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Anti - hesitant fuzzy module

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Abstract: In this paper, we study the definition of hesitant fuzzy set, with some its properties . We introduced the definition of anti-hesitant fuzzy module with the result equivalent of definition . Also, we study the relationship between this concept and the concept hesit-ant fuzzy module with prove some results . We establish anti-image of hesitant fuzzy set, and through which we have the image and the inverse image of the anti-hesitant fuzzy module with respect the homomorphism between modules.

Keywords: hesitant fuzzy set , hesitant fuzzy module , Anti - hesitant fuzzy module**.**

1- Introduction

The concept of fuzzy sets was first introduced by Zadeh in 1965 as an extension of the classical notion of a set [12] **.** Torra (2010) proposed a new generalized of fuzzy set called hesitant fuzzy set (HFS) and he defined the complement, union inter- section of HFS [8] **.** Xia and Xu (2011a) originally gave the mathematical expressions of HFS, and some operational laws for HFS, such as the addition and multiplication [9]. Afterw- ards, Liao and Xu (2014a) introduced the subtraction and division operations over HFSs [11] **.** Bin Zhu (2012) introduced dual hesitant fuzzy set (DHFS) , is an emerging research area due to its capability of representing opposite epistemic degr- ees together with hesitancy [10] **.**The concept of fuzzy module was introduced by Negoita and Relescu [5] in (1975)**.** sharma (2011) introduced the notion of anti-fuzzy modules of a module and studied some of their properties [7]. Deepak and John (2014) introduced the notion of hesitant fuzzy subgroups [2] . Deepa et al. (2017) introduced the notion of dual hesitant fuzzy subring , dual hesitant fuzzy ideal and the image, pre-image of dual hesitant fuzzy subring under a homomorphism are discussed [3]**.** Kim et al. (2019) defined hesitant fuzzy subgroupoid, hesitant fuzzy subring , Hesitant fuzzy ideal [4]**.** Ali and Mohammed (2021) introduce the notions of hesitant fuzzy ideal, hesitant fuzzy prime ideal , hesitant fuzzy strongly prime ideal and hesitant fuzzy 3-prime ideals of a ring R [1] **.** In this paper, we introduced the notion of anti-hesitant fuzzy module and studied some of their properties. Also, we proved the relationship between anti-hesitant fuzzy module and the hesitant fuzzy module with some characteristics around it .

2-Preliminaries

In this section, we shall give the concept of hesitant fuzzy set with some basic definitions and properties about it which are used in the next section.

Definition (2.1) [8]

$Let\, X$ be a reference set a hesitant fuzzy set (HFS) A on X is defined in terms of a function $h_A(x)$ when applied to X returns a finite subset of [0 , 1] i.e. $A=\{(x\,,h_A(x))\big|\, x\in X\}$ where $h_A(x)$ is a set of some differences values in [0 , 1], representing the possible membership degrees of the element $x\in X\; to\; A$ for convenience, we call $h_A(x)$ a hesitant fuzzy element (HFE).

Example (2.2)[8]

Let $X=\{x_1, x_2, x_3\}$ be reference set $h_A(x_1)=\{0.2, 0.4, 0.5\}, h_A(x_2)=\{0.3, 0.4\}$, $h_A(x_3)=\{0.2, 0.3, 0.5, 0.6\}$ be the HFES of $x_i\ (x_i=1,2,3)$ to a set A, respectively. then A can be considered as a HFS, i.e. $A=\{(x_1, h_A(x_1)), (x_2, h_A(x_2)), (x_3, h_A(x_3)))\}$.

