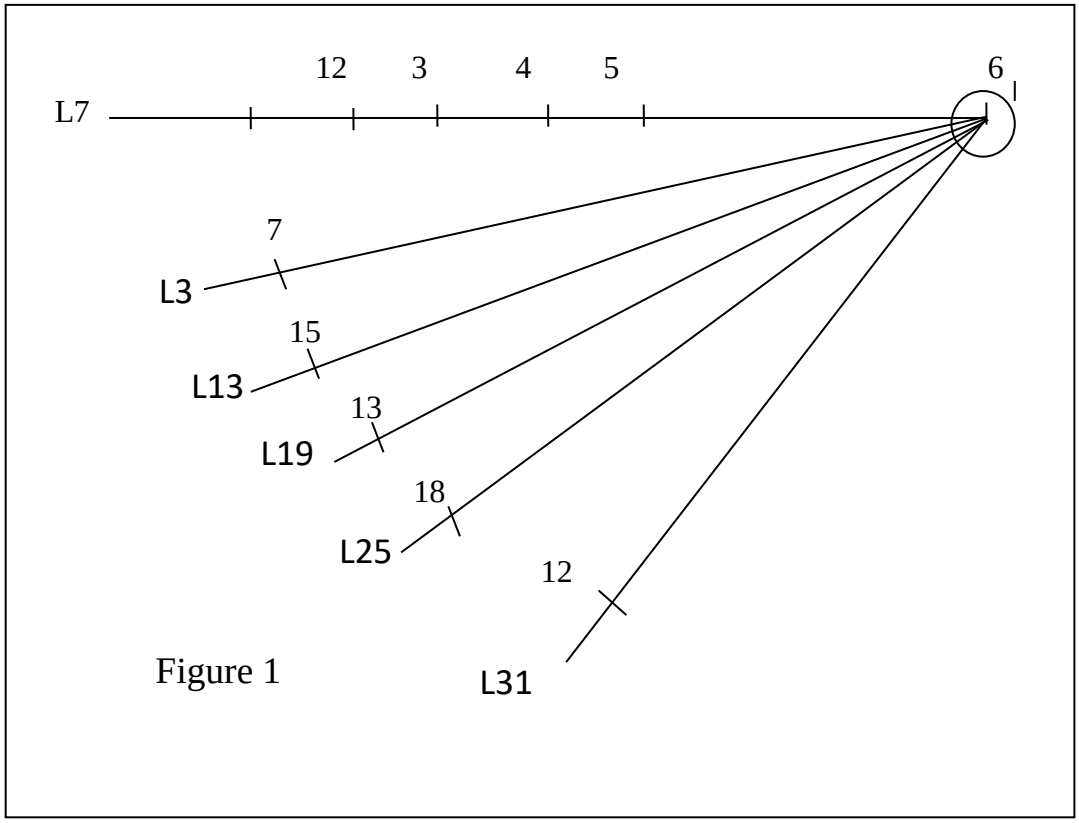


Figure 1

Table(2.2)

i	$\beta_1 \cap L_i$	$K_5 \cap L_i$	$\lvert\beta_1 \cap L_i\rvert$	$\lvert k_5 \cap L_i\rvert$
1	{2,7,12}	{17,22,27}	3	3
2	{1,7}	{8,9,10,11}	2	4
3	{7}	{6,16,20,24,28}	1	5
4	{4,7}	{14,21,23,30}	2	4
5	{5,7,15,18}	{26,29}	4	2
6	{3,7,13}	{19,25,31}	3	3
7	{1,2,3,4,5}	{6}	5	1
8	{2}	{11,16,21,26,31}	1	5
9	{2}	{9,14,19,24,29}	1	5

10	{2,15}	{10,20,25,30}	2	4
11	{2,13,18}	{8,23,28}	3	3
12	{1}	{27,28,29,30,31}	1	5
13	{15}	{6,11,19,23,27}	1	5
14	{4,18}	{9,16,18,25,27}	2	4
15	{5,13}	{10,21,24,27}	2	4
16	{3}	{8,14,20,26,27}	1	5
17	{1,18}	{17,19,20,21}	2	4
18	{5}	{11,14,17,25,28}	1	5
19	{13}	{6,17,26,30}	1	5
20	{3}	{10,16,17,23,29}	1	5
21	{4,15}	{8,17,24,31}	2	4
22	{1}	{22,23,24,25,26}	1	5
23	{4,13}	{11,20,22,29}	2	4
24	{3,15}	{9,21,22,28}	2	4
25	{18}	{6,10,14,22,31}	2	4
26	{5}	{8,16,19,22,30}	1	5
27	{1,12,13,15}	{14,16}	4	2
28	{3,12,18}	{11,24,30}	3	3
29	{5,12}	{9,20,23,31}	2	4
30	{4,12}	{10,19,26,28}	2	4
31	{12}	{6,8,21,25,29}	1	5

3.1.2 Construction of a Minimal (b,2)-Blocking Set.

From section 3.1.1 we take β_1= {1,2,3,4,5,7,12,13,15,18}be the minimal (10,1)-blocking set of Redei-type.

We construct another (10,1)-blocking set we say α_1:

α_1= L_{12}\ {31}U {3,8,16,20,22}, were L_{12} ={1,27,28,29,30,31} and a non-collinear points which are on the lines through the point 27 other than L_{12} (see figure 2) ,then: α_1= {1,3,8,16,20,22,27,28,29,30} is a minimal (10,1)-blocking set of Redei-type. Next, a minimal (b,2)-blocking set is constructed from the sets $\beta_1$and α_1 as follows: Let A = $\alpha_1 \cup \beta_1$= {1,2,3,4,5,7,8,12,13,15,16,18,20,22,27,28,29,30}

A must satisfies the following condition:

1. Every line in π intersects A in at least two points.
2. Each point of A belongs to at least one bisecant of the set.

To satisfy these conditions, the points 9 and 11 are added to A, while the points 2,5,7,8, and 20 are removed, then:

β_2 = A ∪ {9,11}\ {2,5,7,8,20}

β_2={1,3,4,9,11,12,13,15,16,18,22,27,28,29,30}is a minimal (15,2)-blocking set of size 3q, (see table (2.3)).

Let K_4= π \ β_2 = {2,5,6,7,8,10,14,17,19,20,21,23,24,25,26,31}is complete (16,4)-arc

Therefore, applying theorem (1.12), we obtain a projective linear code with parameters [16,3,12].

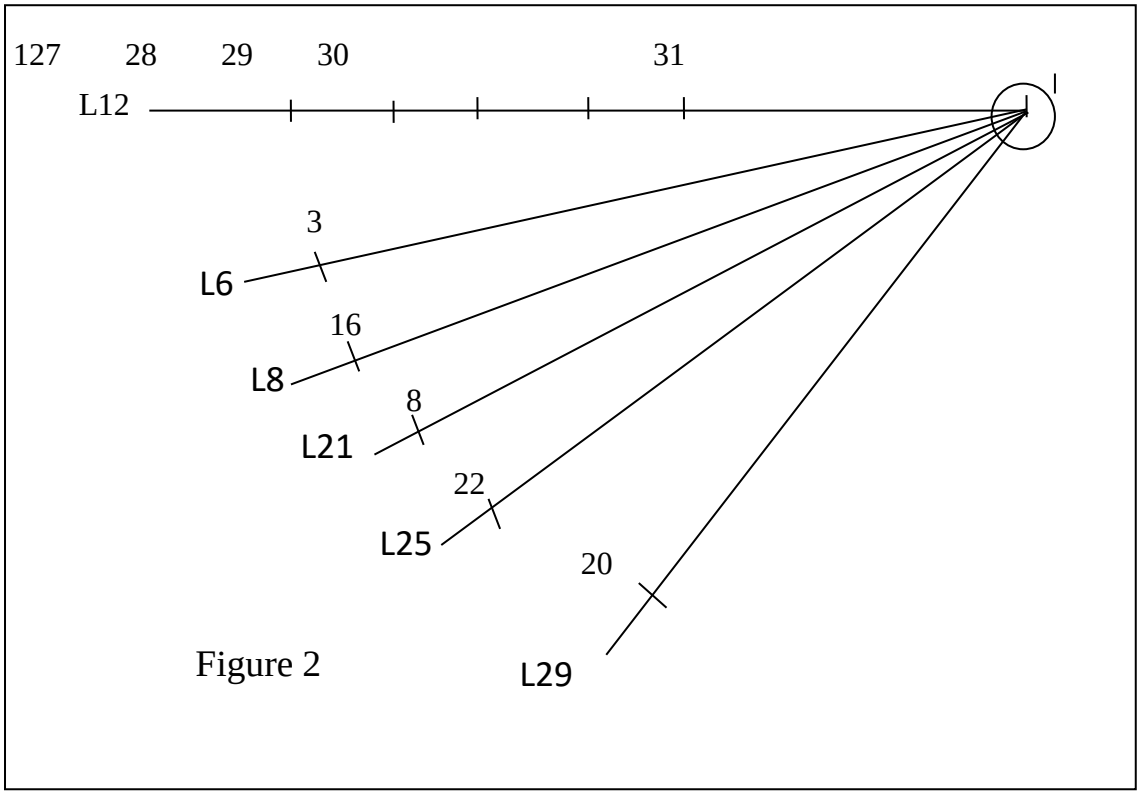


Figure 2

Table (2.3)

