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Some topologies on d - algebra

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Abstract: We will give a filter base generating a $d^{\#}$-algebra topology which is a fundamental neighborhood system of zero for that topology and find some properties of that $d^{\#}$-algebra topology and we use the left(right) stabilizers of a $d^{\#}$- algebra $(D,*,0)$ and produce basis for topology. Then we show that the generated topological space by this basis are connected, locally connected and separable. Also we study the other properties of these topological space.

Introduction

Y. Imai and K. Iseki [4] and K. Iseki [5] introduced two classes of abstract algebras: namely, BCK-algebras and BCI-algebras. It is known that the class of BCK- algebras is a proper subclass of the class of BCI-algebras. In [2], [3] Q. P. Hu and X. Li introduced a wide class of abstract algebras: BCH-algebras. They have shown that the class of BCI-algebras is a proper subclass of the class of BCH-algebras. J. Neggers and H. S. Kim [6] introduced the notion of d-algebras which is another generalization of BCK-algebras, and investigated relations between d-algebras and BCK-algebras. However, no attempts have been made to study the topological structures making the star operation of d – algebra continuous. Theories of topological groups, topological rings and topological modules are well known and still investigated by many mathematicians. Even topological universal algebraic structures have been studied by some authors.

In this paper we initiate the study of some topologies on $d^{\#}$ – algebras. In section 1 We need some preliminary materials that are necessary for the development of the paper and define a new algebra which denoted by $d^{\#}$ - algebra . In section 2 we will find a filter base generating a topology which making the star operation of $d^{\#}$ – algebra continuous. In fact such filter base is a fundamental neighborhood system of zero with respect to the topology generated by that filter base. We will study some properties of topological $d^{\#}$ - algebra which is generated by such filter base introduced in this paper. In section 3, we use the left(right) stabilizers of a $d^{\#}$- algebra $(D,*,0)$ and produce basis for topology. Then we show that the generated topological space by this basis are connected, locally connected and separable. Also we study the other properties of these topological space.

1. Preliminaries

In this section, we examine the definition of topological d-algebra and some issues and examples related to the subject.

1.1 Definition[7]: A non-empty set D together with a binary operation $*$ and a zero element 0 is said to be a d – algebra if the following axioms are satisfied for all $x, y \in D$:

1)) $x * x = 0$
2) $0 * x = 0$
3) $x * y = 0$ and $y * x = 0$ imply that $x = y$.

1.2 Definition[2]: An element e of D is called a left identity if $e* a=a$, a right identity if $a* e = a$ for all $a\in D$ and $a\neq e$. If e is both left and right identity then we called e is an identity element. Also we say that $(D,*)$ is d – algebra with identity element.

1.3 Example:

i) Let D be any non – empty set and P(D) is power set of D then (P(D),-) is d – algebra and ϕ is right identity in (P(D) ,-).

ii) let D={ 0,a,b,c} and define the binary operation $*$ on D by the following table:

$*$	0	a	b	c
0	0	0	0	0
a	0	0	b	c
b	0	b	0	a
c	0	c	a	0

Table (1)

Then the pair (D,$*$) is d – algebra with identity element a .

1.4 Definition[6]: Let (D, $*$,0) be a d-algebra and $\phi \neq I \subseteq D$. I is called a d-sub algebra of Dif $x * y \in I$ whenever $x \in I$ and $y \in I$. I is called BGK – ideal of D if :

1) $0 \in I$

2) $y \in I$ and $x * y \in I$ imply that $x \in I$.

1.5 Definition[2] : Let (D,$*$) be a d – algebra and T be a topology on D. The triple (D,$*$,T) is called a topological d – algebra (denoted by Td – algebra) if the binary operation * is continuous.