Definition (2.3) [6]

Let X be a reference set, then we define some types of hesitant fuzzy set. empty set: $h^0(x)=\{(x,\{0\})\ \forall x\in X\}=\{0\}$, full set : $h^1(x)=\{(x,\{1\})\ \forall x\in X\}=\{1\}$, complete ignorance for $x\in X$ (all is possible) : $h^{[0,1]}(x)=\{(x,[0,1])\ \forall x\in X\}=[0,1]$ and set for a nonsense $h^{\emptyset}(x)=\{(x,\emptyset)\ \forall x\in X\}=\emptyset$ such that $h^{\emptyset}(x)\subseteq h(x)\subseteq h^{[0,1]}(x)$

Definition (2.4) [2]

Let A , B *be a* hesitant fuzzy set of a set X and Y respectively and let $f: X\to Y$ be a mapping. Then the image of A under f , denoted by $f(A)$, is a hesitant fuzzy set in Y defined as follows: for each $y\in Y$

$$f(A)(y)=\begin{cases}\cup_{x\in f^{-1}(y)} A(x) & iF\ \ f^{-1}(y)\neq\emptyset\\ \{0\} & \text{otherwise}\end{cases}$$

the invers of B under *f*, denoted by $f^{-1}(B)$, is a hesitant fuzzy set in X defined as follow : for each $x\in X$, $f^{-1}(B)(x)=B\left(f(x)\right)$.

Definition (2.5) [6+8+9]

For any $\mathrm{h}, \mathrm{h}_1, \mathrm{h}_2$ in HFSs , some operations on them can be described as follows

(1) $(h_1\cup h_2)(x)==\cup_{y_1\in h_1, y_2\in h_2}\ max\{y_1, y_2\}$
(2) $(h_1\cap h_2)(x)=\ \cup_{y_1\in h_1, y_2\in h_2} min\{y_1, y_2\}$
(3) complement $h^c(x)=1-h(x)=\{1-y\mid y\in h(x)\}$
(4) $h^{\lambda}(x)=\{y^{\lambda}| y\in h(x)\}$
(5) $\lambda h(x)=\{1-(1-y)^{\lambda}\mid y\in h(x)\}$
(6) $(h_1\otimes h_2)(x)=\{y_1y_2\mid y_1\in h_1(x), y_2\in h_2(x)\}$
(7) $(h_1\oplus h_2)(x)=\{y_1+y_2-y_1y_2\mid y_1\in h_1(x), y_2\in h_2(x)\}$

Theorem (2,6) [9]:

For three HFEs $h, h_1, h_2$the followings are valid

(1) $\mathrm{h}_1^c\cup\mathrm{h}_2^c=(\mathrm{h}_1\cap\mathrm{h}_2)^c$
(2) $\mathrm{h}_1^c\cap\mathrm{h}_2^c=(\mathrm{h}_1\cap\mathrm{h}_2)^c$
(3) $(\mathrm{h}^c)^{\lambda}=(\lambda\mathrm{h})^c$
(4) $\lambda(\mathrm{h}^c)=(\mathrm{h}^{\lambda})^c$
(5) $\mathrm{h}_1^c\oplus\mathrm{h}_2^c=(\mathrm{h}_1\otimes\mathrm{h}_2)^c$
(6) $\mathrm{h}_1^c\otimes\mathrm{h}_2^c=(\mathrm{h}_1\oplus\mathrm{h}_2)^c$.
(7)

Definition (2.7) [2]

Let G be a group and h is hesitant fuzzy set . Then h is called a hesitant fuzzy subgroup (in short, HFG) in G, if it satisfies the following conditions: for any x, y ∈ G,

(i) $h(x-y) \supseteq h(x) \cap h(y)$ (ii) $h(x^{-1}) \supseteq h(x)$

We will denote the set of all HFGs in G by HFG(G).

Definition (2.8) [4]

Let R be a ring and h be hesitant fuzzy set. Then h is a hesitant fuzzy subring of R if and only if for all $x, y \in R$.
(i) $h(x-y) \supseteq h(x) \cap h(y)$ (ii) $h(xy) \supseteq h(x) \cap h(y)$.