i	$\beta_2 \cap \mathrm{L_i}$	$K_4 \cap \mathrm{L_i}$	$\lvert\beta_2 \cap L_i\rvert$	$\lvert K_4 \cap \mathrm{L_i}\rvert$
1	{12,22,27}	{2,7,17}	3	3
2	{1,9,11}	{7,8,10}	3	3
3	{16,28}	{6,7,20,24}	2	4
4	{4,30}	{7,14,21,23}	2	4
5	{15,18,29}	{5,7,26}	3	3
6	{3,13}	{7,19,25,31}	2	4
7	{1,3,4}	{2,5,6}	3	3
8	{11,16}	{2,21,26,31}	2	4
9	{9,29}	{2,14,19,24}	2	4
10	{15,30}	{2,10,20,25}	2	4
11	{13,18,28}	{2,8,23}	3	3
12	{1,27,28,29, 30}	{31}	5	1
13	{11,15,27}	{6,19,23}	3	3
14	{4,9,16,18,2 7}	{25}	5	1
15	{13,27}	{5,10,21,24}	2	4
16	{3,27}	{8,14,20,26}	2	4
17	{1,18}	{17,19,20,21}	2	4

18	{11,28}	{5,14,17,25}	2	4
19	{9,13,30}	{6,17,26}	3	3
20	{3,16,29}	{10,17,23}	3	3
21	{4,15}	{8,17,24,31}	2	4
22	{1,22}	{23,24,25,26}	2	4
23	{4,11,13,22,29}	{20}	5	1
24	{3,9,15,22,28}	{21}	2	4
25	{18,22}	{6,10,14,31}	2	4
26	{16,22,30}	{5,8,19}	3	3
27	{1,12,13,15,16}	{14}	5	1
28	{3,11,12,18,30}	{24}	5	1
29	{9,12}	{5,20,23,31}	2	4
30	{4,12,28}	{10,19,26}	3	3
31	{12,29}	{6,8,21,25}	2	4

3.1.3 Construction of a Minimal (b,3)-Blocking Set.

From section 3.1.2 we take two minimal (10,1)-blocking set of Redei-type β_1,α_1: β_1= {1,2,3,4,5,7,12,13,15,18} and α_1= {1,3,8,16,20,22,27,28,29,30}.

We introduce another (10,1)-blocking set of Redei-type, denoted by ε_1, and defined by ε_1= L_{22}\ {1}U {3,7,14,17,28}, were L_{22} ={1,22,23,24,25,26}. The added points are chosen to be non-collinear and lie on the lines passing through the point 1 excluding the line L_{12} (see figure 3).

Hence, ε_1= {3,7,14,17,22,23,24,25,26,28} is a minimal blocking set with the parameters (10,1). We now obtain a minimal (b,3)-blocking set by combining the sets $\beta_1, \alpha_1$and $\varepsilon_1$in the following manner:

Let B= $\beta_1 \cup \alpha_1 \cup \varepsilon_1$={1,2,3,4,5,7,8,12,13,14,15,16,17,18,20,22,23,24,25,26,27,28,29,30}

The set B is required to satisfy the following properties:

1. Each line in π contains at least three points of the set B.
2. Each element of B belongs to at least one (3-secant).

To ensure that the required conditions are satisfied, the point 31 is incorporated into the set B, while the points 3,12,14,20,30 are excluded. Accordingly, we define:

β_3 = B ∪ {31}\{3,4,13,14,16}

β_3={1,2,5,7,8,12,15,17,18,20,22,23,24,25,26,27,28,29,30,31}. Hence, β_3 is a minimal trivial (20,3)-blocking set of size 4q, since β_3 includes a whole line of π.

Let K_3= π \ β_3 = {3,4,6,9,10,11,13,14,16,19,21}is complete (11,3)-arc.(see table (2.4))

Therefore, applying theorem (1.12), we obtain a projective linear code with parameters [11,3,8].

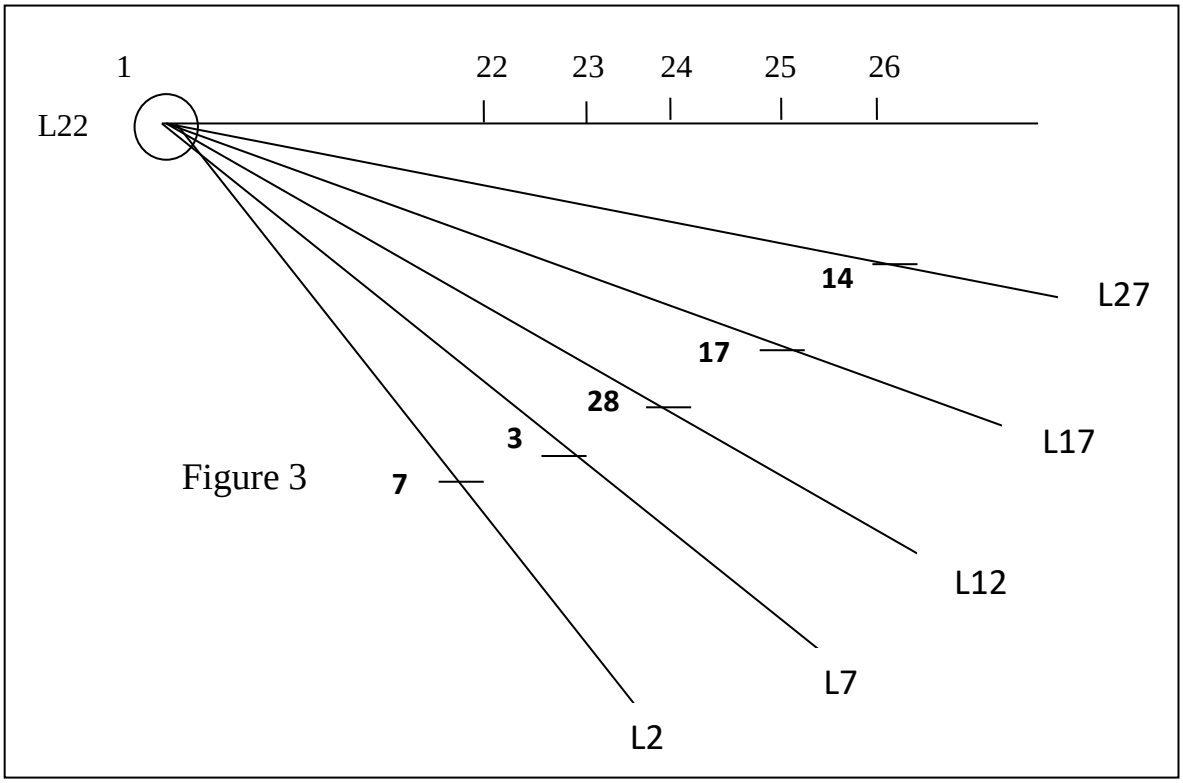


Figure 3

Table (2.4)

i	$\beta_3 \cap \mathrm{L_i}$	$K_3 \cap \mathrm{L_i}$	$\lvert\beta_3 \cap L_i\rvert$	$\lvert K_3 \cap \mathrm{L_i}\rvert$
1	{2,7,12,17,22,27}	Φ	6	0
2	{1,7,8}	{9,10,11}	3	3
3	{7,20,24,28}	{6,16}	4	2
4	{7,23,30}	{4,14,21}	3	3
5	{5,7,15,18,26,29}	Φ	6	0
6	{7,25,31}	{3,13,19}	3	3
7	{1,2,5}	{3,4,6}	3	3
8	{2,26,31}	{11,16,21}	3	3
9	{2,24,29}	{9,14,19}	3	3
10	{2,15,20,25}	{10,30}	4	2
11	{2,8,18,23,28}	{13}	5	1
12	{1,27,28,29,30,31}	Φ	6	0
13	{15,23,27}	{6,11,19}	3	3
14	{18,25,27}	{4,9,16}	3	3
15	{5,24,27}	{10,13,21}	3	3
16	{8,20,26,27}	{3,14}	4	2
17	{1,17,18,20}	{19,21}	4	2
18	{5,17,25,28}	{11,14}	4	2
19	{17,26,30}	{6,9,13}	3	3
20	{17,23,29}	{3,10,16}	3	3
21	{8,15,17,24,31}	{4}	5	1

