On σ-Convergent Triple Sequences Spaces Defined by Triple Sequence of Young Functions

Authors Names	ABSTRACT
Aqeel Mohammed Hussein ***Publication data***: 31 /8 /2023 ***Keywords***: Triple sequence space , σ-convergent , young function , linear space , ideal sequence algebra	In this article, we introduce the concept of σ-convergent triple sequences spaces, which are defined by triple sequences of young functions $(c_0)^{3\sigma}(\mho), (c)^{3\sigma}(\mho), (\ell_\infty)^{3\sigma}(\mho)$, $(\mathbb{m}_0)^{3\sigma}(\mho)$, and $(\mathbb{m})^{3\sigma}(\mho)$ and we examine some of their topological and algebraic properties, such as linear space and sequence algebra. Inclusion relations involving these sequence spaces are also proved by us.

1. Introducation

Fast [4] and Schoenberg [11] independently each presented the idea of statistical convergence for the first time. This idea was expanded upon by Kumar [6] in probabilistic normed space. The σ-convergence double sequence spaces were first introduced by Khan and Khan ([7],[8]). Tripathy [13] established the concept of statistical convergent double sequence and developed it in σ-convergent double sequence.

The double sequence space connected to multiplier sequences was researched by Tripathy and Sen [14]. A generalization of statistical convergence is the idea of σ-convergence. Kostyrko, Salat, and Wilczynski [5] established the concept of σ-convergence of real sequence in its earliest stage. Later, Salat, Tripathy, and Ziman [12] and other other scholars investigated it. Utilizing triple difference sequences of real numbers, Tripathy and Goswami [15] expanded this idea in probabilistic normed space. Sahiner, Gurdal, and Duden [10] introduced and studied the many conceptions of triple sequences in the beginning.

This idea is generalized by Dutta, Esi, and Tripathy [1] using the Orlicz function. In their study of σ-related features in triple sequence spaces, Sahiner and Tripathy [9] produced several intriguing findings. Recently, Tripathy and Goswami ([16],[17]) explored multiple sequences in probabilistic normed spaces and vector valued multiple sequences using the Orlicz function, respectively. By utilizing the difference operator, Debnath, Sharma, and Das [3] and Debnath and Das [2] generalized these ideas.

2. Definitions and Preliminaries

$\Omega : [0,\infty) \to [0,\infty)$ is a continuous, non-decreasing , and convex with $\Omega(0) = 0, \Omega(\mathfrak{A}) \succ 0$ as $\mathfrak{A} \succ 0$ and $\Omega(\mathfrak{A}) \to \infty$ as $\mathfrak{A} \to \infty$ implies Ω is an Orlicz function .

$\mathcal{H}: [0,\infty) \to [0,\infty) \ni \mathcal{H}(\mathfrak{A}) = \frac{\Omega(\mathfrak{A})}{\mathfrak{A}}$, $\mathfrak{A} \succ 0$ and $\mathcal{H}(0) = 0, \mathcal{H}(\mathfrak{A}) \succ 0$ as $\mathfrak{A} \succ 0$ and $\mathcal{H}(\mathfrak{A}) \to 0$ as $\mathfrak{A} \to \infty$ tend to $\mathcal{H}$ is a young function .

The conditions are holds :

(i) $\phi \in \sigma$.

(ii) $\mathbb{A}, \mathbb{B} \in \sigma$ implies $\mathbb{A} \cup \mathbb{B} \in \sigma$.

(iii) $\mathbb{A} \in \sigma$, $\mathbb{B} \subset \mathbb{A}$ implies $\mathbb{B} \in \sigma$ lead to $\sigma \subset \mathbb{X} \neq \phi$ is an ideal in $\mathbb{X}$.

University of Al-Qadisiyah , Faculty of Education, Department of Mathematics, Iraq. E-mail: aqeel.Hussein@qu.edu.iq

$\forall\ \varepsilon \succ 0, \{(\mathfrak{h}, \mathfrak{g}, \mathfrak{f}) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : |\mathfrak{S}_{\mathfrak{hgf}} - \mathbb{L}| \succcurlyeq \varepsilon\} \in \sigma$ pointing to a sequence $(\mathfrak{S}_{\mathfrak{hgf}})$ is σ-Convergence to a number $\mathbb{L}$.