Definition (2.9) [4]

Let R be a ring and h be a hesitant fuzzy subring of R. Then $h \in HFI(R)$ [resp. HFLI(R) and HFRI(R)] if and only if for any $x, y \in R$,
(i) $h(x-y) \supseteq h(x) \cap h(y)$
(ii) $h(xy) \supseteq h(x) \cup h(y)[resp. h(xy) \supseteq h(y)\ and\ h(xy) \supseteq h(x)]$

Definition (2.10)
Let X be a reference set and h be an hesitant fuzzy set of X . Then
(1) the $E-$ upper $(level\ set)$ of h defined $h_E=\{x \in M: h(x) \supseteq E\}$.Such that the $E \subseteq [0,1]$
(2) the E^* $-$ lower $(level\ set)$ of h defined $h_{E^*} = \{x \in M: h(x) \subseteq E^*\}$. Such that the $E^* \subseteq [0,1]$

3 – Anti - hesitant fuzzy module

In this section , we define the concept of anti-hesitant module and study the relationship between this concept and the concept of hesitant fuzzy module with prove some results about it .

Definition (3.1)
Let h be an hesitant fuzzy set over R−module M then h is said to be hesitant fuzzy module (in short, HFM) over R-module M if for all $x, y \in M, r \in R$
(i) $h(x-y) \supseteq h(x) \cap h(y)$ (ii) $h(rx) \supseteq h(x)$.

Example (3.2)
Let h be a hesitant fuzzy set of Z_2-module $(Z_2, +_2)$ such that
$h = \{(0, h(0)), (1, h(1))\}$, $h(0) = \{0.1, 0.3, 0.6, 0.8\}$, $h(1) = \{0.1, 0.3\}$
Then h is hesitant fuzzy module of Z_2-module Z_2.

Proposition (3.3)
Let h, h_1 and h_2 are two hesitant fuzzy module of R-module M, then $h^\lambda, \lambda h$, $h_1 \cap h_2$, $h_1 \otimes h_2$ and $h_1 \oplus h_2$ are hesitant fuzzy module of R-module M .
It can be easily Prove.

Theorem (3.4)

Let h be a hesitant fuzzy set of left $R-$ module M. Then h is hesitant fuzzy module of R-module M iff h_E , $E \subseteq [0,1]$ is submodule of R-module M.
It can be easily Prove

Proposition (3.5)

Let f be an epimorphism from R-module M into R-module N, if B hesitant fuzzy module of N then $f^{-1}(B)$ hesitant fuzzy module of M.
It can be easily Prove.

Proposition (3.6)

Let f be an onto homomorphism from R-module M into R-module N if A hesitant fuzzy module of M then $f(A)$ hesitant fuzzy module of R-module N.
It can be easily Prove.

Definition (3.7)

Let h be an hesitant fuzzy set of an R−module M then h is called an anti-hesitant fuzzy module (in short, A-HFM) of M
if for all $x, y \in M, r \in R$, we have
(i) $h(x-y) \subseteq h(x) \cup h(y)$ (ii) $h(rx) \subseteq h(x)$.

Example (3.8)

Let h be hesitant fuzzy set of Z_2-module $(Z_2, +_2)$ such that
$h = \{(0, h(0)), (1, h(1))\}$, $h(0) = \{0.2\ ,\ 0.5\}$, $h(1) = \{0.2\ , 0.3\ , 0.5\ , 0.9\}$ Then h is anti-hesitant fuzzy module of Z_2-module Z_2.

Proposition (3.9)

Let R be a Ring with unity and h be an hesitant fuzzy set over R−module M then h is anti-hesitant fuzzy module of M if and only if $h(rx - ry) \subseteq h(x) \cap h(y)$ for all $x, y \in M, r \in R$.

Proof :

Suppose that h is anti- hesitant fuzzy module of M
$h(rx - ry) = h(r(x - y)) \subseteq h(x - y) \subseteq h(x) \cup h(y)$
suppose that the condition $h(rx - ry) \subseteq h(x) \cup h(y)$ holds. Putting $r = 1$ we get
$h((1)\,x - (1)y) = h(\ x -\ y) \subseteq h(x) \cup h(y)$
Again, putting $y = 0$
$h(r\,x - (r)(0)) = h(rx - 0) \subseteq h(x) \cup h(0) \subseteq h(x)$
So, h is anti-hesitant fuzzy module of R−module M. □

Proposition (3.10)

Every anti - hesitant fuzzy module of R-module M. Then for any $x \in M$
(1) $h(-x) = h(x)$ (2) $h(0) \subseteq h(x)$.