22	{1,22,23,24,25,26}	Φ	6	0
23	{20,22,29}	{4,11,13}	3	3
24	{15,22,28}	{3,9,21}	3	3
25	{18,22,31}	{6,10,14}	3	3
26	{5,8,22,30}	{16,19}	4	2
27	{1,12,15}	{13,14,16}	3	3
28	{12,18,24,30}	{3,11}	4	2
29	{5,12,20,23,31}	{9}	5	1
30	{12,26,28}	{4,10,19}	3	3
31	{8,12,25,29}	{6,21}	4	2

3.1.4 Construction of a Minimal (b,4)-Blocking Set.

From section 3.1.3 we take three minimal (10,1)-blocking set of Redei-type β_1,α_1,$and\ \varepsilon_1$:

β_1={1,2,3,4,5,7,12,13,15,18}

α_1={1,3,8,16,20,22,27,28,29,30}

ε_1={3,7,14,17,22,23,24,25,26,28}

We introduce another (10,1)-blocking set of Redei-type, denoted by γ_1, and defined by: γ_1= L_2\ {11}∪ {2,5,15,20,24}, were L_2 ={1,7,8,9,10,11}.The selected points are non-collinear and belong to the lines passing through the point 11, excluding the line L_2 (see figure 4). Consequently, γ_1= {1,2,5,7,8,9,10,15,20,24}, forms a minimal blocking set of Redei-type with the parameters (10,1).

We now obtain a minimal (b,4)-blocking set by combining the sets β_1, α_1, $\varepsilon_1$and $\gamma_1$in the following manner:

Let C= β_1∪ α_1∪ε_1∪γ_1, that is:

C = {1,2,3,4,5,7,8,9,10,12,13,14,15,16,17,18,20,22,23,24,25,26,27,28,29,30}

The set C is required to satisfy the following conditions:

1. Each line in π contains at least four points of the set C.
2. Each element of C belongs to at least one (4-secant).

To meet the first condition, three points namely 3,10,30 are removed from C, while two points, 11 and 31 are added to it to ensure the second condition is satisfied, then:

β_4 = A ∪ {11,31}\ {3,10,30}

β_4={1,2,4,5,7,8,9,11,12,13,14,15,16,17,18,20,22,23,24,25,26,27,28,29,31}is a minimal (25,4)-blocking set of cardinality 5q which is a trivial because β_4 includes a whole line of π.

Let K_2= π \ β_4 = {3,6,10,19,21,30} is complete (6,2)-arc,(see table (2.5)).

Therefore, applying theorem (1.12), we obtain a projective linear code with the parameters [6,3,4].

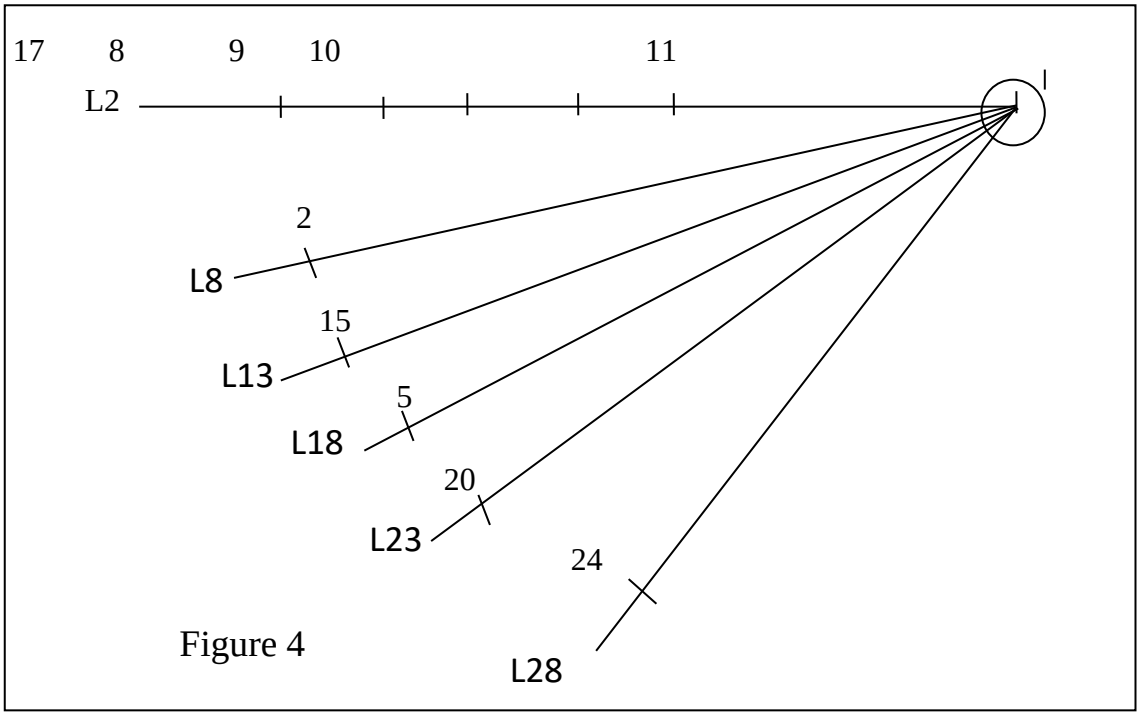


Figure 4

Table (2.5)

i	$\beta_4 \cap \mathrm{L_i}$	$K_2 \cap \mathrm{L_i}$	$\lvert\beta_4 \cap L_i\rvert$	$\lvert K_2 \cap \mathrm{L_i}\rvert$
1	{2,7,12,17,22,27}	Φ	6	0
2	{1,7,8,9,11}	{10}	5	1
3	{7,16,20,24,28}	{6}	5	1
4	{4,7,14,23}	{21,30}	4	2
5	{5,7,15,18,26,29}	Φ	6	0
6	{7,13,25,31}	{3,19}	4	2
7	{1,2,4,5}	{3,6}	4	2
8	{2,11,16,26,31}	{21}	5	1
9	{2,9,14,24,29}	{19}	5	1
10	{2,15,20,25}	{10,30}	4	2
11	{2,8,13,18,23,28}	Φ	5	1
12	{1,27,28,29,31}	{30}	5	1
13	{11,15,23,27}	{6,19}	4	2
14	{4,9,16,18,25,27}	Φ	6	0
15	{5,13,24,27}	{10,21}	4	2
16	{8,14,20,26,27}	{3}	5	1
17	{1,17,18,20}	{19,21}	4	2
18	{5,11,14,17,25,28}	Φ	4	2
19	{9,13,17,26}	{6,30}	4	2
20	{16,17,23,29}	{3,10}	4	2
21	{4,8,15,17,24,31}	Φ	6	0
22	{1,22,23,24,25,26}	Φ	6	0
23	{4,13,20,22,29}	Φ	6	0
24	{9,15,22,28}	{3,21}	4	2

25	{14,18,22,31}	{6,10}	4	2
26	{5,8,16,22}	{19,30}	4	2
27	{1,12,13,14,15,16}	Φ	6	0
28	{11,12,18,24}	{3,30}	4	2
29	{5,9,12,20,23,31}	Φ	6	0
30	{4,12,26,28}	{10,19}	4	2
31	{8,12,25,29}	{6,21}	4	2

3.1.5 Construction of a Minimal (b,5)-Blocking Set.

From section 3.1.4 we take four minimal (10,1)-blocking set of Redei-type β_1,α_1,ε_1,γ_1:

β_1={1,2,3,4,5,7,12,13,15,18}

α_1={1,3,8,16,20,22,27,28,29,30}

ε_1={3,7,14,17,22,23,24,25,26,28}

γ_1={1,2,5,7,8,9,10,15,20,24}

We introduce another (10,1)-blocking set of Redei-type, denoted by δ_1, and defined by: δ_1= L_{31}\ {29}U {4,7,16,19,28}, were L_{31} ={6,8,12,21,25,29}. The added points are chosen to be non-collinear and lie on the lines passing through the point 29 excluding the line L_{31} (see figure 5). Hence, δ_1= {4,6,7,8,12,16,19,21,25,28} is a minimal blocking set with the parameters (10,1).