1.6 Example:

i) Let D={ 0,a,b,c} and $*$ be define by the following table:

$*$	0	a	b	c
0	0	0	0	0
a	a	0	0	a
b	b	b	0	b
c	c	c	c	0

Table (2)

It is clear that (D,$*$) is d – algebra and T={ϕ,{b},{c},{0,a},{b,c},{0,a,b},{0,a,c},D} is topology on D such that the triple (D,$*$,T) is a topological d – algebra.

ii) Let R be a set of real number and $*$ is a binary operation which define by $a * b = a.(a-b)^2$ then (R,$*$) is d – algebra and (R,$*$,T) is Td – algebra where T is usual topology on R.

Proposition 1.7: The product of any two topological d-algebra is again a topological d-algebra.

Proof: Let $(D_1, \tau_1, *_1, 0_1)$ and $(D_2 , \tau_1, *_2 , 0_2)$ be topological d-algebra, let (x_1, y_1) and (x_2 , y_2) is any two element in D, and defines:

$$\begin{cases} D = D_1 \times D_2 \\ o =: (x_1, y_1) o (x_2, y_2) = (x_1 * x_2, y_1 * y_2) \\ 0 = (0_1, 0_2) \end{cases}$$

Now suppose that U is a open neighborhood of the point $(x_1 * x_2, y_1 * y_2)$ in product topology τ, so we conclude that V_1 is an open neighborhood of $x_1 *_1 x_2$ in D_1, and V_2 is an open neighborhood of $y_1 *_2 y_2$ in D_2 , we have $V_1 \times V_2 \subseteq U$. Then we have B_1 is an open neighborhood of x_1 , and B_2 is an open neighborhood of x_2 . Also, B_3 is an open neighborhood of y_1 , and B_4 is a open neighborhood of y_2 , such that

$$B_1 * B_2 \subseteq V_1 \text{ and } B_3 * B_4 \subseteq V_2.$$

Then

$$(B_1 \times B_3)\, o\, (B_2 \times B_4) \subseteq (V_1 \times V_2) \subseteq U,$$

and we claim that in fact

$$(B_1 * B_2) \times (B_3 * B_4) = (B_1 \times B_3)\, o\, (B_2 \times B_4) \subseteq U$$

Where $B_1 \times B_3$ is a open neighborhood of (x_1, y_1)in D , and $B_2 \times B_4$ is a open neighborhood of (x_2,y_2) in D . Thus o is a continuous and (D, τ ,o ,0) is a topological d-algebra.

Proposition 1.8: Let D be a topological d – algebra. If {0} is closed (open). Then {a} is closed (open) for all $a \in D$.
Proof: Let $g : D \times D \times D \to D \times D$ be given by $g(a, b, c) = (a* b, b * c)$. Then g is continuous. If {0} is closed, then so is $\{(0,0)\} \subseteq D \times D$. Let $a \in D$, and let $h : D \to D \times D$ be given by $h(b) = g(a, b, a) = (a * b, b* a)$. Then h is the restriction of g to $\{a\} \times D \times \{a\}$ and hence is continuous. Now,
$h^{-1}\{(0,0)\} = \{b | a * b = 0 \text{ and } b * a = 0\} = \{a\}$.
Thus {a} is also closed. Similarly for the case where {0} is open.

Definition 1.9: The d-algebra is said to be a $d^{\#}$-algebra if the following axioms are satisfied for all $x, y, z \in D$.:
i) $x* 0 = x$
ii) $x* (y* z) = (x* z)* y$
iii) $[(z* x) * (z * y)] * (x * y) = 0$ and $[(x * z) * (y * z)] * (x * y) = 0$.

Example 1.10: Let D = {0,a,b,c} and ∗ operation be given by the following table:

∗	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

Table (3)

Then (D,∗ ,0) is a $d^{\#}$- algebra.

Clearly, a $d^{\#}$- algebra is a d - algebra, but the converse need not be true.

Example 1.11: let D={ 0,a,b,c} and define the binary operation ∗ on D by the following table:

∗	0	a	b	c
0	0	0	0	0
a	0	0	b	c
b	0	b	0	a
c	0	c	a	0

Table (4)

Then the pair (D,*) is d – algebra. But it is not $d^{\#}$ – algebra because $(a * c)*b = c* b = a \neq 0 = a * a = a* (b * c)$.