Proof (1)

$h(-x) = h(0 - x)$
Since h is anti - hesitant fuzzy module of R-module M
$h(-x) \subseteq h(0) \cup h(x) = h(x)$ implies $h(x) = h(0 - (-x))$

Since h is anti - hesitant fuzzy module of R-module M
$h(x) \subseteq h(0) \cup h(-x) = h(-x)$ Thus $h(-x) = h(x)$. □

Proof (2)
$h(0) = h(x-x)$
Since h is anti - hesitant fuzzy module of R-module M
Thus $h(0) \subseteq h(\mathrm{x}) \cup h(x) = h(x)$. □

Proposition (3.11)
Let R be a Ring and h is anti - hesitant fuzzy module of R-module M if $h(x-y) = h(0)$ then $h(x) = h(y)$ for any $x, y \in M$

Proof :
$h(y) = h(x-(x-y) \subseteq h(x) \cup h(x-y)$
$h(y) \subseteq h(\mathrm{x}) \cup h(0)$ implies $h(y) \subseteq h(x)$
$h(x) = h(y-(y-x) \subseteq h(y) \cup h(y-x)$
$h(x) \subseteq h(\mathrm{y}) \cup h(0)$ implies $h(x) \subseteq h(y)$
Thus $h(x) = h(y)$. □

Theorem (3.12)
Let h be a anti - hesitant fuzzy module of R-module M, then $A = \{x \in M \mid h(x) = h(0)\}$ is submodule of M.

Proof :
Let $x, y \in A$, we get $h(x) = h(0)$, $h(y) = h(0)$
Since h is anti - hesitant fuzzy module of R-module M
$h(x-y) \subseteq h(\mathrm{x}) \cup h(y) \subseteq h(0) \cup h(0) \subseteq h(0)$implies from proposition (3.11) (2) it is obvious that $h(x-y) \supseteq h(0)$
So that $h(x-y) = h(0)$, Thus $x - y \in A$
Let $x \in A, r \in R$, we get $h(x) = h(0)$
Since $h(rx) \subseteq h(x)$, implies $h(rx) \subseteq h(0)$
From proposition (3.11) (2) it is obvious that $h(rx) \supseteq h(0)$
So that $h(rx) = h(0)$, thus $rx \in A$
Hence, $A\ is\ submodule\ of\ M$. □

Theorem (3.13)
Let h be hesitant fuzzy set of R-module M then h is A-HFM of M iff h^c is HFM of M .

proof :
suppose that h *is* *A-HFM* of M and Let $x, y \in M$, $r \in R$.
Since $h(x-y) \subseteq h(x) \cup h(y)$
Thus $1 - h^c(x-y) \subseteq \left(1 - h^c(x)\right) \cup \left(1 - h^c(y)\right)$
Implies $1 - h^c(x-y) \subseteq 1 - \left(h^c(x) \cap h^c(y)\right)$
$-h^c(x-y) \subseteq -\left(h^c(x) \cap h^c(y)\right)$
Hence $h^c(x-y) \supseteq h^c(x) \cap h^c(y)$ ……………(*)
Since $h(rx) \subseteq h(x)$
Thus $1 - h^c(rx) \subseteq 1 - h^c(x)$

$-h^c(xr) \subseteq -h^c(x)$

Hence $h^c(rx) \supseteq h^c(x)$ ……………………….(**)

From (*) , (**) we get h is HFM of M

suppose that h^c *is* *HFM* of M and Let $x, y \in M$, $r \in R$.