We now obtain a minimal (b,5)-blocking set by combining the sets $\beta_1, \alpha_1, \varepsilon_1, \gamma_1$and δ_1 in the following manner:

Let D = β_1U α_1Uε_1U γ_1Uδ_1, that is:

D={1,2,3,4,5,6,7,8,9,10,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30}

The set D is required to satisfy the following conditions:

1. Each line in π contains at least five points of the set D.
2. Each element of D belongs to at least one (5-secant).

To satisfy the second condition, the point11 is added to D, then:

β_5 = DU {11}

β_5={1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30}.

Thus, β_5 is a trivial minimal (30,5)-blocking set of cardinality 6q, since it includes a whole line of π, (see table (2.6)). Moreover, the complement of β_5 does not represent an arc because a (k,n)-arc cannot exist for n < 2.

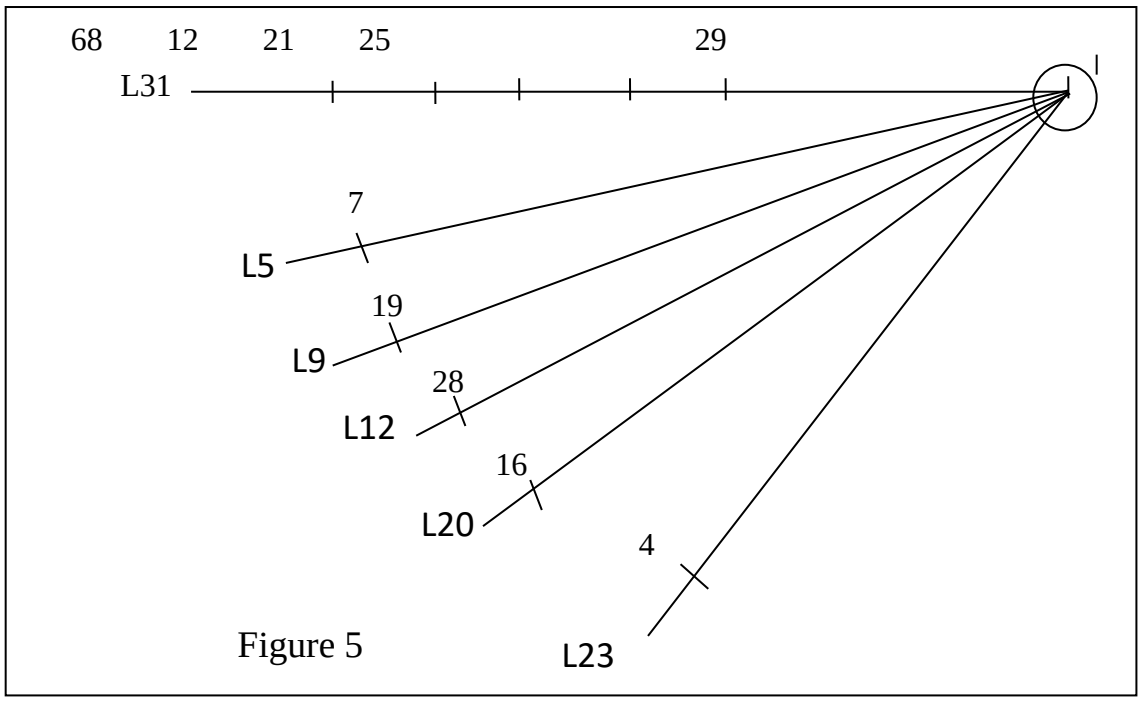


Figure 5

Table (2.6)

i	$\beta_5 \cap \mathrm{L_i}$	$\lvert\beta_5 \cap L_i\rvert$
1	{2,7,12,17,22,27}	6
2	{1,7,8,9,10,11}	6
3	{6,7,16,20,24,28}	6
4	{4,7,14,21,23,30}	6
5	{5,7,15,18,26,29}	6
6	{3,7,13,19,25}	5
7	{1,2,3,4,5,6}	6
8	{2,11,16,21,26}	5
9	{2,9,14,19,24,29}	6
10	{2,10,15,20,25,30}	6
11	{2,8,13,18,23,28}	6
12	{1,27,28,29,30}	5
13	{6,11,15,19,23,27}	6
14	{4,9,16,18,25,27}	6
15	{5,10,13,21,24,27}	6
16	{3,8,14,20,26,27}	6
17	{1,17,18,19,20,21}	6
18	{5,11,14,17,25,28}	6
19	{6,9,13,17,26,30}	6
20	{3,10,16,17,23,29}	6
21	{4,8,15,17,24}	5
22	{1,22,23,24,25,26}	6

23	{4,11,13,20,22,29}	6
24	{3,9,15,21,22,28}	6
25	{6,10,14,18,22}	5
26	{5,8,16,19,22,30}	6
27	{1,12,13,14,15,16}	6
28	{3,11,12,18,24,30}	6
29	{5,9,12,20,23}	5
30	{4,10,12,19,26,28}	6
31	{6,8,12,21,25,29}	6

3.2 The Second Method for Constructing Minimal (b,t)-Blocking Sets of Conic-Type.[4],[9],[12]

Let B = C $\cup \mathscr{L} \cup$ {P}\ { P_1, P_2}, where C denotes a conic and the line $\mathscr{L}$ is a bisecant of C, such that C $\cap \mathscr{L}$ ={ P_1, P_2},the point P denotes the intersection point of the tangents to C at P_1 and P_2. Hence, B is referred to as a conic-type minimal (2q-1,1)-blocking set.

3.2.1 Construction of a Minimal (b,1)-Blocking Set.

We consider a conic C_1 in PG (2,5)defined byC_1= {1,2,7,13,20,26} and a line L_{22} = {1,22,23,24,25,26}. It is observed that $C_1 \cap L_{22}$= {1,26}. The linesL_{12} and L_{30} are tangent to C_1at the points 1 and 26, respectively, and their intersection is $L_{12} \cap L_{30}$= {28},(see figure 6), then:

β_1 = C_1 $\cup L_{22} \cup$ {28}\{1,26} ={2,7,13,20,22,23,24,25,28}represents a minimal (9,1)-blocking set in PG (2,5) as presented in table (2.7).

The set β_1 is required to satisfy the following conditions:

1. Every line of π contains at least one point of β_1.
2. Each element of $\beta_1$belongs to at least one (1-secant).

Moreover, there exists a line $\mathscr{L} \in \pi$ that intersects β_1 only at the point P.

Let K_5= π \ β_1={1,3,4,5,6,8,9,10,11,12,14,15,16,17,18,19,21,26,27,29,30,31} is a complete (22,5)-arc.

Therefore, applying theorem (1.12), we obtain a projective linear code with parameters [22,3,17].