Proposition 1.12 : Let D be a $d^{\#}$-algebra. If $x \leq y$ then $z * x \leq z * y$ and $x * z \leq y * z$ for any $x,y,z \in D$.

Proof: Let $x,y,z \in D$. Since $x \le y$ and $[(z* x) * (z * y)] * (x * y) = 0$,then $[(z* x) * (z * y)] * 0 = 0$.Thus $(z* x) * (z * y)= 0$.Then $z * x \le z * y$. By semi way we prove that $x * z \le y * z$.

Definition 1.13: A $d^{\#}$-algebra D is said to be positive implicative if:

$$(x* y) * z = (x* z) *(y * z) \quad \text{for all } x,y,z \in D.$$

The example (1.10) is a positive implicative $d^{\#}$- algebra.

Definition 1.14: A non-empty subset I of a $d^{\#}$ – algebra D is called a BCK-ideal of D if:

(1) $o \in I$

(2) $x \in I$ and $y * x \in I$ imply $y \in I$. for all $x,y \in D$.

2. Some Neighborhood System and related results

In this section we will give a filter base on D generating a topology on D where D is $d^{\#}$ – algebra.. For arbitrary $a \in X$ and any subset $V \subseteq D$, V (a) will be defined as $V(a) = \{x \in D : x * a \in V \text{ and } a * x \in V\}$. With this, we can get the following theorem .

Theorem 2.1: Let D be a $d^{\#}$-algebra If Ω is a filter base on D satisfying:

(1) For every $v \in V \in \Omega$ there exists $U \in \Omega$ such that $v \in U(v) \subseteq V$.

(2) If $p, q \in V \in \Omega$ and $(x* p)* q = 0$ then $x \in V$ for all $x \in D$.

Then there is a topology on D for which is a fundamental system of neighborhoods of zero and V(a) is open set for every $V \in \Omega$, $a \in D$.

Proof: Let $\tau_\Omega = \{O \subseteq D$: For every $a \in O$, there exists $V \in \Omega$ such that $V(a) \subseteq O\}$. At first we can prove that τ is a topology on D. Clearly D and $\emptyset$ belong to τ. Let $\{O_\alpha\}$ be a family of sets in τ. Then for every $a \in \cup O_\alpha$, $a \in O_\alpha$ for some α. Thus there exists V such that $V(a) \subseteq O_\alpha$, thus $\cup O_\alpha \in \tau$. Assume that O_1 and O_2 belong to τ. Let $a \in O_1 \cap O_2$. Then there exist V_1 and V_2 such that $V_1(a) \subseteq O_1$ and $V_2(a) \subseteq O_2$ respectively. Since is a filter base, there exists V such that $V \subseteq V_1 \cap V_2$. Then we have $V(a) \subseteq (V_1 \cap V_2)(a) \subseteq V_1(a) \cap V_2(a) \subseteq O_1 \cap O_2$ and so $O_1 \cap O_2 \in \tau$.

Next we will show that Ω is the filter base of a neighborhood of 0 with respect to the topology τ . Let $p \in V \in \Omega$. Since $(0 * p)* q =0$ for any $p,q \in V$, then $0 \in V$ by (ii), i.e., every element $V \in \Omega$ contains 0. Let $p \in V$ and if $x \in V(p)$ then $x * p, p * x \in V$ and so $x * p = v$ for some $v \in V$. Hence $(x* p)* v = 0$ which implies that $x * V$. Therefore $V(p) \subseteq V$ and $V \in \tau$. Thus V is a neighborhood of 0. If we let V be a neighborhood of 0, then there exists a $U \in \Omega$ such that $U(0) \subseteq V$. Note that $0 * a \in U$ and $a * 0 \in U$ for some $a \in U$. Hence $a \in U(0)$ and $0 \in U \subseteq U(0) \subseteq V$. Thus Ω is a fundamental system of neighborhoods of zero with respect to the topology τ .