$\forall\ \varepsilon \succ 0, \exists\ \mathfrak{e} = \mathfrak{e}_0, \mathfrak{d} = \mathfrak{d}_0, \mathfrak{c} = \mathfrak{c}_0 \ni \{(\mathfrak{h}, \mathfrak{g}, \mathfrak{f}) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : |\mathfrak{S}_{\mathfrak{hgf}} - \mathfrak{S}_{\mathfrak{edc}}| \succcurlyeq \varepsilon\} \in \sigma$ implies $(\mathfrak{S}_{\mathfrak{hgf}})$ is σ-Cauchy. $\exists\ \mathcal{M} \succ 0 \ni \{(\mathfrak{h}, \mathfrak{g}, \mathfrak{f}) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : |\mathfrak{S}_{\mathfrak{hgf}}| \succ \mathcal{M}\} \in \sigma$ lead to $(\mathfrak{S}_{\mathfrak{hgf}})$ is σ-Bounded.

$(\mathfrak{S}_{\mathfrak{hgf}} * \mathfrak{E}_{\mathfrak{hgf}}) \in \mathbb{E}^3$, whenever $(\mathfrak{S}_{\mathfrak{hgf}}) \in \mathbb{E}^3$ and $(\mathfrak{E}_{\mathfrak{hgf}}) \in \mathbb{E}^3$ tend to a triple sequences space $\mathbb{E}^3$ is a sequence algebra. We provide and define these spaces as follows in this study:

$(c_0)^{3\sigma}(\mho) = \{\mathfrak{S} \in \mathbb{W}^3 : \sigma - \lim \mathcal{H}_{\mathfrak{hgf}}(|\mathfrak{S}_{\mathfrak{hgf}}|) = 0\} \in \sigma.$

$(c)^{3\sigma}(\mho) = \{\mathfrak{S} \in \mathbb{W}^3 : \sigma - \lim \mathcal{H}_{\mathfrak{hgf}}(|\mathfrak{S}_{\mathfrak{hgf}} - \mathbb{L}|) = 0, \text{for some } \mathbb{L}\} \in \sigma.$

$(\ell_\infty)^{3\sigma}(\mho) = \{\mathfrak{S} \in \mathbb{W}^3 : \sup_{\mathfrak{h},\mathfrak{g},\mathfrak{f} \in \mathbb{N}} \mathcal{H}_{\mathfrak{hgf}}(|\mathfrak{S}_{\mathfrak{hgf}}|) \prec \infty\} \in \sigma.$

$(\mathbb{m}_0)^{3\sigma}(\mho) = (c_0)^{3\sigma}(\mho) \cap (\ell_\infty)^{3\sigma}(\mho).$

$(\mathbb{m})^{3\sigma}(\mho) = (c)^{3\sigma}(\mho) \cap (\ell_\infty)^{3\sigma}(\mho).$

3. Main Results

Theorem 3.1: $(c_0)^{3\sigma}(\mho), (c)^{3\sigma}(\mho), (\ell_\infty)^{3\sigma}(\mho), (\mathbb{m}_0)^{3\sigma}(\mho)$, and $(\mathbb{m})^{3\sigma}(\mho)$ are linear spaces.

Proof: Assume that $(\mathfrak{S}_{\mathfrak{hgf}}), (\mathfrak{E}_{\mathfrak{hgf}}) \in (c)^{3\sigma}(\mho)$ and α, β be two scalars such that $|\alpha| \preccurlyeq 1$ and $|\beta| \preccurlyeq 1$. Then

$$\sigma - \lim \mathcal{H}_{\mathfrak{hgf}}(|\mathfrak{S}_{\mathfrak{hgf}} - \mathbb{L}_1|) = 0, \text{for some } \mathbb{L}_1 \in \mathbb{C}$$

$$\sigma - \lim \mathcal{H}_{\mathfrak{hgf}}(|\mathfrak{E}_{\mathfrak{hgf}} - \mathbb{L}_2|) = 0, \text{for some } \mathbb{L}_2 \in \mathbb{C}$$

Now, $\forall\ \varepsilon \succ 0$, we can write

$$\mathbb{A}_1 = \left\{(\mathfrak{h}, \mathfrak{g}, \mathfrak{f}) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \mathcal{H}_{\mathfrak{hgf}}(|\mathfrak{S}_{\mathfrak{hgf}} - \mathbb{L}_1|) \succ \frac{\varepsilon}{2}\right\} \in \sigma. \quad (1)$$

$$\mathbb{A}_2 = \left\{(\mathfrak{h}, \mathfrak{g}, \mathfrak{f}) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \mathcal{H}_{\mathfrak{hgf}}(|\mathfrak{E}_{\mathfrak{hgf}} - \mathbb{L}_2|) \succ \frac{\varepsilon}{2}\right\} \in \sigma. \quad (2)$$