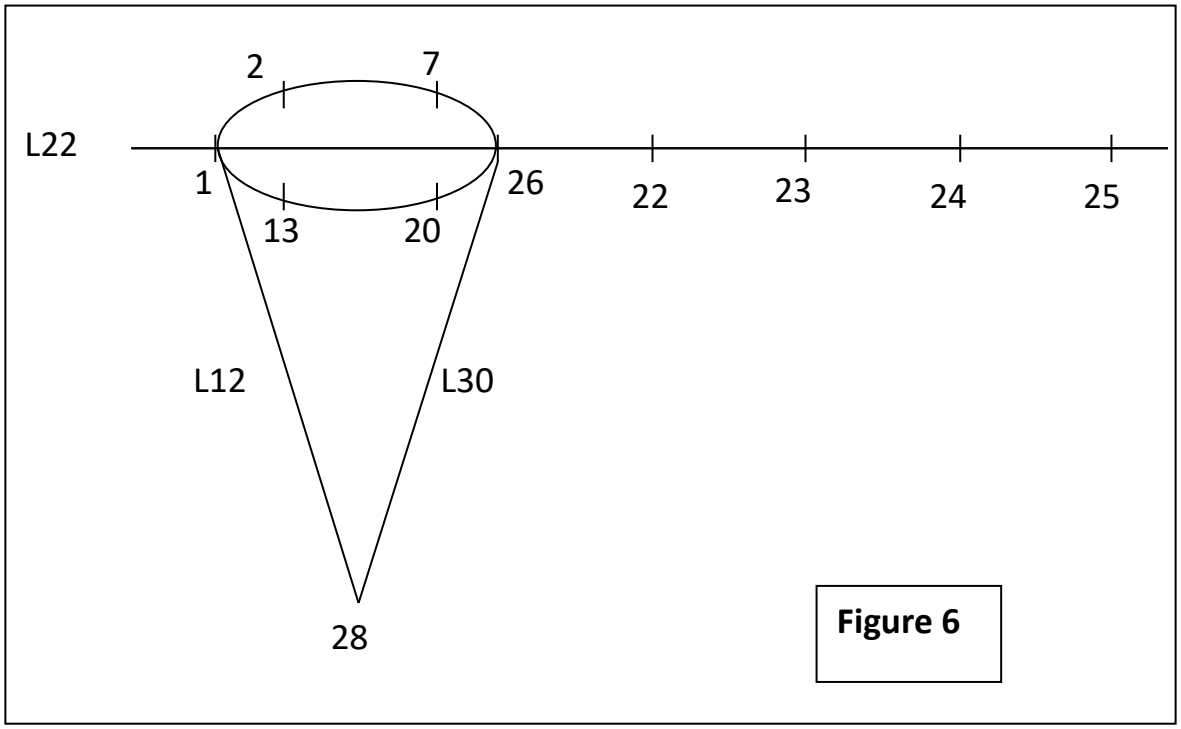


Figure 6

i	$\beta_1 \cap L_i$	$K_5 \cap L_i$	$\lvert\beta_1 \cap L_i\rvert$	$\lvert k_5 \cap L_i\rvert$
1	{2,7,22}	{12,17,27}	3	3
2	{7}	{1,8,9,10,11}	1	5
3	{7,20,24,28}	{6,16}	4	2
4	{7,23}	{4,14,21,30}	2	4
5	{7}	{5,15,18,26,29}	1	5
6	{7,13,25}	{3,19,31}	3	3
7	{2}	{1,3,4,5,6}	1	5
8	{2}	{11,16,21,26,31}	1	5
9	{2,24}	{9,14,19,29}	2	4
10	{2,20,25}	{10,15,30}	3	3
11	{2,13,23,28}	{8,18}	4	2
12	{28}	{1,27,29,30,31}	1	5
13	{23}	{6,11,15,19,27}	1	5
14	{25}	{4,9,16,18,27}	1	5
15	{13,24}	{5,10,21,27}	2	4
16	{20}	{3,8,14,26,27}	1	5
17	{20}	{1,17,18,19,21}	1	5
18	{25,28}	{5,11,14,17}	2	4
19	{13}	{6,17,26,30}	1	5
20	{23}	{3,10,16,17,29}	1	5
21	{24}	{4,8,15,17,31}	1	5
22	{22,23,24,25}	{1,26}	4	2

23	{13,20,22}	{4,11,29}	3	3
24	{22,28}	{3,9,15,21}	2	4
25	{22}	{6,10,14,18,31}	1	5
26	{22}	{5,8,16,19,30}	1	5
27	{13}	{1,12,14,15,16}	1	5
28	{24}	{3,11,12,18,30}	1	5
29	{20,23}	{5,9,12,31}	2	4
30	{28}	{4,10,12,19,26}	1	5
31	{25}	{6,8,12,21,29}	1	5

3.2.2 Construction of a Minimal (b,2)-Blocking Set.

From section 3.2.1 we take β_1= {2,7,13,20,22,23,24,25,28} be a minimal (9,1)-blocking set of Conic-type.

We construct another (9,1)-blocking set we say α_1:

$\alpha_1 = C_2 \cup L_6 \cup \{6\} \setminus \{7,13\}$, where C_2 = {1,2,7,13,21,29}, L_6 = {3,7,13,19,25,31}.It is observed that $C_2 \cap L_6$= {7,13}. The lines L_3 and L_{19} are tangent toC_2 at the points 7 and 13, respectively, and their intersection is $L_3 \cap L_{19}$= {6}, (see figure 7) then: α_1= {1,2,3,6,19,21,25,29,31} which forms a minimal (9,1)-blocking set in PG (2,5). Next, a minimal (b,2)-blocking set is obtained from the sets $\beta_1$and $\alpha_1$by setting

A= $\alpha_1 \cup \beta_1$={1,2,3,6,7,13,19,20,21,22,23,24,25,28,29,31}

The set A is required to satisfy the following conditions:

1. Every line of π contains at least two points of A.
2. Each element of A lies on at least one (2-secant).

To satisfy the first condition, the point 31 is removed from A, whereas the point 4 is added so that the second condition remains satisfied. Hence,

β_2 = A ∪ {4}\ {31}

β_2={1,2,3,4,6,7,13,19,20,21,22,23,24,25,28,29}is a minimal (16,2)-blocking set of cardinality 3q+1.

Let K_4= π \ β_2 ={5,8,9,10,11,12,14,15,16,17,18,26,27,30,31} is complete (15,4)-arc.(see table (2.8)). Therefore, applying theorem (1.12), we obtain a projective linear code with parameters [15,3,11].

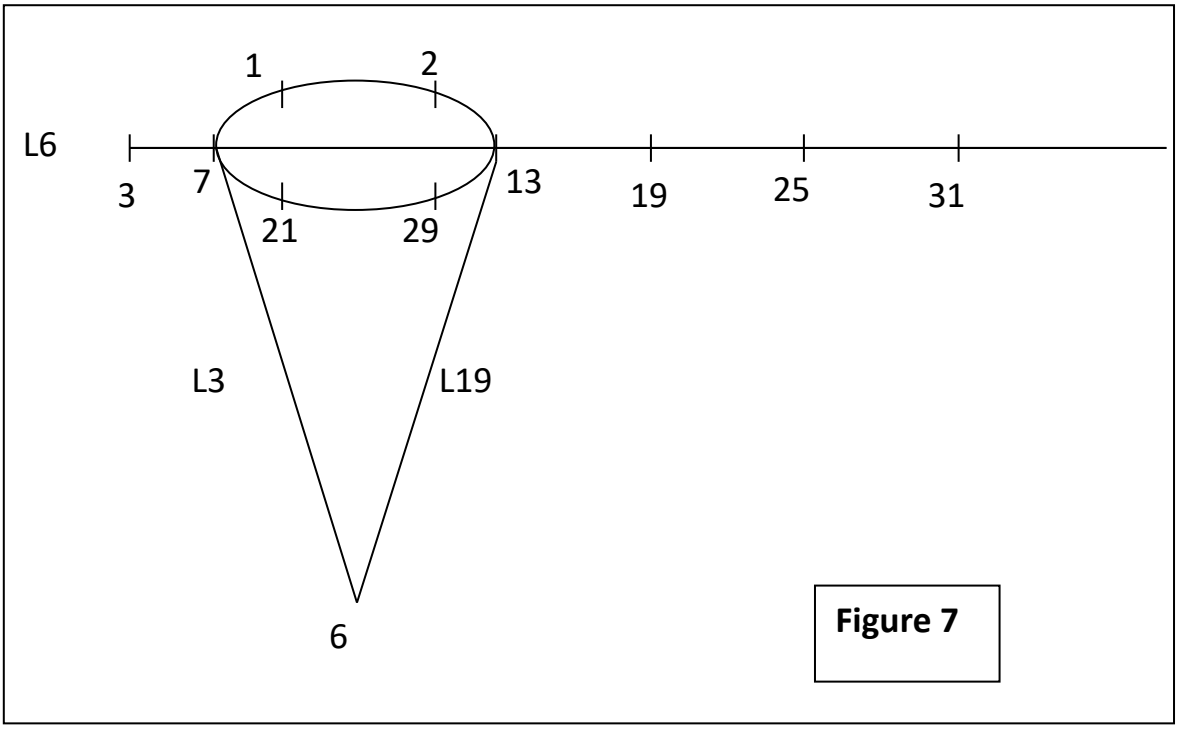


Figure 7

Table (2.8)

i	$\beta_2 \cap L_i$	$K_4 \cap L_i$	$\lvert\beta_2 \cap L_i\rvert$	$\lvert k_4 \cap L_i\rvert$
1	{2,7,22}	{12,17,27}	3	3
2	{1,7}	{8,9,10,11}	2	4
3	{6,7,20,24,28}	{16}	5	1
4	{4,7,21,23}	{14,30}	4	2
5	{7,29}	{5,15,18,26}	2	4
6	{3,7,13,19,25}	{31}	5	1
7	{1,2,3,4,6}	{5}	5	1
8	{2,21}	{11,16,26,31}	2	4
9	{2,19,24,24}	{9,14}	4	2
10	{2,20,25}	{10,15,30}	3	3
11	{2,13,23,28}	{8,18}	4	2
12	{1,28,29}	{27,30,31}	3	3
13	{6,19,23}	{11,15,27}	3	3
14	{4,25}	{9,16,18,27}	2	4
15	{13,21,24}	{5,10,27}	3	3
16	{3,20}	{8,14,26,27}	2	4
17	{1,19,20,21}	{17,18}	4	2
18	{25,28}	{5,11,14,17}	2	4
19	{6,13}	{9,17,26,30}	2	4
20	{3,23,29}	{10,16,17}	3	3
21	{4,24}	{8,15,17,31}	2	4
22	{1,22,23,24,25}	{26}	5	1