Now we will prove that V (a) is open. Let $x \in V(a)$. We know that $x * a \in V$ and $a * x \in V$. Then by (1) of properties of Ω there exits U_1 and U_2 respectively such that $x * a \in U_1 \subseteq V$ and $a * x \in U_2 \subseteq V$. Choose $W \in \Omega$ such that $W \subseteq U_1 \cap U_2$. If $y \in W(x)$ then $x * y \in W$ and $y * x \in W$. Since

$$[(a * y) * (a * x)] * (y * x) = 0$$

and

$$[(y * a) * (x * a)] * (y * x) = 0$$

we know that $(y * a)$ and $(a * y)$ are contained in V by the property (2) of this filter base. Hence and V (a) is open.

Definition 2.2: The topology who owns the filter base Ω is denoted by τ_Ω .

Example 2.3: If D is a $d^{\#}$ – algebra The filter base of BCK-ideals of D is a filter base satisfying conditions of Theorem (2.1). Since for every $x \in I$, $I(x) \subseteq I$ where I is a BCK- ideal of X, we know that condition (1) is satisfied and if $p, q \in I$ then $(x * p)* q = 0 \in I$ implies $x \in I$. Thus (2) in Theorem (2.1) is satisfied.

Proposition 2.4: The triple $(D,*, \tau_\Omega)$ is a topological d – algebra.

Proof: Let x and y be any elements of D. Since every open set containing $x* y$ contains $V(x* y)$ for some $V \in \Omega$. It is sufficient to show that$(V(x))* (V(y)) \subseteq V(x* y)$. Let $u * v \in V(x))* (V(y)$ then $u \in V(x)$ and $v \in V(y)$.Thus $u* x, x* u, v* y, y* v \in V$

$$[(x* y)*(u* v)] * (u* x)$$
$$= (x* y)* [(u* x)* (u * v)$$
$$\le (x* y)* (x* v) \le y * v$$

Then $[((x* y)*(u* v)) * (u* x)]* y * v = 0$. By (2) in Theorem (2.1) $(x* y)*(u* v)\in V$ and by similar method we know that $(u* v)*(x* y)\in V$. Thus $u* v$ is contained in $V(x* y)$ which shows that $V(x) * V(y)\subseteq V(x* y)$. Then $*$ is continuous.

For any subset A of D, we main that $V(A) = \cup_{a\in A}V(a)$ and it is Cleary an open neighborhood of A. We can get the following theorem.

Theorem 2.5: For any subset A of D , $\bar{A} = \cap\{V(A) : V \in\Omega\}$ where $\bar{A}$ is the closure of A.
Proof: Let $b \in \bar{A}$ and $V \in \Omega$. Since V (b) is a neighborhood of b, then $V(b) \cap A \neq \phi$ Thus there exists $a \in A$ such that $a * b \in V$ and $b * a \in V$. Hence $b \in V(a)$ and $b \in \cap\{V(A) : V \in\Omega\}$.
Conversely if $b \in \cap\{V(A) : V \in\Omega\}$then for any $W \in \Omega$, $b \in W(A)$. Thus $W(b) \cap A \neq \phi$.

The following theorem shows that every neighborhood of a compact set contains a neighborhood W(A) for some $W \in \Omega$.

Theorem 2.6: Let A be a compact subset of D, where D is a $d^{\#}$-algebra. If U be a neighborhood of A. Then there exists $V \in \Omega$ such that $A \subseteq V(A) \subseteq U$.
Proof: Since U is a neighborhood of A. By Theorem(2.1) for every $a \in A$ there exists $V_a \in \Omega$ such that $V_a(a) \subseteq U$. Since $A \subseteq \cup_{a\in A} V_a(a)$, there exist $a_1, a_2, \dots a_n$ such that $A \subseteq V_{a_1}(a_1) \cup V_{a_2}(a_2) \cup \dots \cup V_{a_n}(a_n)$
Let $V = \cap V_{a_i}$. It is sufficient to show that $V(a) \subseteq U$ for every $a \in A$. Since $a \in V_{a_i}(a_i)$ for some a_i , $a * a_i \in V_{a_i}$ and $a_i * a \in V_{a_i}$. If $y \in V(a)$ then $a * y \in V$, $y * a \in V$, and since