Since $\mathcal{H}_{\mathfrak{hgf}}$ is a young function, we get

$$\mathcal{H}_{\mathfrak{hgf}}(|(\alpha\mathfrak{S}_{\mathfrak{hgf}} + \beta\mathfrak{E}_{\mathfrak{hgf}}) - (\alpha\mathbb{L}_1 + \beta\mathbb{L}_2)|) = \mathcal{H}_{\mathfrak{hgf}}(|(\alpha\mathfrak{S}_{\mathfrak{hgf}} - \alpha\mathbb{L}_1) + (\beta\mathfrak{E}_{\mathfrak{hgf}} - \beta\mathbb{L}_2)|)$$
$$\preccurlyeq \mathcal{H}_{\mathfrak{hgf}}(|\alpha||\mathfrak{S}_{\mathfrak{hgf}} - \mathbb{L}_1|) + \mathcal{H}_{\mathfrak{hgf}}(|\beta||\mathfrak{E}_{\mathfrak{hgf}} - \mathbb{L}_2|) = |\alpha|\mathcal{H}_{\mathfrak{hgf}}(|\mathfrak{S}_{\mathfrak{hgf}} - \mathbb{L}_1|) + |\beta|\mathcal{H}_{\mathfrak{hgf}}(|\mathfrak{E}_{\mathfrak{hgf}} - \mathbb{L}_2|)$$
$$\preccurlyeq \mathcal{H}_{\mathfrak{hgf}}(|\mathfrak{S}_{\mathfrak{hgf}} - \mathbb{L}_1|) + \mathcal{H}_{\mathfrak{hgf}}(|\mathfrak{E}_{\mathfrak{hgf}} - \mathbb{L}_2|).$$

From (1) and (2), we obtain

$\{(\mathfrak{h}, \mathfrak{g}, \mathfrak{f}) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \mathcal{H}_{\mathfrak{hgf}}(|(\alpha\mathfrak{S}_{\mathfrak{hgf}} + \beta\mathfrak{E}_{\mathfrak{hgf}}) - (\alpha\mathbb{L}_1 + \beta\mathbb{L}_2)|) \succ \varepsilon\} \subset \mathbb{A}_1 \cup \mathbb{A}_2.$

Therefore $\alpha\mathfrak{S}_{\mathfrak{hgf}} + \beta\mathfrak{E}_{\mathfrak{hgf}} \in (c)^{3\sigma}(\mho)$. Thus, $(c)^{3\sigma}(\mho)$ is a linear space.

Other cases are similar.

Theorem 3.2: A sequence $\mathfrak{S} = (\mathfrak{S}_{\mathfrak{hgf}}) \in (\mathbb{m})^{3\sigma}(\mho)$ is σ-convergence $\Leftrightarrow \forall\varepsilon \succ 0, \exists \mathbb{I}_\varepsilon, \mathbb{J}_\varepsilon, \mathbb{K}_\varepsilon \in \mathbb{N} \ni \{(\mathfrak{h}, \mathfrak{g}, \mathfrak{f}) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \mathcal{H}_{\mathfrak{hgf}}|\mathfrak{S}_{\mathfrak{hgf}} - \mathfrak{S}_{\mathbb{I}_\varepsilon\mathbb{J}_\varepsilon\mathbb{K}_\varepsilon}| \preccurlyeq \varepsilon\} \in (\mathbb{m})^{3\sigma}(\mho).$

Proof: Let $\mathbb{L} = \sigma - \lim\mathfrak{S}$. Then we have

$\mathbb{A}_\varepsilon = \left\{(\mathfrak{h}, \mathfrak{g}, \mathfrak{f}) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \mathcal{H}_{\mathfrak{hgf}}|\mathfrak{S}_{\mathfrak{hgf}} - \mathbb{L}| \preccurlyeq \frac{\varepsilon}{2}\right\} \in (\mathbb{m})^{3\sigma}(\mho), \forall\ \varepsilon \succ 0.$