23	{4,13,20,22,29}	{11}	5	1
24	{3,21,22,28}	{9,15}	4	2
25	{6,22}	{10,14,18,31}	2	4
26	{19,22}	{5,8,16,30}	2	4
27	{1,13}	{12,14,15,16}	2	4
28	{3,24}	{11,12,18,30}	2	4
29	{20,23}	{5,9,12,31}	2	4
30	{4,19,28}	{10,12,26}	2	4
31	{6,21,25,29}	{8,12}	4	2

3.2.3 Construction of a Minimal (b,3)-Blocking Set.

From section 3.2.2 we take two minimal (9,1)-blocking set of conic-type:
β_1={2,7,13,20,22,23,24,25,28}
α_1={1,2,3,6,19,21,25,29,31}
We construct another (9,1)-blocking set we say γ_1:
$\gamma_1 = C_3 \cup L_{15} \cup \{4\} \setminus \{13,24\}$, where C_3 = {1,2,7,13,24,30}, L_{15} = {5,10,13,21,24,27}.It is observed that $C_3 \cap L_{15}$= {13,24}. The lines L_{21} and L_{23} are tangents lines to C_3 at the points 13 and 24, respectively, and their intersection is $L_3 \cap L_{19}$= {6}, (see figure 8), then:
γ_1= {1,2,4,5,7,10,21,27,30} which forms a minimal (9,1)-blocking set in PG (2,5).
Next, a minimal (b,3)-blocking set is constructed from the sets β_1, α_1 and $\gamma_1$as follows:
Let B= $\alpha_1 \cup \beta_1 \cup \gamma_1$={1,2,3,4,5,6,7,10,13,19,20,21,22,23,24,25,27,28,29,30,31}.
The set B is required to satisfy the following conditions:
1. Every line of π contains at least three points of B.
2. Each element of B lies on at least one (3-secant).

To satisfy the first condition, the points 19 and 22 are removed from B, whereas the point 16 is added so that the second condition remains satisfied. Hence,

β_3 = A ∪ {16}\ {19,22}, that is:

β_3={1,2,3,4,5,6,7,10,13,16,20,21,23,24,25,27,28,29,30,31}defines a minimal (20,3)-blocking set of size 4q. Moreover, β_3 is trivial since it contains an entire line of π.
Let K_3= π \ β_3 ={8,9,11,12,14,15,17,18,19,22,26} is complete (11,3)-arc.(see table (2.9))
Therefore, applying theorem (1.12), we obtain a projective linear code with parameters [11,3,8].

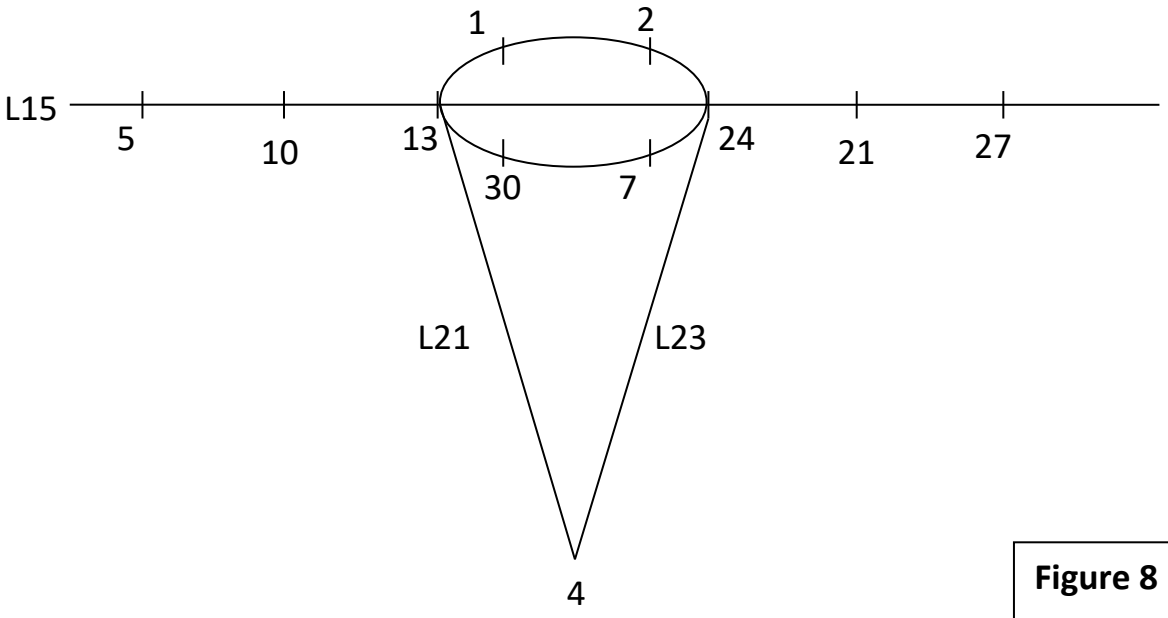


Figure 8

Table (2.9)

i	$\beta_3 \cap L_i$	$K_3 \cap L_i$	$\lvert\beta_3 \cap L_i\rvert$	$\lvert k_3 \cap L_i\rvert$
1	{2,7,27}	{12,17,22}	3	3
2	{1,7,10}	{8,9,11}	3	3
3	{6,7,16,20,24,28}	Φ	6	0
4	{4,7,21,23,30}	{14}	5	1
5	{5,7,29}	{15,18,26}	3	3
6	{3,7,13,25,31}	{19}	5	1
7	{1,2,3,4,5,6}	Φ	6	0
8	{2,16,21,31}	{11,26}	4	2
9	{2,24,29}	{9,14,19}	3	3
10	{2,10,20,25,30}	{15}	5	1
11	{2,13,23,28}	{8,18}	4	2
12	{1,27,28,29,30,31}	Φ	6	0
13	{6,23,27}	{11,15,19}	3	3
14	{4,16,25,27}	{9,18}	4	2
15	{5,10,13,21,24,27}	Φ	6	0
16	{3,20,27}	{8,14,26}	3	3
17	{1,20,21}	{17,18,19}	3	3
18	{5,25,28}	{11,14,17}	3	3
19	{6,13,30}	{9,17,26}	3	3
20	{3,10,16,23,29}	{17}	5	1
21	{4,24,31}	{8,15,17}	3	3
22	{1,23,24,25}	{22,26}	4	2

23	{4,13,20,29}	{11,22}	4	2
24	{3,21,28}	{9,15,22}	3	3
25	{6,10,31}	{14,18,22}	3	3
26	{5,16,30}	{8,19,22}	3	3
27	{1,13,16}	{12,14,15}	3	3
28	{3,24,30}	{11,12,18}	3	3
29	{5,20,23,31}	{9,12}	4	2
30	{4,10,28}	{12,19,26}	3	3
31	{6,21,25,29}	{8,12}	4	2