$$[(y * a_i) * (a * a_i)] * (y * a) = 0$$

Thus $(y * a_i) \in V_{a_i}$ By Theorem (2.1) and similarly

$$[(a_i * y) * (a_i * a)] * (y * a) = 0$$

Hence $(y * a_i) \in V_{a_i}$. It shows that $y \in V_{a_i}(a_i) \subseteq U$ and $V(a) \subseteq U$. Thus $V(A) \subseteq U$.

Theorem 2.7: Let K be compact subset of D and F be closed subset of D where D is a $d^{\#}$-algebra . If $K \cap F = \phi$ then there exists $V \in \Omega$ such that $V(K) \cap V(F) = \phi$.
Proof: Since $D \setminus F$ is a neighborhood of K, there exists $V \in \Omega$ such that $V(K) \subseteq D \setminus F$ by Theorem (2.6). suppose that for every $V\in \Omega$. Then $V(K) \cap V(F) \neq \phi$. Then there exist y is contained in $V(K) \cap V(F)$. $y \in V(k)$ for some $k \in K$ and $y \in V(f)$ for some $f \in F$. Since

$$[(k * f) * (k * y)] * (f * y) = 0$$

and

$$[(f * k) * (y * k)] * (f * y) = 0$$

we know that $k * f \in V$. Similarly $f * k \in V$ by Theorem (2.1). Thus $f \in V(k)$. But it is contradiction on the fact that $V(K) \subseteq D \setminus F$.

3. Closure operator on $d^{\#}$-algebras

We will define A topology on $d^{\#}$ - algebras via left and right stabilizers and some property of this topology.

Definition 3.1: Let D be a $d^{\#}$-algebra and A be a nonempty subset of D. Then the sets $A^{*}{}_l = \{x\in D \mid a* x = a, \forall a \in A\}$
and

$$A^{*}{}_r = \{x\in D \mid x* a = x, \forall a \in A\}$$

are called the left and right stabilizers of A, respectively.

Theorem 3.2: Let A be a nonempty subset of a positive implicative $d^{\#}$-algebra D. Then:
(i) $A^{*}{}_l$ is an ideal of D.
(ii) $A^{*}{}_r$ is a subalgebra of D.
Proof:
i) Let $x \in D$ and $y \in A^{*}{}_l$ such that $x* y\in A^{*}{}_l$ then $a* y = a$ and $a*(x* y) = a$ [By Definition (3.1)]. Since $a = a*(x* y) = (a* y)*x = a* x$. Thus $x\in A^{*}{}_l$.
ii) Let $x , y \in A^{*}{}_r$ then $x* a = x$ and $y * a = y$ Since $(x* y) * a= (x * a)* (y * a)= x *y$ Thus $x* y\in A^{*}{}_r$.

Definition 3.3[9]: Consider A as a nonempty set, a mapping $\varphi : P(A) \to P(A)$ is called a closure operator on A, if for all $X,Y \in P(A)$ the following holds:

(1) $X \subseteq \varphi(X)$,
(2) $\varphi^2(X)= \varphi(X)$,
(3) $X \subseteq Y$ implies $\varphi(X) \subseteq \varphi(Y)$.

Notion 3.4: Note that all definitions and notations on a given topological space (X,τ) are stated from [1] .

Theorem 3.5: Let A and B be two nonempty subsets of D. Then:
(i) $0 \in A^*_l \cap A^*_r$,
(ii) $A \subseteq (A^*_l)^*_r \cap (A^*_r)^*_l$,
(iii) If $A \subseteq B$, then $B^*_l \subseteq A^*_l$ and $B^*_r \subseteq A^*_r$,
(iv) $A^*_l = ((A^*_l)^*_r)^*_l$ and $A^*_r = ((A^*_r)^*_l)^*_r$,
(v) $(\bigcup_{j\in J} A_j)^*_l = \bigcap_{j\in J}((A_j)^*_l)$.