Next fix $\mathbb{I}_\varepsilon, \mathbb{J}_\varepsilon, \mathbb{K}_\varepsilon \in \mathbb{A}_\varepsilon$ then we have

$$|\mathfrak{S}_{\mathfrak{hgf}} - \mathfrak{S}_{\mathbb{I}_\varepsilon\mathbb{J}_\varepsilon\mathbb{K}_\varepsilon}| \preccurlyeq |\mathfrak{S}_{\mathfrak{hgf}} - \mathbb{L}| + |\mathbb{L} - \mathfrak{S}_{\mathbb{I}_\varepsilon\mathbb{J}_\varepsilon\mathbb{K}_\varepsilon}| \preccurlyeq \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon, \forall \mathfrak{h}, \mathfrak{g}, \mathfrak{f} \in \mathbb{A}_\varepsilon.$$

Thus, $\{(\mathfrak{h}, \mathfrak{g}, \mathfrak{f}) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \mathcal{H}_{\mathfrak{hgf}}|\mathfrak{S}_{\mathfrak{hgf}} - \mathfrak{S}_{\mathbb{I}_\varepsilon\mathbb{J}_\varepsilon\mathbb{K}_\varepsilon}| \preccurlyeq \varepsilon\} \in (\mathbb{m})^{3\sigma}(\mho).$

Conversely, suppose that $\{(\mathfrak{h}, \mathfrak{g}, \mathfrak{f}) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \mathcal{H}_{\mathfrak{hgf}}|\mathfrak{S}_{\mathfrak{hgf}} - \mathfrak{S}_{\mathbb{I}_\varepsilon\mathbb{J}_\varepsilon\mathbb{K}_\varepsilon}| \preccurlyeq \varepsilon\} \in (\mathbb{m})^{3\sigma}(\mho)$,we get

$\{(\mathfrak{h},\mathfrak{g},\mathfrak{f}) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \mathcal{H}_{\mathfrak{hgf}}|\mathfrak{S}_{\mathfrak{hgf}} - \mathfrak{S}_{\mathbb{I}_\varepsilon\mathbb{J}_\varepsilon\mathbb{K}_\varepsilon}| \preccurlyeq \varepsilon\} \in (\mathfrak{mm})^{3\sigma}(\mho), \forall\, \varepsilon \succ 0$, then we can find the set $\mathbb{B}_\varepsilon = \{(\mathfrak{h},\mathfrak{g},\mathfrak{f}) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \mathfrak{S}_{\mathfrak{hgf}} \in [\mathfrak{S}_{\mathbb{I}_\varepsilon\mathbb{J}_\varepsilon\mathbb{K}_\varepsilon} - \varepsilon, \mathfrak{S}_{\mathbb{I}_\varepsilon\mathbb{J}_\varepsilon\mathbb{K}_\varepsilon} + \varepsilon]\} \in (\mathfrak{mm})^{3\sigma}(\mho)$.
$\forall\, \varepsilon \succ 0$, consider $\mathcal{N}_\varepsilon = [\mathfrak{S}_{\mathbb{I}_\varepsilon\mathbb{J}_\varepsilon\mathbb{K}_\varepsilon} - \varepsilon, \mathfrak{S}_{\mathbb{I}_\varepsilon\mathbb{J}_\varepsilon\mathbb{K}_\varepsilon} + \varepsilon]$.
Now we have $\mathbb{B}_\varepsilon \in (\mathfrak{mm})^{3\sigma}(\mho)$ as well as $\mathbb{B}_{\frac{\varepsilon}{2}} \in (\mathfrak{mm})^{3\sigma}(\mho)$, hence $\mathbb{B}_\varepsilon \cap \mathbb{B}_{\frac{\varepsilon}{2}} \in (\mathfrak{mm})^{3\sigma}(\mho)$ which implies $\mathbb{B}_\varepsilon \cap \mathbb{B}_{\frac{\varepsilon}{2}} \neq \phi$. Then $\{(\mathfrak{h},\mathfrak{g},\mathfrak{f}) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \mathfrak{S}_{\mathfrak{hgf}} \in \mathbb{N}\} \in (\mathfrak{mm})^{3\sigma}(\mho)$ which implicates diam $\mathbb{M} \preccurlyeq$ diam $\mathbb{M}_\varepsilon$ where the length of the interval $\mathbb{N}$ is indicated by the diam of $\mathbb{M}$.
Thus, using the principle of induction, we discovered the series of closed intervals