3.2.4 Construction of a Minimal (b,4)-Blocking Set.

From section 3.2.3 we take three minimal (9,1)-blocking set of conic-type:
β_1={2,7,13,20,22,23,24,25,28}
α_1={1,2,3,6,19,21,25,29,31}
γ_1={1,2,4,5,7,10,21,27,30}
We construct another (9,1)-blocking set we say ε_1:
$\varepsilon_1 = C_1 \cup L_{16} \cup \{12\} \setminus \{20,26\}$, where C_1 = {1,2,7,13,20,26}, L_{16} = {3,8,14,20,26,27}.It is observed that $C_1 \cap L_{16}$= {20,26}. The lines L_{29} and L_{30} are tangents lines to C_1 at the points 20 and 26, respectively, and their intersection is $L_{29} \cap L_{30}$= {12}, (see figure 9). Then, ε_1={1,2,3,7,8,12,13,14,27} thus obtaining a minimal (9,1)-blocking set in PG (2,5).
Next, a minimal (b,4)-blocking set is constructed from the sets β_1, α_1, $\gamma_1$and $\varepsilon_1$as follows:
Let C =$\alpha_1 \cup \beta_1 \cup \gamma_1 \cup \varepsilon_1$ that is:
C = {1,2,3,4,5,6,7,8,10,12,13,14,19,20,21,22,23,24,25,27,28,29,30,31}.
The set C is required to satisfy the following conditions:
1. Every line of π contains at least four points of C.
2. Each element of C lies on at least one (4-secant).
The set A is modified by adding the points 16 and 26 to it and removing the point 2 from it. Consequently, the required conditions are satisfied and the following set is obtained:
β_4 = C$\cup$ {16,26}\ {2}, that is:
β_4={1,3,4,5,6,7,8,10,12,13,14,16,19,20,21,22,23,24,25,26,27,28,29,30,31}is a minimal (25,4)-blocking set of cardinality 5q, which is a trivial because β_4 includes a whole line of π.
Let K_2= π \ β_4 = {2,9,15,17,18,15} is complete (6,2)-arc.(see table (2.10))
Therefore, applying theorem (1.12), we obtain a projective linear code with parameters [6,3,4].

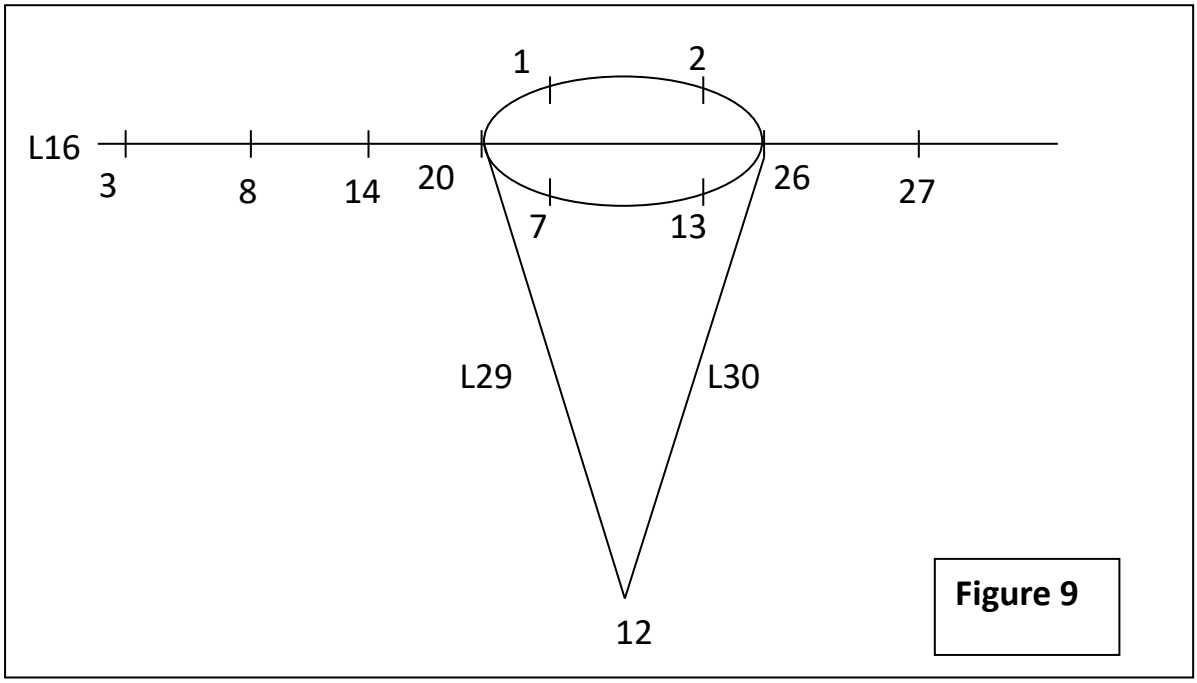


Figure 9

Table (2.10)

i	$\beta_4 \cap L_i$	$K_2 \cap L_i$	$\lvert\beta_4 \cap L_i\rvert$	$\lvert k_2 \cap L_i\rvert$
1	{7,12,22,27}	{2,17}	4	2
2	{1,7,8,10}	{9,11}	4	2
3	{6,7,16,20,24,28}	Φ	6	0
4	{4,7,14,21,23,30}	Φ	6	0
5	{5,7,26,29}	{15,18}	4	2
6	{3,7,13,19,25,31}	Φ	6	0
7	{1,3,4,5,6}	{2}	5	1
8	{16,21,26,31}	{2,11}	4	2
9	{14,19,24,29}	{2,9}	4	2
10	{10,20,25,30}	{2,15}	4	2
11	{8,13,23,28}	{2,18}	4	2
12	{1,27,28,29,30,31}	Φ	6	0
13	{6,19,23,27}	{11,15}	4	2
14	{4,16,25,27}	{9,18}	4	2
15	{5,10,13,21,24,27}	Φ	6	0
16	{3,8,14,20,26,27}	Φ	6	0
17	{1,19,20,21}	{17,18}	4	2
18	{5,14,25,28}	{11,17}	4	2
19	{6,13,26,30}	{9,17}	4	2
20	{3,10,16,23,29}	{17}	5	1
21	{4,8,24,31}	{15,17}	4	2
22	{1,22,23,24,25,26}	Φ	6	0
23	{4,13,20,22,29}	{11}	5	1

24	{3,21,22,28}	{9,15}	4	2
25	{6,10,14,22,31}	{18}	5	1
26	{5,8,16,19,22,30}	Φ	6	0
27	{1,12,13,14,16}	{15}	5	1
28	{3,12,24,30}	{11,18}	4	2
29	{5,12,20,23,31}	{9}	5	1
30	{4,10,12,19,26,28}	Φ	6	0
31	{6,8,12,21,25,29}	Φ	6	0

3.2.5 Construction of a Minimal (b,5)-Blocking Set.

From section 3.2.4 we take four minimal (9,1)-blocking set of Conic-type: β_1={2,7,13,20,22,23,24,25,28}, α_1={1,2,3,6,19,21,25,29,31
γ_1={1,2,4,5,7,10,21,27,30}, ε_1= {1,2,3,7,8,12,13,14,27}
We construct another (9,1)-blocking set we say δ_1:
$\delta_1 = C_2 \cup L_{23} \cup \{17\} \setminus \{13,29\}$, where C_2 = {1,2,7,13,21,29}, L_{23} = {4,11,13,20,22,29}.It is observed that $C_2 \cap L_{23}$= {13,29}. The lines L_{19} and L_{20} are tangent lines to C_2 at the points 13 and 29, respectively, and their intersection is $L_{19} \cap L_{20}$={17}, (see figure 10) then: δ_1= {1,2,4,7,11,17,20,21,22} which forms a minimal (9,1)-blocking set in PG (2,5). Next, a minimal (b,5)-blocking set is constructed from the sets β_1, $\alpha_1, \gamma_1, \varepsilon_1$and $\delta_1$as follows:
Let D =$\alpha_1 \cup \beta_1 \cup \gamma_1 \cup \varepsilon_1 \cup \delta_1$, that is:
D={1,2,3,4,5,6,7,8,10,11,12,13,14,17,19,20,21,22,23,24,25,27,28,29,30,31}
The set D is required to satisfy the following conditions:
1. Every line of π contains at least five points of D.
2. Each element of D lies on at least one (5-secant). In order to satisfy the required conditions, the points 9,15,18, and 26 are incorporated into D. Hence, β_5 = D ∪ {9,15,18,26}
β_5={1,2,3,4,5,6,7,8,9,10,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31}
defines a minimal (30,3)-blocking set of size 6q. Moreover, β_5 is trivial since it contains an entire line of π, (see table (2.11)).
Furthermore, the complement of β_5 fails to be an arc because no (k,n)-arc exist when n < 2.