∗	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

Proof: (i) Since $0 * x = 0$ and $x * 0 = x$, for all $x \in D$, then $0 \in A^*_r \cap A^*_l$

(ii) Let $a \in A$. Then $x * a = x$, $\forall x \in A^*_r$ and $a * y = a$, $\forall y \in A^*_l$. So $a \in (A^*_r)^*_l \cap (A^*_l)^*_r$.

(iii) Let $x \in B^*_l$. Then $b * x = b$, $\forall b \in B$. Since $A \subseteq B$ and $b * x = b$, $\forall b \in A$. So $x \in A^*_l$. Similarly $B^*_r \subseteq A^*_r$.

(iv) By (ii) we get that $A^*_l \subseteq ((A^*_l)^*_r)^*_l)$ and $A^*_r \subseteq ((A^*_r)^*_l)^*_r)$. Also by (ii) and (iii) we have $((A^*_r)^*_l)^*_r) \subseteq A^*_r$ and $((A^*_l)^*_r)^*_l) \subseteq A^*_l$. Therefore $((A^*_r)^*_l)^*_r) = A^*_r$ and $((A^*_l)^*_r)^*_l) = A^*_l$.

(v)Let $x \in (\bigcup_{j\in J} A_j)^*_j \Leftrightarrow a * x = a$, $\forall a \in \bigcup A_j$ and $j \in J \Leftrightarrow a * x = a$, $\forall a \in A_j \Leftrightarrow x \in A^*_l, \forall j \in J \Leftrightarrow \bigcap_{j\in J}((A_j)^*_l)$.

Notion 3.6: we define $\emptyset^*_l = \emptyset$ and $\emptyset^*_r = \emptyset$.

Theorem 3.7: The function $\alpha : P(D) \to P(D)$, where $\alpha(A)= (A^*_l)^*_r$ is a closure operator on D.

Proof: By Theorem (3.5,ii), $A \subseteq \alpha(A)$, for all $A \in P(D)$. Also by Theorem (3.5,iv), $\alpha(A)= (A^*_l)^*_r = (((A^*_l)^*_r)^*_l)^*_r = \alpha^2(A)$, for all $A \in P(D)$.

Let $A \subseteq B$. Then by Theorem (3.5,iii), $\alpha(A) \subseteq \alpha(B)$. Therefore α is a closure operator on X.

Theorem 3.8: Consider the function α given in Theorem 3.7. Then we can obtain that $\beta_\alpha =\{A \in P(D)|\alpha(A)=A\}$ is a basis for a topology on D.
Proof: It is easy to see that $D^*_l=\{0\}$ and also $\{0\}^*_r= D$. Then $\alpha(D)=D$ and so $D \in \beta_\alpha$.. Let $x \in A \cap B$, for $A,B \in \beta_\alpha$. Since α is a closure operator, then we can obtain that $\alpha(A\cap B)=A\cap B$, i.e. $A\cap B \in \beta_\alpha$ containing x. Therefore β_α is a basis topology on D.

Remark 3.9: From Theorem 3.2 elements of β_α are subalgebras of D.

Definition 3.10: The topology who owns the basis β_α is denoted by τ_α .

Example 3.11: Let D = {0,a,b,c} and ∗ operation be given by the following table:

Table(5)

Then $(D,*,0)$ is a $d^{\#}$-algebra. We see that $0 \in \alpha(A)$, for all nonempty sub sets A of D, so if $0 \in A \subseteq D$, we have $A \in \beta_\alpha$. By some manipulations we get that $\beta_\alpha=\{\emptyset, D, \{0\}, \{0,a\}, \{0,b\}, \{0,c\}, \{0,a,b\}, \{0,a,c\}, \{0,b,c\}\}$. Thus $\tau_\alpha=\{\emptyset, D, \{0\}, \{0,a\}, \{0,b\}, \{0,c\}, \{0,a,b\}, \{0,a,c\}, \{0,b,c\}\}$.