$$\mathbb{M}_\varepsilon = \sigma_0 \supseteq \sigma_1 \supseteq \sigma_2 \supseteq \cdots \supseteq \sigma_{\mathbb{s}} \supseteq \cdots$$

with the help of the property that diam $\sigma_{\mathbb{s}} \preccurlyeq \frac{1}{2}$ diam $\sigma_{\mathbb{s}-1}$, $\forall\, \mathbb{s} = 1,2,3,4, \ldots.$ and $\{(\mathfrak{h},\mathfrak{g},\mathfrak{f}) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \mathfrak{S}_{\mathfrak{hgf}} \in \sigma_{\mathfrak{hgf}}\} \in (\mathfrak{mm})^{3\sigma}(\mho)$, $\forall\, \mathfrak{h},\mathfrak{g},\mathfrak{f} = 1,2,3,4, \ldots.$
Then $\exists\, \xi \in \cap\, \sigma_{\mathbb{s}}$ where $\mathbb{s} \in \mathbb{N} \ni \xi = \sigma - \lim \mathfrak{S}$, so that $\mathcal{H}_{\mathfrak{hgf}}(\xi) = \sigma - \lim\mathcal{H}_{\mathfrak{hgf}}(\mathfrak{S})$ therefore $\mathbb{L} = \sigma - \lim\mathcal{H}_{\mathfrak{hgf}}(\mathfrak{S})$.

Theorem 3.3: $(c_0)^{3\sigma}(\mho) \subset (c)^{3\sigma}(\mho) \subset (\ell_\infty)^{3\sigma}(\mho)$.
Proof: We consider $(\mathfrak{S}_{\mathfrak{hgf}}) \in (c)^{3\sigma}(\mho)$. Then $\exists\, \mathbb{L} \in \mathbb{C} \ni \sigma - \lim\mathcal{H}_{\mathfrak{hgf}}(|\mathfrak{S}_{\mathfrak{hgf}} - \mathbb{L}|) = 0$,
we get $\mathcal{H}_{\mathfrak{hgf}}(|\mathfrak{S}_{\mathfrak{hgf}}|) \preccurlyeq \mathcal{H}_{\mathfrak{hgf}}(|\mathfrak{S}_{\mathfrak{hgf}} - \mathbb{L}|) + \mathcal{H}_{\mathfrak{hgf}}(|\mathbb{L}|)$.
On taking supremum over $\mathfrak{h}, \mathfrak{g}$, and $\mathfrak{f}$ on both sides gives $(\mathfrak{S}_{\mathfrak{hgf}}) \in (\ell_\infty)^{3\sigma}(\mho)$.
Therefore $(c)^{3\sigma}(\mho) \subset (\ell_\infty)^{3\sigma}(\mho)$. The direction $(c_0)^{3\sigma}(\mho) \subset (c)^{3\sigma}(\mho)$.
Thus, $(c_0)^{3\sigma}(\mho) \subset (c)^{3\sigma}(\mho) \subset (\ell_\infty)^{3\sigma}(\mho)$.

Theorem 3.4: $(c_0)^{3\sigma}(\mho), (c)^{3\sigma}(\mho), (\ell_\infty)^{3\sigma}(\mho), (\mathfrak{mm}_0)^{3\sigma}(\mho)$, and $(\mathfrak{mm})^{3\sigma}(\mho)$ are sequence algebras.
Proof: Let $(\mathfrak{S}_{\mathfrak{hgf}}), (\mathfrak{E}_{\mathfrak{hgf}}) \in (c_0)^{3\sigma}(\mho)$, then we have $\sigma - \lim\mathcal{H}_{\mathfrak{hgf}}(|\mathfrak{S}_{\mathfrak{hgf}}|) = 0$ and $\sigma - \lim\mathcal{H}_{\mathfrak{hgf}}(|\mathfrak{E}_{\mathfrak{hgf}}|) = 0$.
Now we obtain $\sigma - \lim\mathcal{H}_{\mathfrak{hgf}}(|\mathfrak{S}_{\mathfrak{hgf}}.\mathfrak{E}_{\mathfrak{hgf}}|) = 0$. It implies that $(\mathfrak{S}_{\mathfrak{hgf}}.\mathfrak{E}_{\mathfrak{hgf}}) \in (c_0)^{3\sigma}(\mho)$
Thus, $(c_0)^{3\sigma}(\mho)$ is a sequence algebra.
Other cases are similar.

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