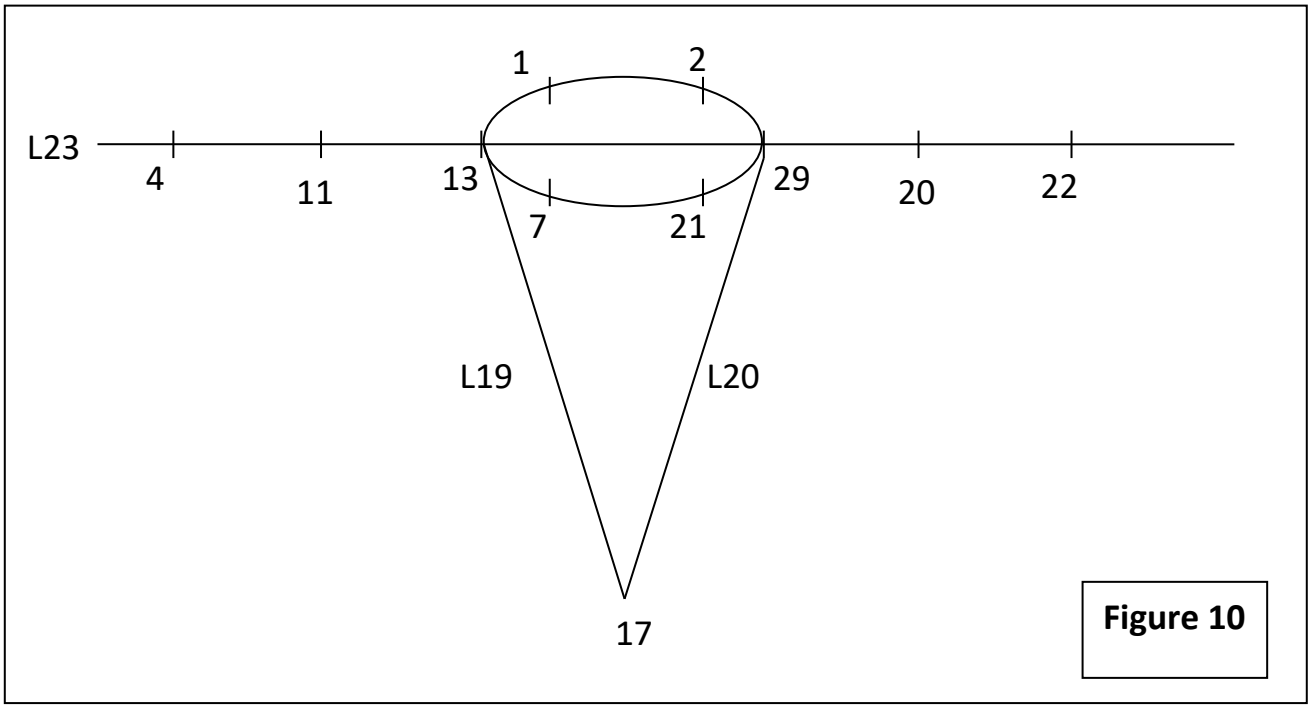


Figure 10

Table (2.11)

i	$\beta_5 \cap L_i$	$\lvert\beta_4 \cap L_i\rvert$
1	{2,7,12,17,22,27}	6
2	{1,7,8,9,10,11}	6
3	{6,7,20,24,28}	5
4	{4,7,14,21,23,30}	6
5	{5,7,15,18,26,29}	6
6	{3,7,13,19,25,31}	6
7	{1,2,3,4,5,6}	6
8	{2,11,21,26,31}	5
9	{2,9,14,19,24,29}	6
10	{2,10,15,20,25,30}	6
11	{2,8,13,18,23,28}	6
12	{1,27,28,29,30,31}	6
13	{6,11,15,19,23,27}	6
14	{4,9,18,25,27}	5
15	{5,10,13,21,24,27}	6
16	{3,8,14,20,26,27}	6
17	{1,17,18,19,20,21}	6
18	{5,11,14,17,25,28}	6
19	{6,9,13,17,26,30}	6
20	{3,10,17,23,29}	5
21	{4,8,15,17,24,31}	6
22	{1,22,23,24,25,26}	6

23	{4,11,13,20,22,29}	6
24	{3,9,15,21,22,28}	6
25	{6,10,14,18,22,31}	6
26	{5,8,19,22,30}	5
27	{1,12,13,14,15}	5
28	{3,11,12,18,24,30}	6
29	{5,9,12,20,23,31}	6
30	{4,10,12,19,26,28}	6
31	{6,8,12,21,25,29}	6

4. Conclusions

1. Using both methods, the minimal (b,t)-blocking sets were obtained as presented in the following table:

t	$\beta_t$obtained by the Redei-type method	β_t obtained by the Conic- type method	Minimal β_t
1	10	9	9
2	15	16	15
3	20	20	20
4	25	25	25
5	30	30	30

.

2. Using both methods, the maximal (k,n)-arcs were obtained as presented in the following table:

n	K_{n} obtained by the Redei-type method	K_{n} obtained by the Conic-type method	Maximal K_{n}
5	21	22	22
4	16	15	16
3	11	11	11
2	6	6	6

3. In both methods, the minimal (b,1)-blocking set and the minimal (b,2)-blocking set are non-trivial, while, minimal (b,3)-blocking set , minimal (b,4)-blocking set and minimal (b,5)-blocking set are trivial.

4. Through the construction of blocking sets, we obtained four linear projective codes as follows:
[21,3,16],[16,3,12],[11,3,8],[6,3,4]

References:

1. Ruman Daskalov, (2008), A Geometric Construction of (38,3)-Blocking set in PG (2,13) and the Related [145,3,133] Code, Discrete Mathematics Technical University of Gabrovo, Bulgaria, 308 (1341-1345).
2. Bruen, A. A. (1971). Blocking sets in finite projective planes. Siam Journal on Applied Mathematics, 21(3), 380-392.
3. Szőnyi, T. (1993). Blocking sets in finite projective planes. Finite Fields and Their Applications, 3(3), 187-202.
4. Blokhuis, A. (1994). On the size of a blocking set in PG (2, q).Combinatorica, 14(1), 111-114.
5. Hoder Leanne D., (2001), "Blocking sets of Conics", Ph. D. Thesis, University of Colorado, Denver.
6. Al-Mukhtar , A.S., Ahmed, and Hassan, A.A.,(2014), "The Construction of Complete (kn,n)-arcs in The Projective plane PG(2,5) By Geometric Method ,With the Related Blocking sets and Projective Cods" Baghdad Science Journal, Vol. 11, No. 2, PP.242-248.
7. Al-Mukhtar , A.S., Ahmed, and kareem,F,F.,(2004), "Complete (k,4)-arcs on Projective plane PG(2,7) over Galois Field", Journal of Al-Qadisiya Pure, Vol.9, pp.1-11.
8. Hirschfeld, J.W.P., (1998), Projective Geometries Over Finite Second Edition, Oxford University Press.
9. Abdul-Hussein, M. A,(1997),"Classification of (k,4)-Arcs In Projective Plane of Order Five " , University of Basrah, Iraq .
10. Ball, S., (1995), Multiple Blocking set and Arcs in Finite Plane , school of Mathematical and Physical Sciences, of Sussex, Brighton,QH, U.K, pp.1-16 .
11. Hassan, U.A.,(2013), "The Reverse Construction for the Complete Arcs in the Projective Plane PG(2,p) Over Galois Field GF(p) by Using Geometric Method , M.Sc. Thesis, University of Baghdad , Iraq.
12.Adrian, A. and Reuben Sander, (1976),"Introduction of Finite Projective Plane ", University of Chicago.
13. Al-Helo, A.J., (2000),"Construction of STS(V) and SQS(V)", M.Sc. Thesis University of Baghdad, Iraq.
14. Al-Mukhtar, A.S., Ahmed, and Hassan, A.A., (2014), "The Construction of Complete (kn, n)-arcs in The Projective plane PG (2,5) By Geometric Method, With the Related Blocking sets and Projective Cods" Baghdad Science Journal, Vol. 11, No. 2, pp. 242-248.
15. Abdul-Hussein, M. A, (1997),"Classification of (k,4)-Arcs in Projective Plane of Order Five ", University of Basrah, Iraq.
16. Hassan, U.A., (2013), "The Reverse Construction for the Complete Arcs in the Projective Plane PG(2,p) Over Galois Field GF(p) by Using Geometric Method , M.Sc. Thesis, University of Baghdad , Iraq.
17. Burton, D.M.,(1972),"Abstract of Linear Algebra", University of New Hampshire. 18. Burton, D.M.,(1967),"Introduction to Modern Abstract Algebra", University of New Hampshire.