Since $h^c(x-y) \supseteq h^c(x) \cap h^c(y)$

Thus $1-h(x-y) \supseteq (1-h(x)) \cap (1-h(y))$

Implies $1-h(x-y) \supseteq 1-(h(x) \cup h(y))$

$-h(x-y) \supseteq -(h(x) \cup h(y))$

Hence $h(x-y) \subseteq h(x) \cup h(y)$ ……………(***)

Since $h^c(rx) \supseteq h^c(x)$

Thus $1-h(rx) \supseteq 1-h(x)$

$-h(rx) \supseteq -h(x)$

Hence $h(rx) \subseteq h(x)$ ……………(****)

From (***) , (****) we get h is A-HFM of M . □

Theorem (3.14)

Let h be a hesitant fuzzy set of $R-$ module M. Then h is anti-hesitant fuzzy module of R-module M iff $h_{E^{**}}$ is submodule of M.

proof :

Suppose that h is A-HFM of M

Thus h^c is HFM of M (by Theorem 3.13)

$h^c{}_E$ is submodule of M (by Theorem 3.4)

We get $h^c{}_E = h^c(x) \supseteq E$

$= 1-h(x) \supseteq E$

$= h(x) \subseteq 1-E$

$= h(x) \subseteq E^* = h_{E^*}$

Hence h_{E^*} is submodule of M .

Suppose that h_{E^*} is submodule of M

By a bove $h^c{}_E = h_{E^*}$

Thus h^c is HFM of M

Implies h is A-HFM of M . □

Proposition (3.15)

Let h_1 *and* h_2 be a anti-hesitant fuzzy module of R-module M, then $h_1 \cup h_2$ is anti-hesitant fuzzy module of R-module M

Proof (1) :

Since h_1 *and* h_2 are anti-hesitant fuzzy module of M

Hence $h_1{}^c$ *and* $h_2{}^c$, are HFM of M (by Theorem 3.13)

Since $h_1{}^c \cap h_2{}^c$ *is* HFM of M (by Proposition 3.3)

Since $(h_1 \cup h_2)^c = h_1{}^c \cap h_2{}^c$, (by Proposition 2.6)

Thus $(h_1 \cup h_2)$ is anti-hesitant fuzzy module of M. □

Remark (3.16)

Note that the $h_1 \cap h_2$ is not necessarily be a anti-Hesitant fuzzy module of M as in the following Example**.**

Let $M = Z$ as a $Z -$ module and $h, K : Z \rightarrow P[0,1]$ defined by

$$h_1(x) = \begin{cases} \{0, 0.4\} & if \quad x \text{ is multiple } 3 \\ \{0, 0.4, 0.8\} & \text{otherwise} \end{cases}$$

$$h_2(x) = \begin{cases} \{0, 0.4\} & if \quad x \text{ is even} \\ \{0, 0.4, 0.6\} & \text{otherwise} \end{cases}$$

It can be verified that h_1 and h_2 are anti-hesitant fuzzy module of M . Now, take x = 9 and y = 4. We see that

$h_1(9) = \{0, 0.4\}$, $h_1(4) = \{0, 0.4, 0.8\}$

$h_2(9) = \{0, 0.4, 0.6\}$, $h_2(4) = \{0, 0.4\}$

Thus $h_1(9-4) = h_1(5) = \{0, 0.4, 0.8\}$ and

$h_2(9-4) = h_2(5) = \{0, 0.4, 0.6\}$

Hence $(h_1 \cap h_2)(9) = \{0, 0.4\}$ and $(h_1 \cap h_2)(4) = \{0, 0.4\}$

Implies $(h_1 \cap h_2)(9-4) = (h_1 \cap h_2)(5) = \{0, 0.4, 0.6\}$

$(h_1 \cap h_2)(9) \cup (h_1 \cap h_2)(4) = \{0, 0.4\}$

Thus $(h_1 \cap h_2)(9-4) \nsubseteq (h_1 \cap h_2)(9) \cup (h_1 \cap h_2)(4)$

So , $h_1 \cap h_2$ *is not is* anti-hesitant fuzzy module of M

Proposition (3.17)