Proposition 3.12: (D,τ_α) is a Hausdorff space if and only if $D=\{0\}$.
Proof: Suppose that $D \neq \{0\}$, Since for any $U \in \tau_\alpha$, we have $0 \in U$, then $U \cap V \neq \emptyset$ for any two arbitrary elements U,V of τ_α. Thus (D,τ_α) is not Hausdorff. Conversely, let $D = \{0\}$. Then $\tau_\alpha = \{\emptyset, D\}$. Thus it is clear that (D,τ_α) is a Hausdorff space.

Proposition 3.13: (D, τ_α) is connected.
Proof: Since $0 \in U$, for any nonempty open set of D, then there are not nonempty open subsets U and V of D such that $U \cap V = \phi$. Thus (D,τ_α) is connected space.

Corollary 3.14: Let U be a nonempty open subset of (X,τ_α) . Then U is a connected set of D.
Proof: It is similar to the proof of Proposition (3.13).

Corollary 3.15: Let U be a nonempty disconnected subset of (D, τ_α). Then $0 \in U$.
Proof: It is clear.

Corollary 3.16: Let $A \neq D$ and $A \neq \emptyset$ be a closed subset of (D, τ_α) . Then A is a connected set of D.
Proof: Since $A \neq D$ is a closed set of (D, τ_α), then $\emptyset \neq D - A$ is an open set of (D,τ_α). By Theorem (3.5,i) we have $0 \in D-A$, therefore $0 \notin A$. Thus by Corollary 3.15, we get that A is a connected set of D.

Remark 3.17:All proper subsets of (D,τ_α) are connected, whenever they are closed or open.

Theorem 3.18: Let A be a subset of topological space (D, τ_α) and $0 \in A$. Then $\overline{A} = D$.
Proof: Since $0 \in U$, for any nonempty open subset of D, then $U \cap A \neq \phi$, for any open set containing x. Therefore $\overline{A} = D$

By the above theorem we can get that the following corollary.

Corollary 3.19: Let U be a nonempty open subset of D. Then $\overline{U} = D$.
Proof: Since U is a non – empty open set of D , then $0 \in \overline{U} = D$(By Theorem 3.18).

Corollary 3.20: Let A be a non-empty subset of the topological space (D,τ_α). Then $0 \in \overline{A}$ if and only if $\overline{A} = D$.
Proof: Since $0 \in \overline{A}$. Then by Theorem (3.18) $\overline{\overline{A}} = D$. Thus $\overline{A} = D$. The proof of the converse is clear.

Lemma 3.21: $\{0\}$is an open subset of the topological space (D,τ_α).
Proof: By Definition 3.10 we can get that $\{0\}_l^* = D$ and $D_r^* = \{0\}$, then $\{0\} \in \beta_\alpha$. Thus $\{0\}$ is an open set of the topological space (D,τ_α).

Theorem 3.22: (D,τ_α) is separable.
Proof: By Corollary 3.20 and Lemma 3.21 we get that $\overline{\{0\}} = D$. Then (D,τ_α) is separable.

Theorem 3.23. $(D,\tau_\alpha)$is locally connected.
Proof: Let x be an arbitrary element of D and U be an open set containing x. By Theorem 3.13, we get that U is connected and also containing x. Therefore (D, τ_α) is locally connected.

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الخلاصة:

في هذا البحث قدمنا قاعد ة راشحيه لتوليد توبولوجيا يجعل العملية الثنائية في جبر- د مستمرة وهذه القاعدة تكون نظام جوارات أساسي لصفر و لهذا التوبولوجيا. ودرسنا بعض الخصائص لذلك التوبولوجيا واستخدمنا المثبتات اليسارية (اليمينية) لجبر – د$^{\#}$ (D، *، 0) لإنتاج أساسا للتبوبولوجيا. ثم وضحنا أن الفضاء التوبولوجي المولّدة من هذا الأساس متصلة ،متصل محليًا وقابلة للانفصال. كما ندرس الخصائص الأخرى لهذا الفضاء التوبولوجي.

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