Let h, h_1 *and* h_2 be a anti-hesitant fuzzy module of R-module M , then

1) $(h_1 \otimes h_2), (h_1 \oplus h_2)$ are anti-hesitant fuzzy module of M
2) $h^{\lambda}, \lambda h$ are anti-hesitant fuzzy module of M

Proof (1) :

Since h_1 *and* h_2 are anti-hesitant fuzzy module of M

Thus ${h_1}^c$ *and* ${h_2}^c$, are HFM of M (by Theorem 3.13)

Hence ${h_1}^c \oplus {h_2}^c$ *and* ${h_1}^c \otimes {h_2}^c$ are HFM of M , (by Proposition 3.3)

Since $(h_1 \otimes h_2)^{\ c} = {h_1}^c \oplus {h_2}^c$ (by Proposition 2.6)

$(h_1 \oplus h_2)^{\ c} = {h_1}^c \otimes {h_2}^c$

Hence $(h_1 \otimes h_2)$ *and* $(h_1 \oplus h_2)$ are A- HFM of M. □

Proof (2) :

Since h is anti-hesitant fuzzy module of M

then h^c is HFM of M (by Theorem 3.13)

Hence $(h^c)^{\lambda}$ *and* λh^c are HFM of M , (by Proposition 3.3)

Since $(h^c)^{\lambda} = (\lambda h)^c$, $\lambda h^c = (h^{\lambda})^c$ (by Proposition 2.6)

Hence h^{λ} *and* λh are anti-hesitant fuzzy module of M . □

Definition (3.18)

Let A , B be hesitant fuzzy set of a set X and Y respectively and let $f : X \rightarrow Y$ be a mapping. Then the anti-image of A under f , denoted by $f_{-}(A)$, is hesitant fuzzy set in Y defined as follows: for each $y \in Y$

$$f_{-}(A)(y) = \begin{cases} \bigcap_{x \in f^{-1}(y)} A(x) & iF \quad f^{-1}(y) \neq \emptyset \\ \{1\} & \text{otherwise} \end{cases}$$

Theorem (3.19)

Let f be be a mapping from R-module M onto R-module N. Then we have that

1- *If A is* hesitant fuzzy set of N, then
$f^{-1}(A^C)(x) = (f^{-1}(A)(x))^C$
2- *If B is* hesitant fuzzy set of in M, then
$f(B^C)(y) = (f_{-}(B)(y))^C$
3- If K is A-HFM of M then $f_{-}(K)$ is *A-HFM* of N
4- If L is A-HFM of N then $f^{-1}(L)$ is *A-HFM* of M

Proof (1) :

$f^{-1}(A^C)(x) = A^C(f(x)) = \{1 - y \mid y \in A(f(x))\}$
$= \{1 - y \mid y \in f^{-1}(A)(x)\} = (f^{-1}(A)(x))^C$ ◻

Proof (2) :

$f(B^C)(y) = \bigcup\{B^C(x) \mid x \in f^{-1}(y)\} = \bigcup\{1 - y \mid y \in B(x)\}$

$= (\bigcap\{B(x)\})^c = (f_{-}(B)(y))^C$

Similarity $f_{-}(B^C) = (f(B))^C$ ◻

Proof (3) :

Let K is *A-HFM of M then K^C is HFM of M* (by Theorem 3.13)
Thus $f(K^C)$ *is HFM of N* (by Proposition 3.5)
By above $f(K^C) = (f_{-}(K))^C$
Implies $f_{-}(K)$ *is A-HFM of N* . ◻

Proof (4) :

Let L is *A-HFM of N then L^C is HFM of N* (by Theorem 3.13)
Thus $f^{-1}(L^C)$ *is HFM of M* (by Proposition 3.6)
By above $f^{-1}(L^C) = (f(L))^C$
Implies $f^{-1}(L)$ *is A-HFM of M.* ◻

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